The Kaon B-parameter from Two-Flavour Dynamical Domain Wall Fermions.

C. Dawson \(^{a}\) [RBC Collaboration] \(^{\ast}\)

\(^{a}\)RIKEN-BNL Research Center, Bldg 510a, Upton, NY 11973-5000

We report on the calculation of the kaon B-parameter using two dynamical flavours of domain wall fermions. Our analysis is based on three ensembles of configurations, each consisting of about 5,000 HMC trajectories, with a lattice spacing of approximately 1.7 GeV for \(16^3 \times 32\) lattices; dynamical quark masses range from approximately the strange quark mass to half of that. Both degenerate and non-degenerate quark masses are used for the kaons.

1. INTRODUCTION

The kaon B-parameter, \(B_K\), is one of the simplest quantities that may be calculated on the lattice which may be used to directly constrain the unitarity triangle. However, its calculation on the lattice is greatly complicated when either flavour or chiral symmetry is broken by the lattice action used. When using the domain wall fermion lattice action, flavour symmetry is exact, and the explicit breaking of chiral symmetry is small enough to be neglected. Because of this it is natural to use domain wall fermions for the calculation of \(B_K\), and, indeed, this has been done to great success in the quenched approximation \[^{[1]}\]. Here we report on the calculation of \(B_K\) as part of the first large-scale dynamical (two-flavour) domain wall fermion simulations. As well as studying the effects of un-quenching, we report on the effects of including non-degenerate masses \((m_s \neq m_u = m_d)\) in both the lattice calculation and the extrapolation to the physical point.

2. DYNAMICAL ENSEMBLES

The results of this work are based on three ensembles of two-flavour dynamical domain wall fermion configurations. Each ensemble utilises a lattice of size \(16^3 \times 32\), a domain wall height of 1.8, a fifth dimension of extent 12, and consists of 94 configurations with the separation of 50 HMC trajectories between each configuration. The bare quark masses of these ensembles are 0.02, 0.03 and 0.04 respectively, the latter of which roughly corresponds to the strange quark mass. Details of lattice generation, and further physical results can be found in \[^{[2,3,4]}\]; here we will touch on only a couple quantities of relevance to the extraction of the kaon B-parameter: the residual mass and the lattice spacing. All quantities are quoted with a jackknife error (although, where relevant, the average correlation matrix was used).

The residual mass, \(a m_{\text{res}}\), is the additive shift in the mass due to the residual chiral symmetry breaking of domain wall fermions. We calculate this from the breaking term in the ward identity \[^{[5]}\], extracting our final value of \(a m_{\text{res}} = 0.001372(49)\) by linearly extrapolating to zero dynamical mass. The lattice spacing we use is fixed by comparing a dynamical extrapolation of the vector meson to the physical point to the experimental mass of the rho meson. This gives value of \(a^{-1} = 1.691(53)\)GeV. It is interesting to note, however, that should we fix the lattice spacing from the Sommer parameter, the result is consistent \[^{[6]}\].

3. \(B_K\)

In the continuum, \(B_K\) is defined as

\[
B_K = \frac{\langle \bar{K}^0 | O_{LL} | K^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}.
\]  

(1)

\[
= \frac{\langle K^0 | O_{LL} | K^0 \rangle}{\frac{8}{3} \langle K^0 | A_4 | 0 \rangle \langle 0 | A_4 | K^0 \rangle}.
\]  

(2)
where $A_4$ is the time component of the axial current, and
\[ O_{LL} = \bar{s}\gamma_\mu(1 - \gamma_5)d\bar{s}\gamma_\mu(1 - \gamma_5)d. \] (3)
When using a lattice action which breaks chiral symmetry, however, $O_{LL}$ may mix under renormalisation with four wrong chirality operators, which we denote $O_{\text{mix},i}$ for $i = 1, \cdots, 4$. Leading order chiral perturbation theory predicts that
\[ \langle K^0|O_{LL}|K^0\rangle \propto M_K^2 \] (4)
and, unfortunately, that
\[ \langle K^0|O_{\text{mix},i}|K^0\rangle \propto 1. \] (5)
This means that as the chiral limit is approached, these wrong chirality operators will dominate the calculation. However, for practical lattice calculations we do not work in the chiral limit and, in fact, we argue that for this calculation the mixing with such operators may be entirely neglected. This may be seen by studying the pattern of chiral symmetry breaking using the spurion field technique introduced in [7]; a simple application of this method leads to the conclusion that the wrong chirality operators are suppressed by a factor of $O((am_{\text{res}})^2) \approx 10^{-6}$. A numerical study, in the quenched approximation, to bound the size of the contributions of such operators has also been made. As reported on elsewhere [1], these effects were, indeed, found to be negligible.

To calculate $B_K$ both the strange quark mass, $m_s$, and the average of the up and down quark masses, $\overline{m}$, must be known. We extract these by fitting the quark mass dependence of the pseudo-scalar and vector mesons to the predictions of NLO partially-quenched chiral perturbation theory (pq-$\chi$pt) and a linear ansatz, respectively. Requiring that the pseudo-scalar and vector mesons have masses equal to the experimental value for the pion and rho in the limit
\[ m_{\text{dyn}} = m_{\text{valence}} = \overline{m}, \] (6)
and the pseudo-scalar meson mass be equal to the kaon mass in the limit
\[ m_{\text{dyn}} = m_{\text{valence},1} = \overline{m} ; \ m_{\text{valence},2} = m_s, \] (7)
gives a set of equations that we iteratively solve to find $a\overline{m} = 0.00017(11)$, $am_s = 0.0446(29)$ and the lattice spacing previously quoted. (Note these are bare quark masses; the renormalised quark mass is defined as $Z_m(m + m_{\text{res}})$, where $Z_m$ is a scheme and scale dependent renormalisation factor.) Figure 1 shows the square of the pseudo-scalar meson versus (degenerate) valence quark mass, together with the results of a fit to NLO partially-quenched chiral perturbation theory.

Figure 2. Bare $B_K$ versus timeslice for the $m_{\text{dyn}} = 0.04$ ensemble.

the lattice spacing previously quoted. (Note these are bare quark masses; the renormalised quark mass is defined as $Z_m(m + m_{\text{res}})$, where $Z_m$ is a scheme and scale dependent renormalisation factor.) Figure 1 shows the square of the pseudo-scalar meson versus (degenerate) valence quark mass, together with the results of the fit to NLO pq-$\chi$pt. For the calculation of $B_K$ using only degenerate valence quarks, we will also make use of the strange quark mass fixed by comparing the pseudo-scalar mass to the kaon mass in the limit
\[ m_{\text{dyn}} = \overline{m} ; \ m_{\text{valence}} = m_s/2, \] (8)
which leads to a value which is $\approx 10\%$ lower than the previous one.

We extract the bare value of $B_K$ from the ratio given in Eq. 2 using (Coulomb) gauge-fixed wall-sources for the kaons at time-slices 4 and 28, varying the position of the operator. As in $\overline{\text{MS}}$, a combination of propagators with periodic and anti-periodic boundary conditions are used to avoid contamination due to the periodicity of the lattice. Figure 2 shows this ratio for the the $m_{\text{dyn}} = 0.04$ ensemble for various degenerate valence quark masses. We take our final values from an error-weighted average of the points between time-slices 14 and 17, but see little change in our answer if this range is varied.

To extrapolate/interpolate to the physical point we again use the results of a fit to pq-$\chi_{\text{pt}}$, restricting the bare quark masses use to be $\leq 0.04$ in an attempt to stay in the region of validity of NLO pq-$\chi_{\text{pt}}$. Also, we do not include the $m_{\text{valence}} = 0.01$ data in our analysis due to the bad quality of the plateau. Figure 3 shows the data for degenerate valence quark masses together with the NLO pq-$\chi_{\text{pt}}$ fit. Completely taking the limit given in Eq. 8 leads to a final result, when considering only degenerate valence quark masses, of

$$B_{K}^{\text{deg}} = 0.509(18)$$

while including non-degenerate valence quark masses, and taking the limit given in Eq. 7 gives

$$B_K = 0.495(18).$$

While these value differ by less than the quoted error, as these errors are correlated, this 3% effect is statistically well resolved. We quote both of these numbers in the $\overline{\text{MS}}$ scheme at 2GeV; the needed combination of renormalisation factors to convert from the bare value of $B_K$, $Z_{B_K} = Z_{LL}/Z_A^2$, is calculated using the NPR technique of the Rome-Southampton group.

4. CONCLUSIONS

As part of the first large-scale project using dynamical domain wall fermions, we have calculated the kaon B-parameter in two-flavour QCD. We present not only a value using degenerate valence quark masses, of $B_K = 0.509(18)$ ($\overline{\text{MS}}, 2\text{GeV}$), but also a value using non-degenerate masses ($m_s \neq m_u = m_d$), of $B_K = 0.495(18)$ ($\overline{\text{MS}}, 2\text{GeV}$). While this difference is within the individual statistical errors, due to correlations between the quantities, we find this difference to be statistically well resolved. These values are significantly below those found in the quenched approximation. We caution that this work is based at a single lattice spacing, on a single volume, and uses dynamical quark masses which are large compared to the physical up and down quark masses together with a quenched strange quark.

REFERENCES

1. RBC, J. Noaki, these proceedings.
2. RBC, Y. Aoki, forthcoming.
3. RBC, T. Izubuchi, these proceedings.
6. RBC, K. Hashimoto, these proceedings.