Angular distribution and azimuthal asymmetry for pentaquark production in proton-proton collisions

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Angular distributions for production of the Θ+ pentaquark are calculated for the collisions of polarized protons with polarized target protons. We compare calculations based on different assumptions concerning spin and parity ($J = 1/2^\pm, 3/2^\pm$) of the Θ+ state. For a wide class of interactions the spin correlation parameters describing the asymmetric angular distributions are calculated up to 250 MeV above production threshold. The deviations from the near threshold behavior are investigated.

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I. INTRODUCTION

Recently a number of experiments [1, 2, 3, 4, 5, 6, 7] have confirmed the existence of a narrow pentaquark state Θ+ with a mass of about 1.53 GeV. It decays into the $pK^0$ or the $nK^+$ channel with a width of less than about 15 MeV. Its positive strangeness $S = +1$ and the small width give good reason to identify this resonance with the pentaquark state predicted in ref. [8] within the chiral soliton model. According to this approach it belongs to a $J^\pi = 1/2^+$ antidecuplet as an isospin singlet with the five quark configuration $uudd\bar{s}$. However, the spin-parity assignment is experimentally still not verified. First measurements are done to observe this resonance in proton-proton collision $pp \rightarrow p\Sigma^+K^0\Theta^+$ at near threshold energy where the invariant masses of the $pK^0$ pair indicate the existence of the Θ+ resonance.

Several investigations are made to find observables to determine spin and parity. In photo-production processes the cross sections were studied using $K_\pi$ and $K_\pi^*$ exchange Lagrangians. For nucleon-nucleon reactions the cross section [11] and the azimuthal angular distribution [14, 15] were analyzed. Especially in the near threshold region the azimuthal angular distribution is very sensitive to distinguish between different parity assignments of the Θ+ particle. The great advantage of using the threshold region lies in the fact that to a large extent model independent predictions can be made [11, 16, 17, 18].

In this work we are going to extend the study of proton-proton reactions to larger energies using a model dependent interaction. We employ a combination of $K^\mp$ and $K^\pm$ exchange, where the parameters are constraint by assuming a production cross section as was found in the experiment [9]. The azimuthal angular distribution is parametrized by correlation coefficients, which we calculate assuming four different spin-parity states $1/2^\pm, 3/2^\pm$ of the Θ+ resonance. It is the aim to investigate how the threshold behavior of the azimuthal angular distribution changes if the energy is increased above threshold.

II. THE MODEL

The simplest mechanism than can be used to describe the reaction $pp \rightarrow \Sigma^+\Theta^+$ is the exchange of a pseudoscalar kaon given by the interaction Lagrangian

\begin{align}
\mathcal{L}_{KN\Theta} &= ig_{KN\Theta} \bar{\Theta}^\gamma^5(pK^0 + nK^+) + h.c., \\
\mathcal{L}_{KN\Sigma} &= i\sqrt{2}g_{KN\Sigma} \bar{\Sigma}^\gamma^5(pK^0 + nK^+) + h.c.
\end{align}

for the $1/2^+$ state of the Θ+ particle. The symbols $\Theta^+, \Sigma^+, p, n$ stand for the spinors of the participating Fermions and $K^\pm$ for the bosonic wave functions. The factor $\sqrt{2}$ in Eq. (2) comes from the isospin factor in the standard representation [19]. The coupling constant $g_{KN\Theta}$ is related to the decay width of the Θ+ into to the $K^+$ and $K^0$ channels via

\begin{equation}
\Gamma_\Theta = g_{KN\Theta}^2 \frac{p_n(p_n^0 - m_N)}{2\pi m_\Theta},
\end{equation}
TABLE I: Coupling strengths and cut-off parameters assuming two different decay widths \( \Gamma_{\Phi} \). The last two columns give the positive and negative limits of the coupling constants for the \( K^* \) exchange in Eqs. (11,12) which do not increase the cross section by more than a factor of two.

<table>
<thead>
<tr>
<th>( J^\pi )</th>
<th>( \Gamma_{\Phi} = 1 \text{ MeV} )</th>
<th>( \Gamma_{\Phi} = 10 \text{ MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(f) )</td>
<td>( \Lambda(\text{GeV}) )</td>
<td>( g^<em>(f^</em>) )</td>
</tr>
<tr>
<td>( g(f) )</td>
<td>( \Lambda(\text{GeV}) )</td>
<td>( g^<em>(f^</em>) )</td>
</tr>
<tr>
<td>( 1/2^+ )</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>( 3/2^+ )</td>
<td>0.46</td>
<td>0.90</td>
</tr>
<tr>
<td>( 3/2^- )</td>
<td>3.44</td>
<td>0.69</td>
</tr>
</tbody>
</table>

where \( p_N \) denotes the momentum of the emitted proton with \( p_N^0 \) being the energy as the zeroth component. Assuming a width of 10 MeV and a mass of \( \Theta^+ \) of \( m_{\Theta} = 1.53 \text{ GeV} \) we obtain \( g_{K_N\Theta} = 3.27 \). (Another possible approach for the determination of \( g_{K_N\Theta} \) uses SU(3) relations of the pentaquark multiplets \[21\]. Assuming that the \( N(1710) \) baryon resonance is an ideal mixture of antidecuplet and octet pentaquark states, they obtain \( g_{K_N\Theta} = 3.0 \) \[21\].) The coupling constant \( g_{K_N\Sigma} \) was estimated within the framework of SU(3) \[11, 19\] to be \( g_{K_N\Sigma} = -3.78 \). (The actual value depends somewhat on the data the SU(3) parameters are adjusted to.) Furthermore we use a monopol formfactor

\[
F(q^2) = \frac{\Lambda^2 - m_K^2}{\Lambda^2 - q^2} \tag{4}
\]

with \( q \) being the square of the transferred four-momentum.

In the following we also consider the possibilities that spin and parity of the \( \Theta^+ \) could take the values \( J^\pi = 1/2^-, \ 3/2^+, \) or \( 3/2^- \). The corresponding Lagrangians in their simplest form read

\[
\mathcal{L}_{KN\Theta}^{'(1)} = g_{KN\Theta} \overline{\Theta^+} (pK^0 + nK^+) + h.c., \tag{5}
\]

\[
\mathcal{L}_{KN\Theta}^{'(2)} = \frac{f_{KN\Theta}}{m_K} \overline{\Theta^+} \left( p\partial^\mu K^0 + n\partial^\mu K^+ \right) + h.c., \tag{6}
\]

\[
\mathcal{L}_{KN\Theta}^{'(3)} = \frac{f_{KN\Theta}}{m_K} \overline{\Theta^+} \gamma^5 \left( p\partial^\mu K^0 + n\partial^\mu K^+ \right) + h.c., \tag{7}
\]

where \( \Theta^+ \) is the Rarita-Schwinger representation of a spin 3/2 state. The coupling constant \( g', f, f' \) are related to the width via

\[
\Gamma_{\Phi} = g_{KN\Theta}^2 \frac{p_N(p_N^0 + m_N)}{2\pi m_{\Theta}}, \tag{8}
\]

\[
= f_{KN\Theta}^2 \frac{p_N^2(p_N^0 + m_N)}{6\pi m_K^2 m_{\Theta}}, \tag{9}
\]

\[
= f_{KN\Theta}^{'2} \frac{p_N^2(p_N^0 - m_N)}{6\pi m_K^2 m_{\Theta}}, \tag{10}
\]

leading to the values given in Table I.

Also other processes can contribute to the production. As an example we include the \( K^* \) exchange in addition to the \( K \) exchange. For the positive parity state of a spin 1/2 baryon \( B \) (\( \Sigma \) or \( \Theta^+ \)) we use

\[
\mathcal{L}_{1/2^+} = g_{K^*NB} \overline{B} (\gamma^\mu + \frac{\kappa}{m_{\Theta} + m_N} \sigma_{\nu\rho} \partial^\rho) K_{\mu}^{0*} p + h.c. \tag{11}
\]

and for the \( 3/2^+ \) state

\[
\mathcal{L}_{3/2^+} = i \frac{f_{K^*NB}}{m_K} \overline{B} (\gamma^\mu \gamma^5 \partial^\nu p) (\partial_\mu K_{\nu}^{0*} - \partial_\nu K_{\mu}^{0*}) + h.c. \tag{12}
\]

For the negative parity states we insert the factor \( i\gamma^5 \) into Eq. (11) and remove this factor from Eq. (12). The coupling constants for the \( \Sigma N \) coupling in Eq. (11) \( g_{K^*\Sigma}^* = -3.25\sqrt{2} \) and \( \kappa = 1.8 \) are chosen in accordance with \[11, 19\]. For the \( \Theta^+ \) particle the coupling constants will be fixed later.
Accordingly to the diagrams shown in Fig. 1 we calculate the T matrix from which the differential cross sections is calculated in the center-of-mass system

\[
\frac{d\sigma}{d\Omega} = \frac{p^2}{64\pi^2 s p_1} \frac{1}{4} \sum_{\text{spins}} T^*_{s_1, s_2, s_\Sigma, s_\Theta} (1 + P_1 \sigma_1) (1 + P_2 \sigma_2) T_{s_1, s_2, s_\Sigma, s_\Theta} \tag{13}
\]

as a function of the incoming momentum \(p_1\), the outgoing momentum \(p_2\) and the center-of-mass energy \(\sqrt{s}\). The polarization of the incoming (target) proton is described by the vector \(P_{1(2)}\), and \(\sigma_{1(2)}\) is the Pauli matrix which acts on the first (second) spin index, respectively.

For convenience we choose our coordinate system such that the z-axis coincides with the direction of the incoming proton. The y-axis is chosen orthogonal to the reaction plane in the direction of the normal vector \(\vec{p}_1 \times \vec{p}_\Theta\) and the x-axis points to the side direction of \(n \times \vec{p}_1\). From general considerations \[22, 23\] one can show that the differential cross section can only be a function of the following combinations of the polarization vectors

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_0 \left( 1 + A_{y0} P_{1y} + A_{0y} P_{2y} + A_{yy} P_{1y} P_{2y} + A_{xz} P_{1x} P_{2x} + A_{xz} P_{1z} P_{2z} \right) \tag{14}
\]

Here the symbols \(P_x, P_y, P_z\) stand for the components of the polarization vectors \(P_{1(2)}\). The first factor in Eq. (14) is the differential cross section for unpolarized protons. The coefficients \(A\) depend on the polar angle \(\Theta\).

Splitting the polarization vectors into components parallel and perpendicular with respect to the beam direction we rewrite Eq. (14) as a function of the azimuth angle \(\phi\) as

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_0 \left( 1 + A_{y0} P_{1\perp} \sin(\phi - \alpha_1) + A_{0y} P_{2\perp} \sin(\phi - \alpha_2) + A_{yy} P_{1\perp} P_{2\perp} \sin(\phi - \alpha_1) \sin(\phi - \alpha_2) + A_{xz} P_{1\perp} P_{2\perp} \cos(\phi - \alpha_1) \cos(\phi - \alpha_2) + A_{zz} P_{1z} P_{2z} + A_{xz} P_{1\perp} P_{2\perp} \cos(\phi - \alpha_1) + A_{xz} P_{1\perp} P_{2\perp} \cos(\phi - \alpha_2) \right) \tag{15}
\]

Here \(\alpha_1, \alpha_2\) denote the angles of the projections of the polarization vectors onto the x-y plane. The coefficients \(A_{ij}\) are usually called spin correlation parameters which depend on the azimuth angle \(\phi\) only up to second order in \(\cos(\phi)\) or \(\sin(\phi)\). Since the initial state is symmetric with respect to the z-axis the parameters \(A_{xx}, A_{yy}, A_{zz}\) are forward-backward symmetric while the others obey the relations

\[
A_{xx}(\Theta) = -A_{xx}(\pi - \Theta) \tag{16}
\]

\[
A_{yy}(\Theta) = -A_{yy}(\pi - \Theta) \tag{17}
\]

similar to those for \(pp \rightarrow pp\pi\) collisions \[23\]. In our calculations using lowest order perturbation theory the coefficient \(A_{y0}\) vanishes. Thus, we do not obtain azimuthal asymmetry if the proton in the beam or in the target is unpolarized.
FIG. 2: Calculated angular distributions in the center-of-mass system for unpolarized beam and target for different spin-parity assignments of the $\Theta^+$ states at an energy of 0.13 GeV above threshold.

III. THRESHOLD BEHAVIOR

The threshold behavior has widely been discussed in refs. [14, 16]. We summarize the consequences for the correlation parameters. The produced particles can only be in a state with relative orbital momentum $L = 0$ since the higher partial waves are suppressed by the centrifugal barrier. The angular distribution is isotropic implying $A_{xx} = 0$. Furthermore, parity conservation and Pauli principle imply that the total spin of the incoming protons is $S = 0$ for positive parity of $\Theta^+$, and $S = 1$ for negative parity.

If we set the polarization vectors $\mathbf{P}_1 = \mathbf{P}_2$ and average over the direction of the polarization vectors we obtain from Eq. (15) the averaged integrated cross section

$$\langle \sigma \rangle = \sigma_0 \left( 1 + \frac{1}{3} P^2 (A_{xx} + A_{yy} + A_{zz}) \right).$$

The same procedure can be applied to the polarization operator in Eq. (13) leading to

$$\langle \sigma \rangle \sim 1 + \frac{1}{3} P^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2).$$

Comparing these two last equations one arrives at the relation $A_{xx} + A_{yy} + A_{zz} = -3(1)$ for $S = 0(1)$, i.e. for positive (negative) parity of $\Theta^+$. 
Asymmetry

\[ \Theta^+, J^\pi = 1/2^+ \]

\[ \Theta^+, J^\pi = 3/2^+ \]

\[ A_{xx} = A_{yy} = A_{zz} = -1 \]

\[ A_{xx} = A_{yy} = A_{zz} = 1 \]

FIG. 3: Calculated correlation parameters as a function of the polar angle \( \Theta \) for different assumptions on the spin-parity assignments of the \( \Theta^+ \) particle at an excess energy of 0.13 GeV.

This causes the relation \( A_{xx} = A_{yy} = A_{zz} = -1 \) in the former case, while in the latter only the relations \( A_{zz} = 1 - A_{yy} - A_{xx} \) and \( A_{yy} = A_{xx} \) can be derived in a model independent way since in this case different spin projections contribute to the \( T \) matrix. For negative parities one obtains \( A_{xx} = A_{yy} = A_{zz} = \frac{1}{3} \) if the kaon exchange of Eqs. (5, 7) is used. But in general the coefficients depend on the parameters of the interaction used.

IV. RESULTS

A. \( K \) exchange

Here we consider only the \( K \) exchange and calculate the differential cross section for the various possibilities of the \( J^\pi \) assignments. We also investigate the effect of the width of the \( \Theta^+ \) assuming values of 10 MeV and 1 MeV which are related to corresponding values of the coupling constants given in Eqs. (3, 8-10). As a constraint we vary the cut-off parameter \( \Lambda \) such that we obtain a cross section of about 0.4 \( \mu b \) at an excess energy of \( \Delta E = \sqrt{s} - \sqrt{s_{\text{thr}}} = 0.13 \) GeV above threshold. This value corresponds to the result obtained in a recent COSY measurement. The values fulfilling this requirement are given in Table I.

In Fig. 2 we present the angular distribution \( \frac{d\sigma}{d\Omega} \) for unpolarized protons in the forward region. The cut-off parameter influences the shape of the angular distribution. A strong cut-off formfactor (small \( \Lambda \) value) leads to a
FIG. 4: Spin correlation parameters at 90 degrees in the center-of-mass system as a function of the excess energy \( \sqrt{s} - \sqrt{s_{th}} \) for different assumptions on the spin-parity assignments of the \( \Theta^+ \) particle.

rather pronounced maximum in forward and backward direction.

In Fig. 3 we show the correlation parameters \( A_{ij} \) in Eq. (15). These parameters are independent of the used formfactor. Most of them reach their maximum value at 90°. The side-side correlation parameter \( A_{xx} \) and the longitudinal correlation parameter \( A_{zz} \) coincide and \( A_{xz} = -A_{zx} \) holds. Comparing the calculations with the opposite parities one recognizes that the transverse correlations \( A_{yy} \) and \( A_{xx} \) correlate their signs in coincidence with the assumed parity of the \( \Theta^+ \) state. Measuring these coefficients could give a unique signal for the determination of the parity as was already found in refs. [14, 15, 16]. A large positive normal-long correlation parameter \( A_{zx} \) and a negative normal-normal correlation could signalize the \( J^\pi = \frac{3}{2}^+ \).

The energy dependence of the correlation parameters is shown in Fig. 4. It is seen that the characteristics of the threshold extend up to 50 MeV a value that has also been estimated in ref. [14].

B. \( K^* \) exchange

Now we investigate the effect of additionally including the \( K^* \) exchange into the interaction. We treat the coupling constants in the Lagrangians (11,12) as free parameters. To reduce the parameter space we choose the tensor coupling \( \kappa = 0 \). This is not completely unrealistic, see [13]. Furthermore we relate the cut-off parameter to that of the \( K^- \) exchange via \( \Lambda_{K^*} = \Lambda_K + m_{K^*} - m_K \). Then we vary the values of the parameters \( g^* \) and \( f^* \) such that the cross
FIG. 5: Spin correlation parameters at 90 degrees as a function of the excess energy for a combination of $K$ and $K^*$ exchange. The thick (thin) lines present calculations with positive (negative) coupling strengths of the $K^*$ exchange given in Table I.

section of 0.4 $\mu$b is increased to a maximum value of 0.8 $\mu$b. We made calculations with both possibilities of the signs of the coupling constants $f^*$ or $g^*$ leading to constructive or destructive interference between $K$ and $K^*$ exchange. The last two columns in Table I give the two possible values for the coupling coefficients.

In Fig. 5 we show the energy dependence of the spin correlation parameters $A_{ii}$. Comparing Figs. 4 and 5 one recognizes that the correlation parameters depend stronger on energy if the $K^*$ exchange has been included. The coefficients $A_{xx}$ and $A_{zz}$ do not coincide anymore. In particular $A_{xx}$ and $A_{yy}$ of the $1/2^+$ state already change their signs at an excess energy of 100 MeV. A more drastic change of these coefficients is seen for the $3/2^+$ state. This seems to contradict the estimates made in ref. [14]. The reason for this effect lies in the strong destructive interference between the interaction Lagrangians which reduces the cross section much stronger than one expects from the $\sqrt{\Delta E}$ threshold behavior. The constructive interference (thin lines, $f^* = -0.9$) does not show this strong energy dependence and agrees well with the estimates of the behavior of the threshold region given in ref. [14].

The behavior of the correlation coefficients for the negative states depends sensitively on the interaction used as can be seen by comparing Figs. 4 and 5. To identify the parity needs therefore the measurement of all three coefficients $A_{ii}$. 
V. CONCLUSION

We have analyzed the asymmetry of the angular distribution for $\Theta^+$ pentaquark production in collisions of polarized protons. Using a variety of different interactions it was found that the characteristic threshold signals survive at energies up to 50 MeV above threshold. Thus such measurements are a useful tool to determine spin and parity of the $\Theta^+$ particle. In rare cases a more rapid change of the correlation parameters has been found which is accompanied with a stronger energy dependence than the expected $\sqrt{\Delta E}$ behavior of the cross section.

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