Novel Azimuthal Asymmetries in Drell Yan and Semi-inclusive Deep Inelastic Scattering

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Abstract. We consider the leading and sub-leading twist $T$-odd and even contributions to the $\cos 2\phi$ azimuthal asymmetry in unpolarized dilepton production in Drell-Yan Scattering. We estimate the contributions’ effects at 500 GeV, 50 GeV, and 25 GeV energies in the framework of the parton model using a quark diquark-spectator model of the nucleon to approximate the soft contributions.

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INTRODUCTION

One of the most interesting results in spin physics has been the discovery of a class of chirally odd quark distribution functions. Considerable attention has been focused on the transversity or covariant transverse spin distribution $h_1$ which provides information on the quark transverse spin distribution in a transversely polarized nucleon [1, 2]. Chiral odd distribution functions require a quark helicity flip and are difficult to probe in inclusive deep inelastic scattering due to the helicity conserving property of quantum chromodynamics (QCD) interactions. However, when two hadrons participate in the scattering process, the nucleon’s transversity can be accessed; for example, the double transverse spin asymmetry in Drell-Yan scattering [1, 3]. Alternatively, transversity can be probed in semi-inclusive deep inelastic scattering (SIDIS) where outgoing hadrons are produced in the current fragmentation region [4, 5]. In this case the cross sections and distribution functions are sensitive to the transverse momentum of the quarks. Sensitivity to transverse momentum dependence in parton distribution functions leads to a class of leading twist spin dependent effects which are $T$-odd [6, 7, 8, 9]. The distribution functions exist by virtue of non-zero parton transverse momenta and would vanish at tree-level in any $T$-conserving model of hadrons and quarks. In this sense they are similar to the decay amplitudes for hadrons that involve single spin asymmetries which are non-zero due to final (and/or initial) state strong interactions (FSI) [4].

Such a $T$-odd distribution was proposed by Sivers in the context of Drell-Yan scattering [6]. How such a distribution could actually arise without violating conservation laws remained an open question for some years. Recently, however, a mechanism was introduced that could produce that Sivers effect without violating invariance principles. A reaction mechanism in terms of FSI (and using the quark-diquark model of the nucleon) was used to calculate this effect by Brodsky, Hwang and Schmidt [10] (BHS). That calculation was subsequently recast in terms of a color gauge invariant treatment of
transverse momentum dependent distribution functions \[11, 12, 13\]. Gluon loop corrections to tree-level calculations for the transversity of quarks and hadrons emerge from the gauge link insertion into the formal definitions of the transversity distributions. At first order in the strong coupling \(\alpha_s\) the correction involves the single gluon approximation to FSI, a single loop diagram.

These FSI corrections, implemented in parton model inspired calculations, give rise to novel single spin and azimuthal asymmetries in SIDIS \[10, 9, 14, 15, 16, 17, 18, 19\] and Drell-Yan processes \[20, 21\] resulting in significant asymmetries. This is all the more interesting in light of the fact that in the perturbative QCD regime of quark and gluon dynamics such asymmetries are expected to be small; that is, of the order, \(\alpha_s m_q/\sqrt{s}\) \[22, 23\]. One \(T\)-odd transversity distribution function (introduced by Boer and Mulders \[8\]) is \(h_1^\perp\), which depends on variables \(x, Q^2\) and \(p_\perp\) of the struck quark; it measures the amount of quark transversity from an unpolarized nucleon. At moderate energies such \(T\)-odd effects arising from the non-perturbative \(h_1^\perp\) may be the source of non-trivial \(\cos 2\phi\) asymmetries in SIDIS \[24, 14, 16\] and in Drell-Yan scattering \[25, 20, 21\].

In this letter we will report our results on SIDIS \[14, 15, 16\] and present new results on the Drell-Yan process \[26\]. The latter process is interesting in light of the possibility of accessing this transversity property of quarks in the proposed proton anti-proton experiments at Darmstat GSI \[27\], where an anti-proton beam is an ideal tool for studying transversity due to the dominance of valence quark effects. We demonstrate at proposed energies that both the \(T\)-odd and sub-leading twist \(T\)-even distributions in \(\bar{p} p \rightarrow \ell \ell^+ X\) provide contributions to the \(\cos 2\phi\) asymmetry that are significant. We estimate that the sub-leading twist contribution is a non-trivial fraction of the large leading twist \(T\)-odd contribution using the parton model.

**THE AZIMUTHAL ASYMMETRIES IN UNPOLARIZED SIDIS AND DRELL YAN SCATTERING**

The angular asymmetries that can arise in *unpolarized* Drell-Yan scattering (e.g. \(\bar{p} + p \rightarrow \mu^+ \mu^- + X\)) and SIDIS (\(e + p \rightarrow e'hX\)) are obtained from the differential cross section expressions:

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

and

\[
\frac{d\sigma}{dxdydzdP_{h\perp}^2d\phi_h} = A + B + C \cos \phi + D \cos 2\phi,
\]

respectively \[28, 29, 30\]. In the Drell-Yan process the angles refer to the lepton pair orientation in their rest frame relative to the boost direction and the initial hadron’s plane, the Collins Soper frame \[28\]. The dependence on the other independent variables, \(s, x, m_{h\perp}^2, q_T\), is suppressed, but the asymmetry parameters, \(\lambda, \mu, \nu\), depend on those variables. In SIDIS, the azimuthal angle refers to the relative angle between the hadron production plane and the lepton scattering plane. \(A, B, C, D\) are functions of \(x, y, z, Q^2, |P_{h\perp}|\). It is especially interesting that the \(\cos 2\phi\) azimuthal asymmetry in Drell...
Yan depends on the $T$-odd distribution $h^+_{1(2)}$ and its anti-quark distribution, $\bar{h}^+_{1(2)}$; whereas in SIDIS one essentially replaces the anti-quark distribution with the Collins function, $H^+_{T}$ \cite{4}.

**cos2$\phi$ Azimuthal Asymmetry in Drell Yan**

To leading order in $Q^2$, i.e. leading twist, the asymmetry $\nu$ is given by \cite{20}

$$v_2 = \frac{\sum_a e_a^2 \mathcal{F} \left[ (2\mathbf{h} \cdot \mathbf{k}_\perp - \mathbf{p}_\perp \cdot \mathbf{k}_\perp) h^+_{1(2)}(x,k^\perp_\perp) h^+_{1(2)}(\bar{x},p^\perp_\perp)/(M_1 M_2) \right]}{\sum_a e_a^2 \mathcal{F} f_1(x,k^\perp_\perp) f_1(\bar{x},p^\perp_\perp)}$$

(3)

where the convolution is, $\mathcal{F} \equiv \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_\perp) f^a(x,\mathbf{k}_\perp) \tilde{f}^a(\bar{x},\mathbf{p}_\perp)$. As pointed out by Collins and Soper \cite{28}, well before $h^+_{1(2)}$ was identified, there is a higher twist $T$-even contribution to the cos2$\phi$ asymmetry which is not small at center of mass energies of 50 GeV$^2$

$$v_4 = \frac{\frac{1}{Q^2} \sum_a e_a^2 \mathcal{F} \left[ (2(\mathbf{h} \cdot (\mathbf{k}_\perp - \mathbf{p}_\perp))^2 - (\mathbf{k}_\perp - \mathbf{p}_\perp)^2) f_1(x,k^\perp_\perp) \tilde{f}_1(\bar{x},p^\perp_\perp) \right]}{\sum_a e_a^2 \mathcal{F} [f_1(x,k^\perp_\perp) \tilde{f}_1(\bar{x},p^\perp_\perp)]}$$

(4)

Collins and Soper considered this effect as a possible source for the azimuthal asymmetry that had been measured. It arises from a dependence on the relative quark and anti-quark azimuthal orientation that enters in the convolution.

We estimate the leading twist 2 and twist 4 contributions using a parton model with the quark diquark spectator framework to estimate the $T$-odd and even distribution functions. Previously, we calculated these functions to predict single spin asymmetries and azimuthal dependences in SIDIS \cite{9,14,15,16}; $h^+_1$ is given by

$$h^+_1(x,k^\perp_\perp) = \frac{e_1 e_2 g^2}{2(2\pi)^4} \frac{(m + xM)(1-x)}{k^2} e^{-2b(k^2 - \Lambda(0))} \frac{1}{k^2} e^{-2b(k^2 - \Lambda(0))} \left[ \Gamma(0,2b\Lambda(0)) - \Gamma(0,2b\Lambda(k^2_\perp)) \right],$$

(5)
and the unpolarized quark distribution function is

\[ f(x) = \frac{g^2}{(2\pi)^2} \left( 1 - x \right) \cdot \left\{ \frac{(m + xM)^2 - \Lambda(0)}{\Lambda(0)} - \left[ 2b \left( (m + xM)^2 - \Lambda(0) \right) - 1 \right] \times e^{2b\Lambda(0)} \Gamma(0, 2b\Lambda(0)) \right\} \]

(6)

We have included a Gaussian damping of the quark transverse momenta in the nucleon. This models the known intrinsic \( k_{\perp} \) distribution and also regularizes the convolutions that have to be done to obtain observable cross sections and asymmetries.

In Fig. 1 at center of mass energy of \( s = 50 \text{ GeV}^2 \), the leading order \( T \)-odd contribution contributes about 28% with an additional 10% from sub-leading order \( T \)-even contributions. At center of mass energy of \( s = 500 \text{ GeV}^2 \) the distinction between the leading order \( T \)-odd and additional sub-leading order \( T \)-even contributions are small. This reflects the diminution of the non-leading contribution with increasing \( s \) and \( Q^2 \).

Note that we have not taken account of the evolution of the distribution \( h_{\perp}^1 \) with \( Q^2 \) scale. This evolution has not been worked out in general at this time. In Fig. 2, \( \nu \) is plotted versus \( x \) at \( s = 50 \text{ GeV}^2 \) where \( q_T \) ranges from 2 to 4 GeV. Again the higher twist contribution is significant.

**FIGURE 2.** Left panel: \( \nu \) plotted as a function of \( x \) for \( s = 50 \text{ GeV}^2 \) \( q_T \) ranging from 2 to 4 GeV. Right panel: The \( z \)-dependence of the \( \cos 2\phi \) asymmetry at HERMES \([5]\) kinematics. The full and dotted curves correspond to the \( T \)-even and \( T \)-odd terms of asymmetry, respectively. The dot-dashed and dashed curves are the sum and the difference of those terms, respectively.

\[ \cos 2\phi \text{ Azimuthal Asymmetry in SIDIS} \]

We use the conventions established in \([8]\) for the asymmetries. Being \( T \)-odd, \( h_{\perp}^1 \) appears with the \( H_{1} \), the \( T \)-odd fragmentation function in observable quantities. In particular, the following weighted SIDIS cross section projects out a leading double
\( T \)-odd \( \cos 2\phi \) asymmetry,

\[
\langle \cos 2\phi \rangle_{UU} = \frac{8(1-y)\sum q e_q^2 h_1^{+(1)}(x)\epsilon^2 H_1^{+(1)}(z)}{(1+(1-y)^2)\sum q e_q^2 f_1(x)D_1(z)}, \tag{7}
\]

where the subscript \( UU \) indicates unpolarized beam and target. The Collins fragmentation function is given by [15],

\[
H_1^{+}(z, k_{\perp}) = \frac{f_2^{qq} g^2}{(2\pi)^4} \frac{4\pi}{k_{\perp}^4} e^{-2c(k_{\perp}^2 - \Lambda(0))} \Gamma'(0, 2c\Lambda(0)) - \Gamma'(0, 2c\Lambda(k_{\perp}^2)). \tag{8}
\]

The \( \cos 2\phi \) asymmetry originating from \( T \)-even distribution and fragmentation function appears at order \( 1/Q^2 \) [29, 30]. The \( \langle \cos 2\phi \rangle \) from ordinary sub-sub-leading \( T \)-even and leading double \( T \)-odd (up to a sign) effects to order \( 1/Q^2 \) is given by [16]

\[
\langle \cos 2\phi \rangle_{UU} = \frac{2\langle k_{\perp}^2 \rangle(1-y)f_1(x)D_1(z) \pm 8(1-y)h_1^{+}(x)H_1^{+}(z)}{1+(1-y)^2 + 2\langle k_{\perp}^2 \rangle(1-y)} f_1(x)D_1(z). \tag{9}
\]

The \( z \)-dependence of this asymmetry at HERMES kinematics [5] are shown in the right panel of Fig. 2. The full and dotted curves correspond to the \( T \)-even and \( T \)-odd terms in the asymmetry, respectively. The dot-dashed and dashed curves are the sum and the difference of those terms, respectively. From the figure, one can see that the double \( T \)-odd asymmetry behaves like \( z^2 \), while the \( T \)-even asymmetry is flat in the whole range of \( z \). Therefore, aside from the competing \( T \)-even \( \cos 2\phi \) effect, the experimental observation of a strong \( z \)-dependence (especially at high \( z \) region) would indicate the presence of \( T \)-odd structures in unpolarized SIDIS implying that novel transversity properties of the nucleon can be accessed without involving spin polarization.

**Conclusions**

The interdependence of intrinsic transverse quark momentum and angular momentum conservation are intimately connected with studies of \( T \)-odd effects underlying the \( \cos 2\phi \) azimuthal asymmetries in Drell-Yan and semi-inclusive deep inelastic scattering. Using re-scattering as a mechanism to generate \( T \)-odd distribution functions opens a new window into the theory and phenomenology of transversity in hard processes. We have also demonstrated that at moderate energies sub-leading twist contributions are non-trivial.

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