A SELECTION RULE FOR MULTiquark DECAYS

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Abstract

By assuming $SU(6)_{CS}$ symmetry for pentaquark decays one finds a selection rule, which strongly reduces the number of states able to decay into a baryon and a meson final state and allows an intriguing identification for the $\Theta^+$ particle recently discovered with the prediction of a narrow width.

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The discovery of a narrow $Y = 2 T = 0$ KN resonance $\Theta^+$ at 1540 MeV in different experiments \cite{1} is a great success for the Skyrme model \cite{2}, which predicts the existence of a $(10, 1/2)^+$ state, stable in the non-relativistic limit, in the same group of states of the better-established $(8, 1/2)^+$ and $(10, 3/2)^+$ traditionally classified in the 56-dimensional representation of flavour-spin $SU(6)_{FS}$ \cite{3}. The value of the mass of the state happened to be predicted at the right value \cite{4}. In fact, one of the authors (D.D.) has been very active in promoting the experimental search for that state. This state can be thought to be a pentaquark, consisting of four quarks $uudd$ and a $\bar{s}$. Pentaquarks have been considered many years ago \cite{5} and their relevance for heavy quark systems has been stressed \cite{6}.

To get positive parity states with a $S$-wave $\bar{q}$, one should consider, as in \cite{7} \cite{8} \cite{9} \cite{10}, $L = 1$ four-quark states. In a previous paper \cite{11} all these states were classified in the $126 + 210 + 105 + 105'$ $L = 1$ representation of $SU(6)_{FS} \times SO(3)_L$.

Within the approximation of considering the decay of a pentaquark as a separation process with a $\bar{q}$ forming a meson with one of the $4q$ – in the case of $\Theta^+$, the $\bar{s}$ together with a $u$ or a $d$ forms a $K$ – and the remaining three $q$’s giving rise to the final baryon, the pentaquarks with the $4q$’s transforming as the $105 + 105'$ representations of $SU(6)_{FS}$ are not allowed to decay into a final state with a meson and a $(8, 1/2^+)$ baryon \cite{8}. In fact, as it can be easily seen by considering the Young tableaux associated to those representations, at least two of the three quarks remain in a $SU(6)_{FS}$ antisymmetric state, so that the $3q$’s wave function is orthogonal to that of the totally symmetric 56 representation. An analogous selection rule has been found in \cite{11}, which follows from the fact that the $(10, \frac{3}{2}^+)$ and the $(8, \frac{1}{2}^+)$ transform as the 20 and 70 $SU(6)_{CS}$ representations, respectively. The states of the 210 and of the 105 $SU(6)_{CS}$ cannot decay into meson decuplet states, while the states of the 15 cannot decay into meson octet states. In conclusion, only the states, with their $4q$ transforming as the $105'$ of $SU(6)_{CS}$ may be found by looking for decuplet-meson final states in octet-meson reactions.

$SU(6)_{CS}$ plays an important role in the mass splittings of the $L = 0$ ordinary hadrons ($3q$ baryons and $q\bar{q}$ mesons) and the chromomagnetic interaction, which predicts properly \cite{12} the $\Delta - N$ and $\rho - \pi$ mass splittings, gives a contribution proportional to a combination of the $SU(6)_{CS}$, $SU(3)_{C}$ and $SU(2)_S$ Casimir operators \cite{5}. This fact was the motivation \cite{11} to write a mass formula for pentaquarks, where the mass splittings are provided by the
chromomagnetic interaction of the $4q$'s and the $\bar{q}$ and by a spin-orbit term, as for ordinary hadrons.

In \[11\] the phenomenological mass formula has been proposed:

$$m = m_0 + h \frac{3}{16}(m_{K^*} - m_K) \left[ C_6(p) - C_6(t) - \frac{1}{3} S_p(S_p + 1) + \frac{1}{3} S_t(S_t + 1) \right. $$

$$\left. - \frac{4}{3} \right] + \tilde{h} \frac{1}{4}(m_N - m_{\Delta}) \left[ C_6(t) - \frac{1}{3} S_t(S_t + 1) - \frac{26}{3} \right] + a \vec{L} \cdot \vec{S} \ (1)$$

where $C_6(p)$ and $C_6(t)$ are the Casimir of $SU(6)_{CS}$ and $p$ and $t$ are the representations for pentaquark and $4q$ states, respectively. In \[11\] the values $h = 1/2$, $\tilde{h} = 0$ and $a = 40 MeV$ have been chosen for the positive parity states built with $(4q, L = 1)$ and a $\bar{q}$ in $S$-wave respect to them, $h = \tilde{h} = 1$ for the negative parity states with all the constituent in $S$-wave and $h = 0$, $\tilde{h} = 1$ for the states built with $4q$, $L = 0$ and a $\bar{q}$ in $P$-wave respect to them; in this case, inspired by the spectrum of the mesons of the $(35, L = 1)$ of $SU(6)_{FS}$, we take $a = 100 MeV$ \[13\].

That choice implies that the transformation properties with respect to $SU(6)_{CS}$ of the $L = 1$ $4q$'s and of the pentaquarks, one forms combining them with the $\bar{q}$, play a major role to identify the mass eigenstates. In Table 1 we write for the $(4q, L = 1)$ $SU(3)_{F} \times SU(2)_{S}$ multiplets the transformation properties with respect to $SU(6)_{CS}$ and $SU(6)_{FS}$.

<table>
<thead>
<tr>
<th>$SU(6)_{CS}$</th>
<th>$SU(6)_{FS}$</th>
<th>$SU(3)<em>{F} \times SU(2)</em>{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$105'$</td>
<td>$126$</td>
<td>$(15', S = 2)$</td>
</tr>
<tr>
<td>$105'$</td>
<td>$210$</td>
<td>$(15, S = 2) + (15', S = 1)$</td>
</tr>
<tr>
<td>$105'$</td>
<td>$105$</td>
<td>$(6, S = 2) + (15', S = 0)$</td>
</tr>
<tr>
<td>$105'$</td>
<td>$105'$</td>
<td>$(3, S = 2)$</td>
</tr>
<tr>
<td>$105' + 210$</td>
<td>$126 + 105$</td>
<td>$(6, S = 0)$</td>
</tr>
<tr>
<td>$105' + 210$</td>
<td>$210 + 105'$</td>
<td>$(6, S = 1) + (15 + 3, S = 0)$</td>
</tr>
<tr>
<td>$105' + 210 + 105$</td>
<td>$210 + 105 + 105'$</td>
<td>$(3, S = 1)$</td>
</tr>
<tr>
<td>$105' + 210 + 105 + \bar{15}$</td>
<td>$126 + 210 + 105 + 105'$</td>
<td>$(15, S = 1)$</td>
</tr>
</tbody>
</table>

**TABLE 1**: Transformation properties with respect to $SU(6)_{CS}$, $SU(6)_{FS}$ and $SU(3)_{F} \times SU(2)_{S}$ of the $(4q, L = 1)$ states. For convenience, the $SU(2)$ representations are not denoted by their dimensions - which is the case for
their \(SU(3)\) partners - but by their highest weight. Apart the states, which transform as a \(15'\) of \(SU(3)_F\) or (and) a \(S = 2\) of \(SU(2)_S\), the states with definite transformation properties with respect to \(SU(6)_{CS}\) are a combination of states with definite \(SU(6)_{FS}\) properties. In the case of the \(\bar{6}\) with \(S = 1 \) and 0, the mixing is maximal.

In Tables 2 and 3 we report the mass splittings for the \((Y = 2, I = 0)\) pentaquark states deduced from eq.(1) and the values chosen for the parameters.

<table>
<thead>
<tr>
<th>(SU(6)_{CS} \times S)</th>
<th>(J)</th>
<th>(\Delta M) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((20^*, \frac{3}{2})(105'))</td>
<td>(\frac{5}{2} + \frac{3}{2} + \frac{1}{2})</td>
<td>-150 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((70^*, \frac{1}{2})(210))</td>
<td>(\frac{3}{2} + \frac{1}{2})</td>
<td>-190 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((70^*, \frac{1}{2})(105'))</td>
<td>(\frac{3}{2} + \frac{1}{2})</td>
<td>-48 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((540, \frac{3}{2})(105'))</td>
<td>(\frac{5}{2} + \frac{3}{2} + \frac{1}{2})</td>
<td>+64 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((1134, \frac{3}{2})(210))</td>
<td>(\frac{5}{2} + \frac{3}{2} + \frac{1}{2})</td>
<td>+76 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((540^*, \frac{3}{2})(105'))</td>
<td>(\frac{5}{2} + \frac{3}{2} + \frac{1}{2})</td>
<td>+92 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((1134^*, \frac{1}{2})(210))</td>
<td>(\frac{3}{2} + \frac{1}{2})</td>
<td>+95 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((540^*, \frac{1}{2})(105'))</td>
<td>(\frac{3}{2} + \frac{1}{2})</td>
<td>+95 + 40(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((70, \frac{1}{2})(105))</td>
<td>(\frac{3}{2} + \frac{1}{2})</td>
<td>-98 + 100(\vec{L} \cdot \vec{S})</td>
</tr>
<tr>
<td>((560, \frac{3}{2})(105))</td>
<td>(\frac{5}{2} + \frac{3}{2} + \frac{1}{2})</td>
<td>-98 + 100(\vec{L} \cdot \vec{S})</td>
</tr>
</tbody>
</table>

Table 2 : Mass splittings of the positive parity \((Y = 2, I = 0)\) pentaquarks. The first eight rows correspond to the states with \((4q, L = 1)\) and a \(\bar{q}\) in \(S\)-wave. The last two rows to \((4q, L = 0)\) and a \(\bar{q}\) in \(P\)-wave. The * is put to remind of a mixing between the \(SU(6)_{CS}\) representations and the transformation properties of the \(4q\) state have been written in brackets.

<table>
<thead>
<tr>
<th>(SU(6)_{CS})</th>
<th>(J = S)</th>
<th>(\Delta M) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70(105)</td>
<td>(\frac{7}{2})</td>
<td>-342</td>
</tr>
<tr>
<td>560(105)</td>
<td>(\frac{5}{2})</td>
<td>+24</td>
</tr>
</tbody>
</table>

Table 3 : Mass splittings of the \((Y=2,I=0)\) negative parity pentaquarks built with \(4q\) and a \(\bar{q}\) in \(S\)-wave.
The dynamics for pentaquark decays may be different from the case of $\Delta$ and $\rho$ decays, where one has to create a $q\bar{q}$ pair, with the $\bar{q}$ forming a meson with one of the initial quarks in the first case or with the initial quark in the second one. As long as for pentaquarks all the elementary fermions in the final state are present in the initial one, which makes possible the hypothesis that the decay is a consequence of the separation of its constituents. Also, at difference with what happens for the decay of the previously mentioned ordinary hadrons, with the orbital angular momentum not conserved (changing from 0 to 1) as well as the spin, for the pentaquark decay, to the initial orbital momentum of the 4$q$’s in the initial state corresponds the relative angular momentum of the emitted meson with respect to the final baryon. So $L$ and $S$ may be both conserved. One may even assume that, as in the hypothesis that the amplitude is proportional to the scalar product of the initial and final wave-functions, also $SU(6)_{CS}$ and (or) $SU(6)_{FS}$ are conserved in pentaquark decays. We want first to explore the consequences of $SU(6)_{CS}$ conservation for the decay of a pentaquark into a meson baryon final states, which are very restrictive, since the pseudoscalar mesons are $SU(6)_{CS}$ singlets. According to the previously mentioned transformation properties of the baryon of the 56 of $SU(6)_{FS}$, only the pentaquarks transforming as a 70 (20) of $SU(6)_{CS}$ may decay into a final state containing a pseudoscalar meson and a $(8, 1/2)^+ ((10, 3/2)^+)$ baryon. As long as the $\bar{16}$ multiplets of $SU(3)_F$, they may be obtained by combining the $\bar{6}$ constructed with 4$q$ with the $\bar{q}$ which transforms as a $\bar{3}$ of $SU(3)_F$. According to Table1 only the 105’ and the 210 representations of $SU(6)_{CS}$ contain $\bar{6}$’s of $SU(3)_F$ (the demand of complete antisymmetry for the $L = 1$ 4$q$ wavefunction relates the transformation properties with respect to different groups [11]). Let us therefore consider the $SU(6)_{CS}$ products:

$$210 \times \bar{6} = 1134 + 56 + 70$$  \hspace{1cm} (2)

$$105' \times \bar{6} = 540 + 70 + 20$$  \hspace{1cm} (3)

In conclusion the only $\bar{16}$ states allowed to decay into a pseudoscalar meson and a $(8, 1/2)^+$ baryon should transform as a 70 of $SU(6)_{CS}$, These are the combinations written in [11] for the 4$q$, $L = 1$ states:
\[ |70, \ (1, S = 1/2), S_z = \frac{1}{2} > \leftrightarrow \]

\[ \frac{1}{\sqrt{3}} \ \{ \frac{1}{\sqrt{3}} \ |105'(3, S = 1)S_z = 1 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 >^a \\
-\frac{1}{\sqrt{6}} \ |105'(3, S = 1)S_z = 0 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 >^a \\
+\frac{1}{\sqrt{2}} \ |105'(3, S = 0) >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 >^a \} \]

(4)

\[ |70, \ (1, S = 1/2), S_z = \frac{1}{2} > \leftrightarrow \]

\[ \frac{1}{\sqrt{3}} \ \{ \frac{1}{\sqrt{2}} \ |210(3, S = 1)S_z = 1 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 >^a \\
-\frac{1}{2} \ |210(3, S = 1)S_z = 0 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 >^a \\
+\frac{1}{2} \ |210(3, S = 0) >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 >^a \} \]

(5)

where \( a = 1, 2, 3 \) is a colour index to be saturated to get a colour singlet, and for the \( 4q, L = 0 \) state:

\[ |70, \ (1, S = 1/2), S_z = \frac{1}{2} > \leftrightarrow \]

\[ \frac{1}{\sqrt{3}} \ \{ \frac{\sqrt{2}}{3} \ |105'(3, S = 1)S_z = 1 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = -1/2 >^a \\
-\frac{1}{\sqrt{3}} \ |105'(3, S = 1)S_z = 0 >_a |\bar{6}; (\bar{3}, S = 1/2), S_z = 1/2 >^a \\
+\frac{1}{2} \ |210(3, S = 0) >_a \} \]

(6)

for the \( 4q, L = 0 \) states. The mass eigenstates are approximately given by states with definite \( SU(6)_{CS} \) and the two \((\bar{10}, 1/2)^+\) states, built with \( 4q, L = 1 \) and a \( \bar{q} \) in \( S \)-wave respect to them, have their larger components along the 70’s of \( SU(6)_{CS} \) and masses, which differ by about 140MeV. By identifying the lightest one with the discovered \( \Theta^+ \) state, one predicts the existence of another \((\bar{10}, 1/2^+)\) resonance at about 1680MeV.

As stated in the caption of Table1 the \((\bar{6}, S = 1)\) and \((\bar{\bar{6}}, S = 0)\) states of the
210 representation of $SU(6)_{CS}$ are maximal mixtures of the 210 and 105' and 126 and 105 representations of $SU(6)_{FS}$, respectively. The $SU(6)_{FS}$ selection rule implies that only the first components, the 210 and the 126 of $SU(6)_{FS}$ contribute to the decay into a final state consisting of a pseudoscalar meson and of a $(8, 1/2)^+$ baryon (as $K\bar{N}$). In \[14\] the amplitude for the decay $\Theta^+$ into $K\bar{N}$ is predicted to be suppressed by the small overlap factor $\frac{5\sqrt{2}}{96}$, if one assumes that the 4$q$'s in the $\Theta^+$ transform as the $(126, \bar{6}, S = 0, L = 1)$ of $SU(6)_{FS} \times SU(3)_F \times SU(2)_S \times SO(3)_L$.

Our $SU(6)_{CS}$ selection rules implies that the $S = 1/2$ colour singlet of the 1134 representation of $SU_{CS}(6)$ cannot decay into a pseudoscalar 1/2 octet final state. From the ortogonality of the 70 and 1134 final states and the vanishing coupling to the $K\bar{N}$ final state of the second one, one can relate the couplings of the $S = 1$ and $S = 0$ states and, consequently, the coupling of the state in the left hand side of eq.(5) to the one of the $(\bar{10}, J = 1/2)$ state constructed with the tetraquark in the $(126, 6, S = 0, L = 1)$ of $SU(6)_{FS}$ and the $\bar{q}$: in fact the ratio of the two couplings comes out to be $\sqrt{2}$ and so, by identifying the $\Theta^+$ with the l.h.s of eq.(5), we predict a width twice larger than in \[14\] with a different state with the diagonalization fixed by the mass formula in Eq.(1), where the chromomagnetic interaction, successfully introduced in \[12\] for ordinary hadrons, plays the main role.

By considering $(4q, L = 0)$, we can build positive (negative) parity states by combining them with a $\bar{q}$ with $L = 1(0)$ with respect to them. In both cases the 4$q$-$\bar{6}$ of $SU_F(3)$ should transform as the 105 representation of $SU_{CS}(6)$ with $S = 1$. By combining it with the 6 of $SU_{CS}(6)$:

$$126 \times \bar{6} = 560 + 70 \quad (7)$$

one realizes that only the $\bar{10}$s with $S = 1/2$, which transform as the 70 of $SU(6)_{CS}$, are allowed to decay into a pseudoscalar 1/2 octet state.

Within the approximation first suggested in \[5\] of requiring that the $\bar{q}$ should form a meson only with a $q$ in the some cluster, one expects a narrow width for the $J = 1/2^+$ state built with $(uudd, I = 0, S = 1, L = 0)$ and a $\bar{s}$ in $P$-wave respect to them, transforming as the 70 representation of $SU(6)_{CS}$, which according to Table 2 is $32MeV$ heavier than the state, we have identified with the $\Theta^+$ positive parity. The $J = 1/2^+$ state transforming as the 560 representation of $SU(6)_{CS}$ is even lighter, but is forbidden also by the $SU(6)_{CS}$ selection rule to decay into a $K\bar{N}$ final state.

One may write for the positive parity $(qq\bar{q}\bar{q}, L = 0)$ ($q = u, d$) meson states
a mass formula analogous to eq.(1):

\[ \mu = \mu_0 + \frac{3}{16}(m_\rho - m_\pi) \left[ C_6(\tau) - C_6(2q) - C_6(2\bar{q}) + C_3(2q) - \frac{1}{3}C_2(\tau) \\
+ \frac{1}{3}C_2(2q) + \frac{1}{3}C_2(2\bar{q}) \right] + \frac{1}{4}(m_N - m_\Delta) [C_6(2q) + C_6(2\bar{q}) - C_3(2q) \\
- \frac{1}{3}C_2(2q) - \frac{1}{3}C_2(2\bar{q}) - 8] \]  

(8)

where \( \tau \) is the \( 2q2\bar{q} \) state.

The lightest state, with a contribution of the chromo-magnetic interaction \( \simeq -1\,\text{GeV} \), is a \( (I = 0, 0^+) \) state with quark content \( ud\bar{u}d \) [15], which transforms mostly as a singlet of \( SU(6)_{CS} \), to be identified with the \( f^0(600) \) \( 0^+ \) state [15]. With the appropriate changes for the presence of strange quarks several hundreds \( \text{MeV} \) above that state one predicts a \( (I = 0 + 1, 0^+) \) \( qs\bar{q}s \) multiplet to be identified with the \( (f^0(980) + a^0(980), 0^+) \) states, for which the \( qs\bar{q}s \) content has been already proposed [16].

According to \( SU(6)_{CS} \) symmetry only \( SU(6)_{CS} \) singlets, as the states just mentioned, may decay into two pseudoscalar mesons. For the same reason only the \( qq\bar{q}\bar{q} \) states transforming as the 35 representation of \( SU(6)_{CS} \) should be allowed to decay into a final state consisting of a pseudoscalar and a vector meson.

The OZI selection rule [17] forbids the decay \( f^0(980) \rightarrow \pi\pi \) and accounts for the relevance in that region, despite the larger phase space for \( \pi\pi \), of the \( K\bar{K} \) channel, which plays an important role in the disprove [18] of the lower bound found [19] for the pion radius.

The symmetry with respect to \( SU(6)_{FS} \) would also have important consequences. In fact by eqs.(2-4) and the tensor products:

[\]

\[ 56 \times 35 = 1134 + 700 + 70 + 56 \]  

(9)

\[ 105 \times 6 = 560 + 70 \]  

(10)

we reach the conclusion that the pentaquarks transforming as the exotic \( SU(3)_F \) representations cannot decay into the final state consisting of a pseudoscalar or a vector meson and a baryon of the octet \( 1/2^+ \) or of the decuplet \( 3/2^+ \), if their \( 4q \)’s transform as the 105 + 105’ of \( SU(6)_{FS} \) [8], have their couplings to these states proportional with ratios dictated by \( SU(6)_{FS} \) symmetry if their \( 4q \)’s transform as the 126 or 210.
References

[1] Photon Nucleus
Kaon Nucleus
Photon Deuteron
Photon Proton
Neutrino (D + Ne)


S. Okubo, Phys. Lett. 5 (1963) 165;
