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Topological Vertex Search in Collider Experiments

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1. Introduction

The growing interest in heavy flavour physics together with the possibility of very precise particle trajectory measurement with the help of position sensitive semiconductor detectors has lead to the desire of analysing complicated short range decay topologies. As an ideal one would like to uniquely assign particle trajectories to the in general multi-step decay vertices. Due to the short lifetime of the heavy flavour particles and the limited precision of the back extrapolated tracks many ambiguities in the assignment will remain. Selection of only one assignment by e.g. χ^2 criteria will often lead to wrong results. Therefore it was the aim of this method to retain all possible track-vertex assignments and to select those being compatible with a specific physics channel only afterwards. Analysing the vertex topology in such a general way requires the investigation of a large number of track combinations. Therefore a rather fast method for fitting a vertex to a given set of tracks has been developed as well as a efficient strategy for handling the combinatorial system. Furthermore for tackling the physics analysis a set of modular tools have been developed which may be combined in an appropriate manner so as to select a specific channel.

The methods described in this paper have originally been developed for the NA32 fixed target experiment at CERN where the charm decay topology was investigated in a field free region. Later on they have been modified and developed much further for application in the ALEPH experiment at LEP. This development contains the adaptation of the formalism to the case of a homogeneous magnetic field in which charged tracks are represented as helices as well as the extension of the formalism to neutral tracks. Furthermore kinematic fits have been developed which give as result e.g. the invariant mass of particles combined to a vertex and the incident track parameter as well as all error correlations. These modular tools can easily be adapted to select and to investigate specific physics channels of interest.

In the main part of this paper the choice representation of physical quantities (chapter 2) and the basic fitting methods (chapters 3 to 6) are described. Following are suggestions on the application of the program.

2. Representation of physical quantities

The specific track and vertex representations have been chosen in view of the cylindrical symmetry of colliding beam experiments in which the interaction point is close to the origin of a cartesian coordinate system with the z-axis parallel to the homogeneous magnetic field. The charged and neutral track representations are closely related in order to keep the mathematics rather similar.

2.1. HELIX REPRESENTATION

Charged particles trajectories inside a homogeneous magnetic field are helices. They will be described by the following parameters:

- $\rho = \frac{1}{r}$ signed positive for counterclockwise bending, negative for clockwise bending. r is the radius of the helix projection into the $x - y$ plane.
- $\tau = \frac{\delta z}{\delta s}$ vertical slope
- Φ_0 ($0 \leq \Phi_0 \leq 2 \cdot \pi$) projected particle direction at closest approach of projected circle to origin
- D_0 projected distance of projected circle from origin, signed positive if particle has a positive angular momentum around the origin at this point, negative otherwise.
- z_0 z-coordinate at point of closest approach of projected circle to the origin $x = y = 0$.

Fig. 1: Helix representation.

A point P on the helix is defined by these parameters and the projected distance s along this circle measured from the point of closest approach in the projection from the origin. The sign convention illustrated in Fig. 2 leads to simple mathematical formulations valid for all four possible sign combinations.

2.2. NEUTRAL TRACK REPRESENTATION

The parameters describing the straight trajectories of neutral particles have been chosen as close as possible to those of helices in order to unify the formalism as much as possible:

- p momentum signed positive
- $\tau = \frac{\delta z}{\delta s}$ vertical slope
- Φ_0 ($0 \leq \Phi_0 \leq 2 \cdot \pi$) projected particle direction

Fig. 2: Helix sign convention

- D_0 projected distance of projected track from origin, signed positive if particle has a positive angular momentum around the origin, negative otherwise.
- z_0 z-coordinate at point of closest approach of projected track to origin.

Fig. 3: Neutral track representation and sign convention.

A point P on the neutral track is defined by these parameters and the projected distance s along the track measured from the point of closest approach in the x-y projection.

2.3. NOTATION

Throughout the paper we will use vector respectively matrix notation as far as possible. Input (measured) quantities for the fit will be in normal setting, output (fitted) quantities in bold face. The following notation of basic quantities will be used:

Helix

$h = (h_1 \dots h_5) = (\rho, \tau, \phi_o, D_o, z_o)$ helix parameters

H complete error covariance matrix $H_{i,j} = \langle \Delta h_i \Delta h_j \rangle$

d transverse distance of a point to the helix $d = (d_1, d_2) = (d_r, d_z)$

D ... transverse distance error covariance matrix $D_{i,j} = \langle \Delta d_i \Delta d_j \rangle$

n_h ... number of helices

Neutrals

$n = (n_1 \dots n_5) = (p, \tau, \phi_o, D_o, z_o)$

N complete error covariance matrix $N_{i,j} = \langle \Delta n_i \Delta n_j \rangle$

d transverse distance of a point to the helix $d = (d_1, d_2) = (d_r, d_z)$

D ... transverse error covariance matrix $D_{i,j} = \langle \Delta d_i \Delta d_j \rangle$

n_n ... number of neutral tracks

Vertex

$v = (v_1, v_2, v_3) = (v_x, v_y, v_z)$ vertex coordinates

V error covariance matrix $V_{i,j} = \langle \Delta v_i \Delta v_j \rangle$

Momentum

$p = (p_1, p_2, p_3) = (p_x, p_y, p_z)$ momentum components

P momentum error covariance matrix $P_{i,j} = \langle \Delta p_i \Delta p_j \rangle$

Mass

m mass of particle

In many instances the distinction between charged and neutral tracks will be avoided. Then the following notation will be used:

Tracks

$t = (t_1 \dots t_5)$ track parameters

T corresponding complete 5×5 error covariance matrix $T_{i,j} = \langle \Delta t_i \Delta t_j \rangle$

d transverse distance of a point to the track $d = (d_1, d_2) = (d_r, d_z)$

D ... transverse distance error covariance matrix $D_{i,j} = \langle \Delta d_i \Delta d_j \rangle$

$n_t = n_h + n_n$... number of tracks

3. Constraint fit of tracks to common vertex

The basic task in doing a topological vertex search is the identification of tracks belonging to a specific decay. This requires the fit of tracks to a common vertex. Furthermore one usually will want to know the invariant mass of the decay products as well as the incoming track parameters and all error correlations. Due to the large number of tracks involved a extremely high number of track combinations will have to be tested. Most of these combinations will fail and it is not necessary to have complete information on most tries. Therefore two methods of vertex fitting have been developed. One which makes a refit of all tracks to a common vertex and provides complete information on the decay while the second fast version only looks for a point which is close to all tracks considered simultaneously. While for the complete fit the computer time rises with the square of the number of tracks involved, the fast fitting method shows only a linear increase with n_t .

In this chapter the general fit is described while the fast fitting method is deferred to the following chapter.

The track parameters are varied according to their measurement precision so as to be constrained to a common vertex. In order to treat this problem in more generality the task will be expanded to a common fit of charged and neutral tracks as well as other vertices – which might have been found previously by combining a subset of tracks – to a new common vertex. Thus the formalism may also be applied to some other common tasks as e.g. checking if a track may be attached to a vertex or if two vertices are really separated.

There are $3n_v + 5n_h + 5n_n$ input parameters for n_v vertices n_h helices and n_n neutral tracks. Two constraints for every fitted track requiring passage to the fitted vertex reduce the number of independent parameters for each helix or neutral track to three. With the three common vertex coordinates one thus obtains $3 + 3n_h + 3n_n = 3 + 3n_t$ free parameters to fit. We choose the following independent parameters:

The common vertex: $\mathbf{v} = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z)$ vertex coordinates

For each helix ${}_h\mathbf{q} = (\eta = \mathbf{r} - \mathbf{D}_0, \tau, \Phi_0)$

For each neutral track ${}_n\mathbf{q} = (\mathbf{p}, \tau, \Phi_0)$

This set of $n_f = 3(1 + n_h + n_n) \equiv 3(1 + n_t)$ independent parameters leads to simple relations between standard track parameters and independent parameters which may be immediately read from the geometrical pictures shown in Fig. 4:

helix:

$$\begin{aligned}
 \mathbf{x}_c &= -\eta \sin \Phi_0 & y_c &= \eta \cos \Phi_0 & (3.1) \\
 \mathbf{r} &= \sqrt{(\mathbf{v}_1 - \mathbf{x}_c)^2 + (\mathbf{v}_2 - y_c)^2} \operatorname{sign}(\eta) & \mathbf{D}_0 &= \mathbf{r} - \eta \\
 \sin \Phi &= \frac{\mathbf{v}_1 - \mathbf{x}_c}{\mathbf{r}} & \cos \Phi &= -\frac{\mathbf{v}_2 - y_c}{\mathbf{r}} \\
 \mathbf{s} &= \mathbf{r} \cdot (\Phi - \Phi_0) & \mathbf{z}_0 &= \mathbf{v}_3 - \mathbf{s} \cdot \tau
 \end{aligned}$$

Fig. 4: Dependent and independent helix and neutral track parameters

neutral tracks:

$$\begin{aligned} D_0 &= v_1 \cdot \sin \Phi_0 - v_2 \cdot \cos \Phi_0 \\ s &= v_1 \cdot \cos \Phi_0 + v_2 \cdot \sin \Phi_0 \quad z_0 = v_3 - s \cdot \tau \end{aligned} \quad (3.2)$$

Fitting is done by minimizing a χ^2 which is composed of terms describing the difference between fitted and measured parameters (vertices, helices and neutral tracks).

$$\chi^2 = \sum_{\alpha=1}^{n_v} \alpha \chi_v^2 + \sum_{\beta=1}^{n_h} \beta \chi_h^2 + \sum_{\gamma=1}^{n_n} \gamma \chi_n^2 \equiv \sum_{\alpha=1}^{n_v} \alpha \chi_v^2 + \sum_{\kappa=1}^{n_t} \kappa \chi_t^2 \quad (3.3)$$

The splitting up of the total χ^2 in this form is possible due to the absence of error correlations between different tracks. In the rightmost formulation we have used a uniform notation for charged and neutral tracks. This praxis will be followed whenever possible, but keep in mind that the track parameters have a different meaning for these two species.

Explicitly written in matrix form the contribution of input vertices and tracks to χ^2 have the following form:

$$\begin{aligned} \alpha \chi_v^2 &= (\mathbf{v} - \alpha_v)^T \cdot \alpha_V^{-1} \cdot (\mathbf{v} - \alpha_v) \\ \beta \chi_h^2 &= (\beta_{\mathbf{h}} - \beta_h)^T \cdot \beta_H^{-1} \cdot (\beta_{\mathbf{h}} - \beta_h) \\ \gamma \chi_n^2 &= (\gamma_{\mathbf{n}} - \gamma_n)^T \cdot \gamma_N^{-1} \cdot (\gamma_{\mathbf{n}} - \gamma_n) \\ \kappa \chi_t^2 &= (\kappa_{\mathbf{t}} - \kappa_t)^T \cdot \kappa_T^{-1} \cdot (\kappa_{\mathbf{t}} - \kappa_t) \end{aligned} \quad (3.4)$$

In order to minimize the total χ^2 we have to calculate the first and second derivatives with respect to the independent fit parameters. This requires the knowl-

edge of the derivatives of the helix and neutral track parameters with respect to the independent parameters. Fortunately most of the derivatives will be zero, we therefore write down only the non-zero ones – again omitting the track index.

For helices one finds

$$\begin{aligned}
\frac{\partial \mathbf{h}_1}{\partial \mathbf{v}_1} &= \frac{\partial \rho}{\partial \mathbf{v}_1} = -\rho^2 \cdot \sin \Phi & \frac{\partial \mathbf{h}_1}{\partial \mathbf{v}_2} &= \frac{\partial \rho}{\partial \mathbf{v}_2} = \rho^2 \cdot \cos \Phi & (3.5) \\
\frac{\partial \mathbf{h}_1}{\partial \mathbf{h}\mathbf{q}_1} &= \frac{\partial \rho}{\partial \eta} = -\rho^2 \cdot \cos(\Phi - \Phi_0) & \frac{\partial \mathbf{h}_1}{\partial \mathbf{h}\mathbf{q}_3} &= \frac{\partial \rho}{\partial \Phi_0} = -\rho^2 \eta \cdot \sin(\Phi - \Phi_0) \\
\frac{\partial \mathbf{h}_2}{\partial \mathbf{h}\mathbf{q}_2} &= \frac{\partial \tau}{\partial \tau} = 1 \\
\frac{\partial \mathbf{h}_3}{\partial \mathbf{h}\mathbf{q}_3} &= \frac{\partial \Phi_0}{\partial \Phi_0} = 1 \\
\frac{\partial \mathbf{h}_4}{\partial \mathbf{v}_1} &= \frac{\partial \mathbf{D}_0}{\partial \mathbf{v}_1} = \frac{\partial \mathbf{r}}{\partial \mathbf{v}_1} = \sin \Phi & \frac{\partial \mathbf{h}_4}{\partial \mathbf{v}_2} &= \frac{\partial \mathbf{D}_0}{\partial \mathbf{v}_2} = \frac{\partial \mathbf{r}}{\partial \mathbf{v}_2} = -\cos \Phi \\
\frac{\partial \mathbf{h}_4}{\partial \mathbf{h}\mathbf{q}_1} &= \frac{\partial \mathbf{D}_0}{\partial \eta} = \frac{\partial \mathbf{r}}{\partial \eta} - 1 = \cos(\Phi - \Phi_0) - 1 \\
\frac{\partial \mathbf{h}_4}{\partial \mathbf{h}\mathbf{q}_3} &= \frac{\partial \mathbf{D}_0}{\partial \Phi_0} = \frac{\partial \mathbf{r}}{\partial \Phi_0} = \eta \sin(\Phi - \Phi_0) \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{v}_1} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_1} = -\tau [\sin \Phi \cdot (\Phi - \Phi_0) + \cos \Phi] \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{v}_2} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_2} = \tau [\cos \Phi \cdot (\Phi - \Phi_0) - \sin \Phi] \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{v}_3} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_3} = 1 \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{h}\mathbf{q}_1} &= \frac{\partial \mathbf{z}_0}{\partial \eta} = -\tau [(\Phi - \Phi_0) \cdot \cos(\Phi - \Phi_0) - \sin(\Phi - \Phi_0)] \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{h}\mathbf{q}_2} &= \frac{\partial \mathbf{z}_0}{\partial \tau} = -s \\
\frac{\partial \mathbf{h}_5}{\partial \mathbf{h}\mathbf{q}_3} &= \frac{\partial \mathbf{z}_0}{\partial \Phi_0} = -\tau \{ \eta [(\Phi - \Phi_0) \cdot \sin(\Phi - \Phi_0) + \cos(\Phi - \Phi_0)] - \mathbf{r} \}
\end{aligned}$$

For the neutral tracks one obtains similarly

$$\begin{aligned}
\frac{\partial \mathbf{n}_1}{\partial \mathbf{n}\mathbf{q}_1} &= \frac{\partial \mathbf{p}}{\partial \mathbf{p}} = 1 & \frac{\partial \mathbf{n}_2}{\partial \mathbf{n}\mathbf{q}_2} &= \frac{\partial \tau}{\partial \tau} = 1 & \frac{\partial \mathbf{n}_3}{\partial \mathbf{n}\mathbf{q}_3} &= \frac{\partial \Phi_0}{\partial \Phi_0} = 1 & (3.6) \\
\frac{\partial \mathbf{n}_4}{\partial \mathbf{v}_1} &= \frac{\partial \mathbf{D}_0}{\partial \mathbf{v}_1} = \sin \Phi_0 & \frac{\partial \mathbf{n}_4}{\partial \mathbf{v}_2} &= \frac{\partial \mathbf{D}_0}{\partial \mathbf{v}_2} = -\cos \Phi_0 \\
\frac{\partial \mathbf{n}_4}{\partial \mathbf{n}\mathbf{q}_3} &= \frac{\partial \mathbf{D}_0}{\partial \Phi_0} = \mathbf{v}_1 \cdot \cos \Phi_0 + \mathbf{v}_2 \cdot \sin \Phi_0 = s
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathbf{n}_5}{\partial \mathbf{v}_1} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_1} = -\tau \cdot \cos \Phi_0 & \frac{\partial \mathbf{n}_5}{\partial \mathbf{v}_2} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_2} = -\tau \cdot \sin \Phi_0 \\
\frac{\partial \mathbf{n}_5}{\partial \mathbf{v}_3} &= \frac{\partial \mathbf{z}_0}{\partial \mathbf{v}_3} = 1 \\
\frac{\partial \mathbf{n}_5}{\partial \tau} &= \frac{\partial \mathbf{z}_0}{\partial \tau} = -[\mathbf{v}_1 \cdot \cos \Phi_0 + \mathbf{v}_2 \cdot \sin \Phi_0] = -\mathbf{s} \\
\frac{\partial \mathbf{n}_5}{\partial \Phi_0} &= \frac{\partial \mathbf{z}_0}{\partial \Phi_0} = \tau[\mathbf{v}_1 \cdot \sin \Phi_0 - \mathbf{v}_2 \cdot \cos \Phi_0] = \tau \cdot \mathbf{D}_0
\end{aligned}$$

For the derivatives of the χ^2 terms with respect to the standard vertex and track parameters one obtains in the unified notation for charged and neutral tracks

$$\begin{aligned}
\frac{\partial \alpha \chi_v^2}{\partial \mathbf{v}_i} &= 2 \cdot \sum_{j=1}^3 \alpha V_{i,j}^{-1} \cdot (\mathbf{v}_j - \alpha \mathbf{v}_j) \\
\frac{\partial^2 \alpha \chi_v^2}{\partial \mathbf{v}_i \partial \mathbf{v}_j} &= 2 \cdot \alpha V_{i,j}^{-1} \\
\frac{\partial \kappa \chi_t^2}{\partial \kappa \mathbf{t}_i} &= 2 \cdot \sum_{j=1}^5 \kappa T_{i,j}^{-1} \cdot (\kappa \mathbf{t}_j - \kappa \mathbf{t}_j) \\
\frac{\partial^2 \kappa \chi_t^2}{\partial \kappa \mathbf{t}_i \partial \kappa \mathbf{t}_j} &= 2 \cdot \kappa T_{i,j}^{-1}
\end{aligned} \tag{3.7}$$

First and second derivatives with respect to the independent parameters

$$\begin{aligned}
\lambda &= (\lambda_1, \lambda_2, \dots, \lambda_{n_f}) \\
&\equiv (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, {}^1_h \mathbf{q}_1, {}^1_h \mathbf{q}_2, {}^1_h \mathbf{q}_3, {}^2_h \mathbf{q}_1, \dots, {}^{n_h}_h \mathbf{q}_3, {}^1_n \mathbf{q}_1, \dots, {}^n_n \mathbf{q}_3) \\
&\equiv (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, {}^1_l \mathbf{q}_1, {}^1_l \mathbf{q}_2, {}^1_l \mathbf{q}_3, {}^2_l \mathbf{q}_1, \dots, {}^{n_t}_l \mathbf{q}_3)
\end{aligned} \tag{3.8}$$

are obtained from these equations (3.8) and the derivatives (3.6 and 3.7) of the standard track parameters with respect to the independent parameters:

$$\begin{aligned}
\frac{\partial \chi^2}{\partial \mathbf{v}_k} &= \sum_{\alpha=1}^{n_v} \frac{\partial \alpha \chi_v^2}{\partial \mathbf{v}_k} + \sum_{\kappa=1}^{n_t} \sum_{i=1}^5 \frac{\partial \kappa \chi_t^2}{\partial \kappa \mathbf{t}_i} \frac{\partial \kappa \mathbf{t}_i}{\partial \mathbf{v}_k} \\
&= 2 \sum_{\alpha=1}^{n_v} \sum_{j=1}^3 V_{k,j}^{-1} \cdot (\mathbf{v}_j - \alpha \mathbf{v}_j) + 2 \sum_{\kappa=1}^{n_t} \sum_{i=1}^5 \sum_{j=1}^5 T_{i,j}^{-1} \cdot (\kappa \mathbf{t}_j - \kappa \mathbf{t}_j) \frac{\partial \kappa \mathbf{t}_i}{\partial \mathbf{v}_k} \\
\frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l} &= \sum_{\alpha=1}^{n_v} \frac{\partial^2 \alpha \chi_v^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l} + \sum_{\kappa=1}^{n_t} \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa \mathbf{t}_i}{\partial \mathbf{v}_k} \frac{\partial^2 \kappa \chi_t^2}{\partial \kappa \mathbf{t}_i \partial \kappa \mathbf{t}_j} \frac{\partial \kappa \mathbf{t}_j}{\partial \mathbf{v}_l}
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
&= 2 \sum_{\alpha=1}^{n_v} V_{k,l}^{-1} + 2 \sum_{\kappa=1}^{n_t} \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \mathbf{v}_k} \kappa_{T_{i,j}}^{-1} \frac{\partial \kappa_{\mathbf{t}_j}}{\partial \mathbf{v}_l} \\
\frac{\partial \chi^2}{\partial \kappa_{\mathbf{q}_k}} &= \sum_{i=1}^5 \frac{\partial \kappa_{\chi_l^2}}{\partial \kappa_{\mathbf{t}_i}} \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \kappa_{\mathbf{q}_k}} \\
&= \sum_{i=1}^5 \sum_{j=1}^5 \kappa_{T_{i,j}}^{-1} \cdot (\kappa_{\mathbf{t}_j} - \kappa_{\mathbf{t}_j}) \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \kappa_{\mathbf{q}_k}} \\
\frac{\partial^2 \chi^2}{\partial \kappa_{\mathbf{q}_k} \partial \kappa_{\mathbf{q}_l}} &= \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \kappa_{\mathbf{q}_k}} \frac{\partial^2 \kappa_{\chi_l^2}}{\partial \kappa_{\mathbf{t}_i} \partial \kappa_{\mathbf{t}_j}} \frac{\partial \kappa_{\mathbf{t}_j}}{\partial \kappa_{\mathbf{q}_l}} \\
&= 2 \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \kappa_{\mathbf{q}_k}} \kappa_{T_{i,j}}^{-1} \frac{\partial \kappa_{\mathbf{t}_j}}{\partial \kappa_{\mathbf{q}_l}} \\
\frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \kappa_{\mathbf{q}_l}} &= \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \mathbf{v}_k} \frac{\partial^2 \kappa_{\chi_l^2}}{\partial \kappa_{\mathbf{t}_i} \partial \kappa_{\mathbf{t}_j}} \frac{\partial \kappa_{\mathbf{t}_j}}{\partial \kappa_{\mathbf{q}_l}} \\
&= 2 \sum_{i=1}^5 \sum_{j=1}^5 \frac{\partial \kappa_{\mathbf{t}_i}}{\partial \mathbf{v}_k} \kappa_{T_{i,j}}^{-1} \frac{\partial \kappa_{\mathbf{t}_j}}{\partial \kappa_{\mathbf{q}_l}}
\end{aligned}$$

It may be worth mentioning that in calculating the second derivatives of χ^2 terms of the form $(\kappa_{\mathbf{t}_i} - \kappa_{\mathbf{t}_j}) \cdot \kappa_{T_{i,j}}^{-1} \cdot \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l}$ i.e. those containing second derivatives of the fitted track parameters with respect to the independent parameters have been neglected. This is justified for a good fit where the measured and fitted track parameters are rather close to each other. It is also advisable in case of bad vertex candidates since inclusion of these terms may lead to non-positiveness of matrices and corresponding breakdown of the iterative fitting procedure. This may also be the case for good track combinations if the initial vertex is chosen far off the true value.

First and second derivatives with respect to the independent fit parameters can be written in matrix form in the same order as the free parameters λ . They have then the structure

$$g = \left(\frac{\partial \chi^2}{\partial \lambda_1}, \dots, \frac{\partial \chi^2}{\partial \lambda_{n_f}} \right) \equiv \left(\frac{\partial \chi^2}{\partial \mathbf{v}_1}, \frac{\partial \chi^2}{\partial \mathbf{v}_2}, \frac{\partial \chi^2}{\partial \mathbf{v}_3}, \frac{\partial \chi^2}{\partial \mathbf{q}_1}, \dots, \frac{\partial \chi^2}{\partial \kappa_{\mathbf{q}_k}}, \dots, \frac{\partial \chi^2}{\partial \kappa_{\mathbf{q}_3}} \right) \quad (3.10)$$

$$G = \begin{bmatrix} \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l} & \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \mathbf{q}_l} & \dots & \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \kappa_{\mathbf{q}_l}} \\ \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \mathbf{q}_l} & \frac{\partial^2 \chi^2}{\partial \mathbf{q}_k \partial \mathbf{q}_l} & 0 & 0 \\ \dots & 0 & \dots & 0 \\ \frac{\partial^2 \chi^2}{\partial \mathbf{v}_k \partial \kappa_{\mathbf{q}_l}} & 0 & 0 & \frac{\partial^2 \chi^2}{\partial \kappa_{\mathbf{q}_k} \partial \kappa_{\mathbf{q}_l}} \end{bmatrix}$$

i.e. cross terms between tracks and vertex but not between different tracks.

The deviation of the independent free parameters from the χ^2 minimum is estimated as

$$\delta\lambda = G^{-1} \cdot g \quad (3.11)$$

$\Lambda \equiv G^{-1}$ gives also the error covariance matrix of the n_f fit parameters λ . It may be used to extract the estimated errors of the vertex as well as the standard helix and neutral track parameters, respectively their correlations. For the vertex this is especially simple as it has been chosen as the first three independent parameters. Correspondingly one has

$$V_{k,l} = \langle \Delta \mathbf{v}_k \Delta \mathbf{v}_l \rangle = G_{k,l}^{-1} \quad k, l = 1, 2, 3 \quad (3.12)$$

Formally this may also be written in matrix notation as

$$V = F_v^T G^{-1} F_v \quad \text{with} \quad F_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \end{pmatrix} \quad (3.13)$$

The standard helix and neutral track parameters may be recalculated with the formulas given previously in eq. (3.1) and eq. (3.2). The errors of the track parameters are obtained with the relations

$$\Delta \kappa_{\mathbf{t}_k} = \sum_{i=1}^3 \frac{\partial \kappa_{\mathbf{t}_k}}{\partial \kappa_{\mathbf{q}_i}} \Delta \kappa_{\mathbf{q}_i} + \sum_{i=1}^3 \frac{\partial \kappa_{\mathbf{t}_k}}{\partial \mathbf{v}_i} \Delta \mathbf{v}_i \quad (3.14)$$

Correspondingly one finds the error covariance matrix for fitted helices and neutral tracks as

$$\begin{aligned} \kappa_{\mathbf{T}_{k,l}} = \langle \Delta \kappa_{\mathbf{t}_k} \Delta \kappa_{\mathbf{t}_l} \rangle = & \sum_{i=1}^3 \sum_{j=1}^3 \left[\frac{\partial \kappa_{\mathbf{t}_k}}{\partial \kappa_{\mathbf{q}_i}} G_{m+i,m+j}^{-1} \frac{\partial \kappa_{\mathbf{t}_l}}{\partial \kappa_{\mathbf{q}_j}} + \right. \\ & \left. \frac{\partial \kappa_{\mathbf{t}_k}}{\partial \kappa_{\mathbf{q}_i}} G_{m+i,j}^{-1} \frac{\partial \kappa_{\mathbf{t}_l}}{\partial \mathbf{v}_j} + \frac{\partial \kappa_{\mathbf{t}_k}}{\partial \mathbf{v}_i} G_{i,m+j}^{-1} \frac{\partial \kappa_{\mathbf{t}_l}}{\partial \kappa_{\mathbf{q}_j}} + \frac{\partial \kappa_{\mathbf{t}_k}}{\partial \mathbf{v}_i} G_{i,j}^{-1} \frac{\partial \kappa_{\mathbf{t}_l}}{\partial \mathbf{v}_j} \right] \end{aligned} \quad (3.15)$$

$$\text{with } m = 3 + (\kappa - 1) \cdot 3 = 3 \cdot \kappa$$

Similar expressions can easily be written down for error correlations between track parameters and vertex coordinates.

Besides vertex and refitted track parameters one is usually interested in the mass and the track parameters of the incoming decaying particle. This question will be taken up again in chapter 5 after an excursion to an alternative fast fitting procedure described in the following chapter.

4. Fast vertex fit without refit of tracks

For large numbers of tracks the vertex fitting method described in the previous chapter is rather computer time consuming. It provides refitted parameters of all tracks, quantities which very often are of little interest, especially when one only tries to find out which tracks can be combined to originate from a common vertex.

Therefore an alternative fast method has been developed to find the vertex only but leave the track parameters unchanged. There one tries to find a point whose weighted distance sum from the measured tracks is minimal. The weighting is done in such a way as to lead for the vertex to identical results as the elaborate procedure described in the previous chapter.

In the following we first find the distance of a point from a helix or a neutral track before writing down the χ^2 to be minimized for finding the vertex.

4.1. DISTANCE OF A POINT FROM A HELIX

Fig. 5: Distance of point (A,B,C) from a helix

The following important quantities may be calculated from the helix parameters and a point (A,B,C) as can be read off Fig. 5:

- x_0 ; y_0 coordinates of closest approach in projection to the origin and x_c ; y_c center of projected circle

$$\begin{aligned} x_0 &= D_0 \cdot \sin \Phi_0 & y_0 &= -D_0 \cdot \cos \Phi_0 \\ x_c &= x_0 - r \cdot \sin \Phi_0 & y_c &= y_0 + r \cdot \cos \Phi_0 \end{aligned} \quad (4.1)$$

- distance in $r - \phi$ projection d_r and in z -direction d_z of point (A,B,C) from helix, corresponding point at helix and several related quantities.

$$\begin{aligned}
l &= \text{sign}(r) \cdot \sqrt{(A - x_c)^2 + (B - y_c)^2} \\
\sin\Phi &= \frac{A - x_c}{l} & \cos\Phi &= -\frac{B - y_c}{l} \\
x &= x_c + (A - x_c) \cdot \frac{r}{l} & y &= y_c + (B - y_c) \cdot \frac{r}{l} \\
s &= (\Phi - \Phi_0) \cdot r & z &= z_0 + s \cdot \tau
\end{aligned} \tag{4.2}$$

$$d_r = (l - r) \quad d_z = C - z_0 - s \cdot \tau \tag{4.3}$$

The following derivatives are later on of interest:

$$\begin{aligned}
\frac{\partial d_r}{\partial A} &= \sin\Phi & \frac{\partial d_r}{\partial B} &= -\cos\Phi & \frac{\partial d_r}{\partial C} &= 0 \\
\frac{\partial d_z}{\partial A} &= -\tau \cdot \cos\Phi \cdot \frac{r}{l} & \frac{\partial d_z}{\partial B} &= -\tau \cdot \sin\Phi \cdot \frac{r}{l} & \frac{\partial d_z}{\partial C} &= 1 \\
\frac{\partial l}{\partial A} &= \sin\Phi & \frac{\partial l}{\partial B} &= -\cos\Phi \\
\frac{\partial l}{\partial x_c} &= -\sin\Phi & \frac{\partial l}{\partial y_c} &= \cos\Phi \\
\frac{\partial d_r}{\partial r} &= \frac{\partial l}{\partial x_c} \cdot \frac{\partial x_c}{\partial r} + \frac{\partial l}{\partial y_c} \cdot \frac{\partial y_c}{\partial r} - 1 \\
&= \sin\Phi \cdot \sin\Phi_0 + \cos\Phi \cdot \cos\Phi_0 - 1 = \cos(\Phi - \Phi_0) - 1 \\
\frac{\partial d_r}{\partial \Phi_0} &= \frac{\partial l}{\partial x_c} \cdot \frac{\partial x_c}{\partial \Phi_0} + \frac{\partial l}{\partial y_c} \cdot \frac{\partial y_c}{\partial \Phi_0} = (r - D_0) \cdot \sin(\Phi - \Phi_0) \\
\frac{\partial d_r}{\partial D_0} &= \frac{\partial l}{\partial x_c} \cdot \frac{\partial x_c}{\partial D_0} + \frac{\partial l}{\partial y_c} \cdot \frac{\partial y_c}{\partial D_0} = -\cos(\Phi - \Phi_0) \\
\frac{\partial d_r}{\partial \tau} &= 0 & \frac{\partial d_r}{\partial z_0} &= 0 \\
\frac{\partial d_z}{\partial \tau} &= -s & \frac{\partial d_z}{\partial z_0} &= -1 \\
\frac{\partial d_z}{\partial r} &= 0 & \frac{\partial d_z}{\partial \Phi_0} &= 0 & \frac{\partial d_z}{\partial D_0} &= 0
\end{aligned} \tag{4.4}$$

4.2. DISTANCE OF A POINT FROM A NEUTRAL TRACK

The following important quantities may be calculated from the neutral track parameters and a point (A,B,C):

- x_0 ; y_0 coordinates of closest approach in projection to the origin

$$x_0 = D_0 \cdot \sin\Phi_0 \quad y_0 = -D_0 \cdot \cos\Phi_0 \tag{4.5}$$

- distance in $r - \phi$ projection δ_r and in z-direction δ_z of point (A,B,C) from neutral track and corresponding point at track and several related quantities.

Fig. 6: Distance of point (A, B, C) from neutral track

$$\begin{aligned} s &= A \cdot \cos \Phi_0 + B \cdot \sin \Phi_0 \\ x &= x_0 + s \cdot \cos \Phi_0 \quad y = y_0 + s \cdot \sin \Phi_0 \quad z = z_0 + s \cdot \tau \end{aligned} \quad (4.6)$$

$$d_r = A \cdot \sin \Phi_0 - B \cdot \cos \Phi_0 - D_0 \quad d_z = C - z_0 - s \cdot \tau \quad (4.7)$$

The following derivatives are later on of interest:

$$\begin{aligned} \frac{\partial d_r}{\partial A} &= \sin \Phi_0 & \frac{\partial d_r}{\partial B} &= -\cos \Phi_0 & \frac{\partial d_r}{\partial C} &= 0 \\ \frac{\partial d_z}{\partial A} &= -\tau \cdot \cos \Phi_0 & \frac{\partial d_z}{\partial B} &= -\tau \cdot \sin \Phi_0 & \frac{\partial d_z}{\partial C} &= 1 \\ \frac{\partial d_r}{\partial \Phi_0} &= A \cos \Phi_0 + B \sin \Phi_0 = s \\ \frac{\partial d_r}{\partial D_0} &= -1 \\ \frac{\partial d_r}{\partial \tau} &= 0 & \frac{\partial d_r}{\partial z_0} &= 0 \\ \frac{\partial d_z}{\partial \tau} &= -s & \frac{\partial d_z}{\partial z_0} &= -1 \\ \frac{\partial d_z}{\partial r} &= 0 & \frac{\partial d_z}{\partial \Phi_0} &= 0 & \frac{\partial d_z}{\partial D_0} &= 0 \end{aligned} \quad (4.8)$$

4.3. "TRANSVERSE" ERROR MATRIX

As mentioned previously we attempt to fit a vertex by taking the distance from the tracks as a measure. The transverse distance in $r - \Phi$ projection and the corresponding difference in z between the vertex and the "closest" (in projection) point on the track will be taken as the relevant quantities. We therefore have the task of finding the 2×2 error matrix for these quantities from the 5×5 error covariance matrix of the tracks. This matrix of course is dependent on the position along the track.

The 2×2 error matrix D may be calculated from the complete 5×5 helix error matrix H by

$$D = A^T \cdot H \cdot A \quad (4.9)$$

with A the 2×5 matrix for the helix

$$\begin{aligned} A_{1,1} &= \frac{\partial d_r}{\partial \rho} = -\frac{1}{\rho^2} \cdot [\cos(\Phi - \Phi_0) - 1] \\ A_{1,3} &= \frac{\partial d_r}{\partial \Phi_0} = (r - D_0) \cdot \sin(\Phi - \Phi_0) \\ A_{1,4} &= \frac{\partial d_r}{\partial D_0} = -\cos(\Phi - \Phi_0) \\ A_{2,2} &= \frac{\partial d_z}{\partial \tau} = -s \\ A_{2,5} &= \frac{\partial d_z}{\partial z_0} = -1 \end{aligned} \quad (4.10)$$

all other terms being zero.

Similarly one obtains for neutral tracks

$$D = A^T \cdot N \cdot A \quad (4.11)$$

with A the 2×5 matrix

$$\begin{aligned} A_{1,1} &= \frac{\partial d_r}{\partial p} = 0 \\ A_{1,3} &= \frac{\partial d_r}{\partial \Phi_0} = s \\ A_{1,4} &= \frac{\partial d_r}{\partial D_0} = -1 \\ A_{2,2} &= \frac{\partial d_z}{\partial \tau} = -s \\ A_{2,5} &= \frac{\partial d_z}{\partial z_0} = -1 \end{aligned} \quad (4.12)$$

all other terms being zero.

4.4. VERTEX FIT

The vertex coordinates may be found by the standard procedure of minimizing a χ^2 function which we write down as a function of three terms representing the input vertices, the helices and neutral tracks.

$$\chi^2 = \sum_{\alpha=1}^{n_v} \alpha \chi_v^2 + \sum_{\beta=1}^{n_h} \beta \chi_h^2 + \sum_{\gamma=1}^{n_n} \gamma \chi_n^2 \equiv \sum_{\alpha=1}^{n_v} \alpha \chi_v^2 + \sum_{\kappa=1}^{n_t} \kappa \chi_t^2 \quad (4.13)$$

The splitting up of the total χ^2 in this form is possible due to the absence of error correlations between different tracks.

Explicitly written in matrix form the individual terms of χ^2 have the following form:

$$\begin{aligned} \alpha \chi_v^2 &= (\mathbf{v} - \alpha_v)^T \cdot \alpha V^{-1} \cdot (\mathbf{v} - \alpha_v) \\ \beta \chi_h^2 &= \beta d_h^T \cdot \beta D^{-1} \cdot \beta d_h \\ \gamma \chi_n^2 &= \gamma d_n^T \cdot \gamma D^{-1} \cdot \gamma d_n \\ \kappa \chi_t^2 &= \kappa d_t^T \cdot \kappa D^{-1} \cdot \kappa d_t \end{aligned} \quad (4.14)$$

Following the standard procedure for minimizing the expression for χ^2 with respect to the vertex coordinates $v = (A, B, C)$ we calculate first and second derivatives of χ^2 with respect to these three parameters:

$$\begin{aligned} g \quad ; \quad g_k &= \frac{\partial \chi^2}{\partial v_k} = \sum_{\alpha=1}^{n_v} \frac{\partial \alpha \chi_v^2}{\partial \mathbf{v}_k} + \sum_{\beta=1}^{n_h} \sum_{i=1}^2 \frac{\partial \beta \chi_h^2}{\partial \beta d_i} \frac{\partial \beta d_i}{\partial \mathbf{v}_k} + \sum_{\gamma=1}^{n_n} \sum_{i=1}^2 \frac{\partial \gamma \chi_n^2}{\partial \gamma d_i} \frac{\partial \gamma d_i}{\partial \mathbf{v}_k} \\ &\equiv \sum_{\alpha=1}^{n_v} \frac{\partial \alpha \chi_v^2}{\partial \mathbf{v}_k} + \sum_{\kappa=1}^{n_t} \sum_{i=1}^2 \frac{\partial \kappa \chi_t^2}{\partial \kappa d_i} \frac{\partial \kappa d_i}{\partial \mathbf{v}_k} \\ G \quad ; \quad G_{k,l} &= \frac{\partial^2 \chi^2}{\partial v_k \partial v_l} = \sum_{\alpha=1}^{n_v} \frac{\partial^2 \alpha \chi_v^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l} + \\ &\quad \sum_{\beta=1}^{n_h} \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial^2 \beta \chi_h^2}{\partial \beta d_i \partial \beta d_j} \frac{\partial \beta d_i}{\partial \mathbf{v}_k} \frac{\partial \beta d_j}{\partial \mathbf{v}_l} + \sum_{\gamma=1}^{n_n} \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial^2 \gamma \chi_n^2}{\partial \gamma d_i \partial \gamma d_j} \frac{\partial \gamma d_i}{\partial \mathbf{v}_k} \frac{\partial \gamma d_j}{\partial \mathbf{v}_l} \\ &\equiv \sum_{\alpha=1}^{n_v} \frac{\partial^2 \alpha \chi_v^2}{\partial \mathbf{v}_k \partial \mathbf{v}_l} + \sum_{\kappa=1}^{n_t} \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial^2 \kappa \chi_t^2}{\partial \kappa d_i \partial \kappa d_j} \frac{\partial \kappa d_i}{\partial \mathbf{v}_k} \frac{\partial \kappa d_j}{\partial \mathbf{v}_l} \end{aligned} \quad (4.15)$$

The expressions for g and G may also be written in matrix form as

$$\begin{aligned} g &= 2 \left[\sum_{\alpha=1}^{n_v} \alpha V^{-1} (\mathbf{v} - \alpha_v) + \sum_{\kappa=1}^{n_t} \kappa_B^T \kappa_D^{-1} \kappa (\kappa_t - \kappa_t) \right] \\ G &= 2 \left[\sum_{\alpha=1}^{n_v} \alpha V^{-1} + \sum_{\kappa=1}^{n_t} \kappa_B^T \kappa_D^{-1} \kappa_B \right] \end{aligned} \quad (4.16)$$

with

$$\begin{aligned}
B &= \begin{pmatrix} \frac{\partial d_r}{\partial v_1} & \frac{\partial d_r}{\partial v_2} & \frac{\partial d_r}{\partial v_3} \\ \frac{\partial d_z}{\partial v_1} & \frac{\partial d_z}{\partial v_2} & \frac{\partial d_z}{\partial v_3} \end{pmatrix} \\
&= \begin{pmatrix} \sin\Phi & -\cos\Phi & 0 \\ -\frac{R}{L}t\cos\Phi & -\frac{R}{L}t\sin\Phi & 1 \end{pmatrix} \quad \text{for helices} \\
&= \begin{pmatrix} \sin\Phi_0 & -\cos\Phi_0 & 0 \\ -t\cos\Phi_0 & -t\sin\Phi_0 & 1 \end{pmatrix} \quad \text{for neutral tracks}
\end{aligned} \tag{4.17}$$

as is found with the definition of d (eq. (4.3)) for helices and equation eq. (4.7) for neutral tracks. The displacement of the assumed vertex from the position of the χ^2 minimum is obtained by the relation

$$\Delta v = G^{-1} \cdot g \tag{4.18}$$

It is used for calculation of the next iteration of the vertex

$$v_{i+1} = v_i - \Delta v \tag{4.19}$$

The error covariance matrix for the fitted vertex is given by

$$V = \frac{1}{2} \cdot G^{-1} \tag{4.20}$$

4.5. FINDING OF THE VERTEX STARTING VALUE

In order to save computer time it is advisable to have a starting value not too far off the true vertex. Being close to the true value is even more important in some special cases when one has more than one local minimum in the χ^2 distribution and the result of the iterative procedure depends on the starting value. An obvious example for this situation is the vertex of two helices with similar vertical slope τ . In this case the two crossing points of the projected circles are determined and the one giving smaller separation in z is selected.

This selection procedure has been generalized for n_t tracks by applying the same procedure to several pairs of tracks out of the sample and requiring consistency of the results.

The method of obtaining the vertex starting value is also available for the general vertex fit described in chapter 3. It is however advisable to use the result of the fast fit as input for the complete fit.

Fig. 7: Relation between helix parameters and particle momentum

5. Momenta and invariant mass at vertex

Having in chapter 3 performed a fit of charged and neutral tracks to a common vertex one may be interested in the invariant mass of the system. In order to do get this information one has first to determine the momenta of all particles at the vertex. For doing proper error treatment one furthermore needs the derivatives with respect to vertex coordinates and independent track parameters.

5.1. MOMENTA OF REFITTED TRACKS

The relation between helix parameters vertex and particle momentum can be read from Fig. 7:

$$\begin{aligned} p_T &= K/\rho = K \cdot r & p_z &= p_T \tau = K r \tau \\ p_x &= p_T \cos \Phi = K r \cos \Phi & p_y &= p_T \sin \Phi = K r \sin \Phi \end{aligned} \quad (5.1)$$

where K is the proportionality constant relating momentum and radius of the helix $\mathbf{r}$. It is negative for positive charge and magnetic field aligned along the positive z -direction. The radius r and angle Φ are functions of the vertex coordinates $\mathbf{v} = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ and independent helix parameters $_{h}\mathbf{q} = (\eta, \tau, \Phi_0)$ as given in equations (3.1)

The derivatives with respect to vertex coordinates and independent helix parameters follow from simple algebra with the help of equations eq. (3.1). Only the non zero terms are listed below:

$$\begin{aligned} \frac{\partial p_x}{\partial v_2} &= K & \frac{\partial p_x}{\partial _{h}\mathbf{q}_1} &= \frac{\partial p_x}{\partial \eta} = K \cos \Phi_0 \\ \frac{\partial p_x}{\partial _{h}\mathbf{q}_3} &= \frac{\partial p_x}{\partial \Phi_0} = -K \eta \sin \Phi_0 \end{aligned} \quad (5.2)$$

$$\begin{aligned}
\frac{\partial \mathbf{p}_y}{\partial \mathbf{v}_1} &= K \\
\frac{\partial \mathbf{p}_y}{\partial h\mathbf{q}_1} &= \frac{\partial \mathbf{p}_y}{\partial \eta} = K \sin \Phi_0 & \frac{\partial \mathbf{p}_y}{\partial h\mathbf{q}_3} &= \frac{\partial \mathbf{p}_y}{\partial \Phi_0} = K \eta \cos \Phi_0 \\
\frac{\partial \mathbf{p}_z}{\partial \mathbf{v}_1} &= K \tau \sin \Phi & \frac{\partial \mathbf{p}_z}{\partial \mathbf{v}_2} &= -K \tau \cos \Phi \\
\frac{\partial \mathbf{p}_z}{\partial h\mathbf{q}_1} &= \frac{\partial \mathbf{p}_z}{\partial \eta} = K \tau \cos(\Phi - \Phi_0) & \frac{\partial \mathbf{p}_z}{\partial h\mathbf{q}_2} &= \frac{\partial \mathbf{p}_z}{\partial \tau} = K \mathbf{r} = \mathbf{p}_T \\
\frac{\partial \mathbf{p}_z}{\partial h\mathbf{q}_3} &= \frac{\partial \mathbf{p}_z}{\partial \Phi_0} = K \eta \sin(\Phi - \Phi_0)
\end{aligned}$$

The equivalent relations for the neutral tracks obtained directly from the independent track parameters $n\mathbf{q} = (\mathbf{p}, \tau, \Phi_0)$ are given below:

$$\begin{aligned}
\mathbf{p}_T &= \frac{\mathbf{p}}{\sqrt{1 + \tau^2}} & \mathbf{p}_z &= \mathbf{p}_T \tau \\
\mathbf{p}_x &= \mathbf{p}_T \cos \Phi_0 & \mathbf{p}_y &= \mathbf{p}_T \sin \Phi_0
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
\frac{\partial \mathbf{p}_x}{\partial n\mathbf{q}_1} &= \frac{\partial \mathbf{p}_x}{\partial \mathbf{p}} = \frac{\mathbf{p}_x}{\mathbf{p}} & \frac{\partial \mathbf{p}_x}{\partial n\mathbf{q}_2} &= \frac{\partial \mathbf{p}_x}{\partial \tau} = \mathbf{p}_x \frac{\tau}{1 + \tau^2} \\
\frac{\partial \mathbf{p}_x}{\partial n\mathbf{q}_3} &= \frac{\partial \mathbf{p}_x}{\partial \Phi_0} = -\mathbf{p}_y & & \\
\frac{\partial \mathbf{p}_y}{\partial n\mathbf{q}_1} &= \frac{\partial \mathbf{p}_y}{\partial \mathbf{p}} = \frac{\mathbf{p}_y}{\mathbf{p}} & \frac{\partial \mathbf{p}_y}{\partial n\mathbf{q}_2} &= \frac{\partial \mathbf{p}_y}{\partial \tau} = -\mathbf{p}_y \frac{\tau}{1 + \tau^2} \\
\frac{\partial \mathbf{p}_y}{\partial n\mathbf{q}_3} &= \frac{\partial \mathbf{p}_y}{\partial \Phi_0} = \mathbf{p}_x & & \\
\frac{\partial \mathbf{p}_z}{\partial n\mathbf{q}_1} &= \frac{\partial \mathbf{p}_z}{\partial \mathbf{p}} = \frac{\mathbf{p}_z}{\mathbf{p}} & \frac{\partial \mathbf{p}_z}{\partial n\mathbf{q}_2} &= \frac{\partial \mathbf{p}_z}{\partial \tau} = \frac{\mathbf{p}_T}{1 + \tau^2}
\end{aligned} \tag{5.4}$$

5.2. MOMENTUM SUM

In order to calculate the invariant mass of the tracks forming the (decay) vertex we will attempt to calculate the four momentum sum vector, respectively three momentum sum vector $\vec{\mathbf{p}}^s = (\mathbf{p}^s_x, \mathbf{p}^s_y, \mathbf{p}^s_z)$ and energy $\mathbf{E}^s$

$$\begin{aligned}
\vec{\mathbf{p}}^s &= \sum_{\beta=1}^{n_h} \beta \vec{\mathbf{p}}_h + \sum_{\gamma=1}^{n_n} \gamma \vec{\mathbf{p}}_n \equiv \sum_{\kappa=1}^{n_t} \kappa \vec{\mathbf{p}}_t \\
\mathbf{E}^s &= \sum_{\beta=1}^{n_h} \beta \mathbf{E}_h + \sum_{\gamma=1}^{n_n} \gamma \mathbf{E}_n \equiv \sum_{\kappa=1}^{n_t} \kappa \mathbf{E}_t
\end{aligned} \tag{5.5}$$

Forgetting for the moment the distinction between neutral and charged tracks, which only enter through the different relation between dependent and independent track

parameters one may write down the same relation in component form

$$\mathbf{p}^s_i = \sum_{\kappa=1}^{n_t} \kappa \mathbf{p}_i \quad (5.6)$$

(i=1,2,3 representing the x,y and z components) and obtains for the corresponding error

$$\Delta \mathbf{p}^s_i = \sum_{k=1}^3 \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_i}{\partial \mathbf{v}_k} \Delta \mathbf{v}_k + \sum_{\kappa=1}^{n_t} \sum_{k=1}^3 \frac{\partial \kappa \mathbf{p}_i}{\partial \kappa \mathbf{q}_k} \Delta \kappa \mathbf{q}_k \quad (5.7)$$

As in chapter 3 we define the vector representing the independent fit parameters as

$$\begin{aligned} \lambda &= (\lambda_1, \lambda_2, \dots, \lambda_{n_f}) \\ &\equiv (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, {}^1_h \mathbf{q}_1, {}^1_h \mathbf{q}_2, {}^1_h \mathbf{q}_3, {}^2_h \mathbf{q}_1, \dots, {}^{n_h}_h \mathbf{q}_3, {}^1_n \mathbf{q}_1, \dots, {}^{n_n}_n \mathbf{q}_3) \\ &\equiv (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, {}^1 \mathbf{q}_1, {}^1 \mathbf{q}_2, {}^1 \mathbf{q}_3, {}^2 \mathbf{q}_1, \dots, {}^{n_t} \mathbf{q}_3) \end{aligned} \quad (5.8)$$

The equation relating the momentum sum error with the errors on the independent track parameters λ may then be written in matrix representation as

$$\Delta \vec{\mathbf{p}}^s = \Delta \lambda F_p \quad (5.9)$$

with

$$F_p = \begin{pmatrix} \frac{\partial \mathbf{p}^s_1}{\partial \lambda_1} & \frac{\partial \mathbf{p}^s_2}{\partial \lambda_1} & \frac{\partial \mathbf{p}^s_3}{\partial \lambda_1} \\ \frac{\partial \mathbf{p}^s_1}{\partial \lambda_{n_f}} & \frac{\partial \mathbf{p}^s_2}{\partial \lambda_{n_f}} & \frac{\partial \mathbf{p}^s_3}{\partial \lambda_{n_f}} \end{pmatrix} \equiv \begin{pmatrix} \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_1}{\partial \mathbf{v}_1} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_2}{\partial \mathbf{v}_1} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_3}{\partial \mathbf{v}_1} \\ \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_1}{\partial \mathbf{v}_2} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_2}{\partial \mathbf{v}_2} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_3}{\partial \mathbf{v}_2} \\ \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_1}{\partial \mathbf{v}_3} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_2}{\partial \mathbf{v}_3} & \sum_{\kappa=1}^{n_t} \frac{\partial \kappa \mathbf{p}_3}{\partial \mathbf{v}_3} \\ \frac{\partial {}^1 \mathbf{p}_1}{\partial {}^1 \mathbf{q}_1} & \frac{\partial {}^1 \mathbf{p}_2}{\partial {}^1 \mathbf{q}_1} & \frac{\partial {}^1 \mathbf{p}_3}{\partial {}^1 \mathbf{q}_1} \\ \frac{\partial {}^1 \mathbf{p}_1}{\partial {}^1 \mathbf{q}_2} & \frac{\partial {}^1 \mathbf{p}_2}{\partial {}^1 \mathbf{q}_2} & \frac{\partial {}^1 \mathbf{p}_3}{\partial {}^1 \mathbf{q}_2} \\ \frac{\partial {}^1 \mathbf{p}_1}{\partial {}^1 \mathbf{q}_3} & \frac{\partial {}^1 \mathbf{p}_2}{\partial {}^1 \mathbf{q}_3} & \frac{\partial {}^1 \mathbf{p}_3}{\partial {}^1 \mathbf{q}_3} \\ \dots & \dots & \dots \\ \frac{\partial \kappa \mathbf{p}_1}{\partial \kappa \mathbf{q}_1} & \frac{\partial \kappa \mathbf{p}_2}{\partial \kappa \mathbf{q}_1} & \frac{\partial \kappa \mathbf{p}_3}{\partial \kappa \mathbf{q}_1} \\ \dots & \dots & \dots \\ \frac{\partial {}^{n_t} \mathbf{p}_1}{\partial {}^{n_t} \mathbf{q}_3} & \frac{\partial {}^{n_t} \mathbf{p}_2}{\partial {}^{n_t} \mathbf{q}_3} & \frac{\partial {}^{n_t} \mathbf{p}_3}{\partial {}^{n_t} \mathbf{q}_3} \end{pmatrix} \quad (5.10)$$

The error covariance matrix for the momentum sum thus may be written as

$$P^S = F_p^T \cdot G^{-1} \cdot F_p \quad \text{with} \quad P_{i,k}^S = \langle \Delta \mathbf{p}_i^S \Delta \mathbf{p}_k^S \rangle \quad i, k = 1, 2, 3 \quad (5.11)$$

the error covariance between vertex position and momentum sum similarly may be written as

$$C_{vp} = F_v^T \cdot G^{-1} \cdot F_p \quad (5.12)$$

with F_v and F_p defined in eqs. (3.13 and xV10) . Writing out this equation in individual terms one finds

$$\begin{aligned} (C_{vp})_{i,j} &= \langle \Delta \mathbf{v}_i \Delta \mathbf{p}^s_j \rangle \\ &= \sum_{k=1}^3 \sum_{\kappa=1}^{n_t} G_{ik}^{-1} \frac{\partial \kappa \mathbf{p}_j}{\partial \mathbf{v}_k} + \sum_{\kappa=1}^{n_t} \sum_{k=1}^3 G_{i,3\kappa+k}^{-1} \frac{\partial \kappa \mathbf{p}_j}{\partial \kappa \mathbf{q}_k} \\ &\equiv \sum_{k=1}^3 \sum_{\kappa=1}^{n_t} \langle \Delta v_i \Delta v_k \rangle \frac{\partial \kappa \mathbf{p}_j}{\partial \mathbf{v}_k} + \sum_{\kappa=1}^{n_t} \sum_{k=1}^3 \langle \Delta v_i \Delta \kappa \mathbf{q}_k \rangle \frac{\partial \kappa \mathbf{p}_j}{\partial \kappa \mathbf{q}_k} \end{aligned} \quad (5.13)$$

5.3. INVARIANT MASS

The invariant mass may be calculated from the momentum sum four vector

$$m^2 = \mathbf{E}^s{}^2 - \vec{\mathbf{p}}^s{}^2 = \mathbf{E}^s{}^2 - \sum_{i=1}^3 \mathbf{p}^s_i{}^2 \quad (5.14)$$

with

$$\mathbf{p}^s_i = \sum_{\kappa=1}^{n_t} \kappa \mathbf{p}_i \quad \mathbf{E}^s = \sum_{\kappa=1}^{n_t} \sqrt{\sum_{i=1}^3 \kappa \mathbf{p}_i{}^2 + \kappa m^2} \quad \frac{\partial \mathbf{E}^s}{\partial \kappa \mathbf{p}_i} = \frac{\kappa \mathbf{p}_i}{\kappa E_i} \quad (5.15)$$

The error on the mass is found with the derivatives with respect to the independent parameters:

$$\begin{aligned} \Delta m^2 &= 2m \Delta m = \sum_{\kappa=1}^{n_t} \sum_{i=1}^3 2 \left(\mathbf{E}^s \frac{\partial \mathbf{E}^s}{\partial \kappa \mathbf{p}_i} - \mathbf{p}^s_i \frac{\partial \mathbf{p}^s_i}{\partial \kappa \mathbf{p}_i} \right) \Delta \kappa \mathbf{p}_i \\ &= 2 \sum_{\kappa=1}^{n_t} \sum_{i=1}^3 \left(\frac{\mathbf{E}^s \kappa \mathbf{p}_i}{\kappa E_i} - \mathbf{p}^s_i \right) \sum_{k=1}^3 \left(\frac{\partial \kappa \mathbf{p}_i}{\partial \kappa \mathbf{v}_k} \Delta \kappa \mathbf{v}_k + \frac{\partial \kappa \mathbf{p}_i}{\partial \kappa \mathbf{q}_k} \Delta \kappa \mathbf{q}_k \right) \end{aligned} \quad (5.16)$$

Here we have made use of the fact that the standard track parameters $\kappa \mathbf{p}_i$ are only dependent on the vertex $\mathbf{v}_k$ and the proper three independent track parameters $\kappa \mathbf{q}_k$.

With the definition

$$F_m = \begin{pmatrix} \sum_{\kappa=1}^{n_t} \sum_{i=1}^3 \left(\mathbf{E}^s \frac{\kappa \mathbf{p}_i}{\kappa \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial \kappa \mathbf{p}_i}{\partial \mathbf{v}_1} \\ \sum_{\kappa=1}^{n_t} \sum_{i=1}^3 \left(\mathbf{E}^s \frac{\kappa \mathbf{p}_i}{\kappa \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial \kappa \mathbf{p}_i}{\partial \mathbf{v}_2} \\ \sum_{\kappa=1}^{n_t} \sum_{i=1}^3 \left(\mathbf{E}^s \frac{\kappa \mathbf{p}_i}{\kappa \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial \kappa \mathbf{p}_i}{\partial \mathbf{v}_3} \\ \sum_{i=1}^3 \left(\mathbf{E}^s \frac{1 \mathbf{p}_i}{1 \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial 1 \mathbf{p}_i}{\partial 1 \mathbf{q}_1} \\ \sum_{i=1}^3 \left(\mathbf{E}^s \frac{1 \mathbf{p}_i}{1 \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial 1 \mathbf{p}_i}{\partial 1 \mathbf{q}_2} \\ \dots\dots\dots \\ \dots\dots\dots \\ \sum_{i=1}^3 \left(\mathbf{E}^s \frac{n_t \mathbf{p}_i}{n_t \mathbf{E}} - \mathbf{p}^s_i \right) \frac{\partial n_t \mathbf{p}_i}{\partial n_t \mathbf{q}_3} \end{pmatrix} \quad (5.17)$$

one may write

$$\langle \Delta m^2 \rangle = \frac{1}{m^2} F_m^T G^{-1} F_m \quad (5.18)$$

Similarly one obtains for the correlation between vertex and mass

$$C_{v,m} = \frac{1}{m} F_v^T G^{-1} F_m \quad (5.19)$$

6. Track parameters from momentum sum and vertex

In the previous chapter the momentum sum from tracks originating from a common vertex has been found. Assuming that these particles stem from the decay of a long lived parent particle one may want to know its track parameters. This e.g. is the case for a K^0 where the incoming particle has a straight trajectory or for a D^+ where it is a helix.

6.1. NEUTRAL TRACKS

The neutral track parameters $(n_1, n_2, n_3, n_4, n_5) = (p, \tau, \phi_0, D_0, z_0)$ can be read directly from Fig. 8a:

$$\begin{aligned} p_T &= \sqrt{p_x^2 + p_y^2} & p &= \sqrt{p_T^2 + p_z^2} \\ \sin \phi_0 &= \frac{p_y}{p_T} & \cos \phi_0 &= \frac{p_x}{p_T} \\ D_0 &= v_x \sin \phi_0 - v_y \cos \phi_0 & s &= v_x \cos \phi_0 + v_y \sin \phi_0 \\ \tau &= \frac{p_z}{p_T} & z_0 &= v_z - s \tau \end{aligned} \quad (6.1)$$

In order to calculate the error covariance matrix of the track parameters we calculate

Fig. 8: Calculation of track parameters from particle momentum and vertex

the partial derivatives

$$\begin{aligned}
 \frac{\partial p}{\partial p_x} &= -\frac{p_x}{p} & \frac{\partial p}{\partial p_y} &= -\frac{p_y}{p} & \frac{\partial p}{\partial p_z} &= -\frac{p_z}{p} \\
 \frac{\partial \tau}{\partial p_x} &= -\frac{\tau \cos \phi_0}{p_T} & \frac{\partial \tau}{\partial p_y} &= -\frac{\tau \sin \phi_0}{p_T} & \frac{\partial \tau}{\partial p_z} &= \frac{1}{p_T} \\
 \frac{\partial \phi_0}{\partial p_x} &= -\frac{\sin \phi_0}{p_T} & \frac{\partial \phi_0}{\partial p_y} &= \frac{\cos \phi_0}{p_T} & \frac{\partial \phi_0}{\partial p_z} &= 0 \\
 \frac{\partial D_0}{\partial v_x} &= \sin \phi_0 & \frac{\partial D_0}{\partial v_y} &= -\cos \phi_0 & \frac{\partial D_0}{\partial v_z} &= 0 \\
 \frac{\partial D_0}{\partial p_x} &= s \frac{\partial \phi_0}{\partial p_x} & \frac{\partial D_0}{\partial p_y} &= s \frac{\partial \phi_0}{\partial p_y} & \frac{\partial D_0}{\partial p_z} &= 0 \\
 \frac{\partial z_0}{\partial v_x} &= -\tau \cos \phi_0 & \frac{\partial z_0}{\partial v_y} &= -\tau \sin \phi_0 & \frac{\partial z_0}{\partial v_z} &= 1 \\
 \frac{\partial z_0}{\partial p_x} &= D_0 \tau \frac{\partial \phi_0}{\partial p_x} - s \frac{\partial \tau}{\partial p_x} & \frac{\partial z_0}{\partial p_y} &= D_0 \tau \frac{\partial \phi_0}{\partial p_y} - s \frac{\partial \tau}{\partial p_y} & \frac{\partial z_0}{\partial p_z} &= -\frac{s}{p_T}
 \end{aligned} \tag{6.2}$$

6.2. CHARGED TRACKS

The helix parameters of charged particle $(h_1, h_2, h_3, h_4, h_5) = (\rho, \tau, \phi_0, D_0, z_0)$ can be similarly be found from Fig. 8b:

$$\begin{aligned}
 p_T &= \sqrt{p_x^2 + p_y^2} & \tau &= \frac{p_z}{p_T} \\
 \sin \phi &= \frac{p_y}{p_T} & \cos \phi &= \frac{p_x}{p_T} \\
 r &= -\frac{K}{Q} p_T \text{sign}(B) & \rho &= \frac{1}{r} \\
 x_c &= v_x - r \sin \phi & y_c &= v_y + r \cos \phi \\
 \eta &= \sqrt{x_c^2 + y_c^2} \cdot \text{sign}(r) & D_0 &= r - \eta
 \end{aligned} \tag{6.3}$$

$$\begin{aligned}
\sin\phi_0 &= -\frac{x_c}{\eta} & \cos\phi_0 &= \frac{y_c}{\eta} \\
s &= r(\phi - \phi_0) & z_0 &= v_z - s\tau
\end{aligned}$$

In order to calculate the error covariance matrix of the track parameters we calculate the partial derivatives

$$\begin{aligned}
\frac{\partial r}{\partial p_x} &= \frac{r}{p_T} \cos\phi & \frac{\partial r}{\partial p_y} &= \frac{r}{p_T} \sin\phi & \frac{\partial r}{\partial p_z} &= 0 \\
\frac{\partial \phi}{\partial p_x} &= -\frac{\sin\phi}{p_T} & \frac{\partial \phi}{\partial p_y} &= \frac{\cos\phi}{p_T} & \frac{\partial \phi}{\partial p_z} &= 0 \\
\frac{\partial x_c}{\partial v_x} &= 1 & \frac{\partial x_c}{\partial v_y} &= 0 & \frac{\partial x_c}{\partial v_z} &= 0 \\
\frac{\partial x_c}{\partial p_x} &= 0 & \frac{\partial x_c}{\partial p_y} &= -\frac{r}{p_T} & \frac{\partial x_c}{\partial p_z} &= 0 \\
\frac{\partial y_c}{\partial v_x} &= 0 & \frac{\partial y_c}{\partial v_y} &= 1 & \frac{\partial y_c}{\partial v_z} &= 0 \\
\frac{\partial y_c}{\partial p_x} &= \frac{r}{p_T} & \frac{\partial y_c}{\partial p_y} &= 0 & \frac{\partial y_c}{\partial p_z} &= 0 \\
\frac{\partial \eta}{\partial v_x} &= \frac{x_c}{\eta} = -\sin(\phi_0) & \frac{\partial \eta}{\partial v_y} &= \frac{y_c}{\eta} = \cos(\phi_0) & \frac{\partial \eta}{\partial v_z} &= 0 \\
\frac{\partial \eta}{\partial p_x} &= \frac{r}{p_T} \cos\phi_0 & \frac{\partial \eta}{\partial p_y} &= \frac{r}{p_T} \sin\phi_0 & \frac{\partial \eta}{\partial p_z} &= 0 \\
\\
\frac{\partial 1/r}{\partial p_x} &= -\frac{\cos\phi}{rp_T} & \frac{\partial 1/r}{\partial p_y} &= -\frac{\sin\phi}{rp_T} & \frac{\partial 1/r}{\partial p_z} &= 0 \\
\frac{\partial \tau}{\partial p_x} &= -\frac{\tau \cos\phi}{p_T} & \frac{\partial \tau}{\partial p_y} &= -\frac{\tau \sin\phi}{p_T} & \frac{\partial \tau}{\partial p_z} &= \frac{1}{p_T} \\
\frac{\partial \phi_0}{\partial v_x} &= -\frac{\cos\phi_0}{\eta} & \frac{\partial \phi_0}{\partial v_y} &= -\frac{\sin\phi_0}{\eta} & \frac{\partial \phi_0}{\partial v_z} &= 0 \\
\frac{\partial \phi_0}{\partial p_x} &= -\frac{r \sin\phi_0}{\eta p_T} & \frac{\partial \phi_0}{\partial p_y} &= \frac{r \cos\phi_0}{\eta p_T} & \frac{\partial \phi_0}{\partial p_z} &= 0 \\
\frac{\partial D_0}{\partial v_x} &= \sin\phi_0 & \frac{\partial D_0}{\partial v_y} &= -\cos\phi_0 & \frac{\partial D_0}{\partial v_z} &= 0 \\
\frac{\partial D_0}{\partial p_x} &= \frac{r}{p_T} (\cos\phi - \cos\phi_0) & \frac{\partial D_0}{\partial p_y} &= \frac{r}{p_T} (\sin\phi - \sin\phi_0) & \frac{\partial D_0}{\partial p_z} &= 0 \\
\frac{\partial z_0}{\partial v_x} &= -\frac{r\tau}{\eta} \cos\phi_0 & \frac{\partial z_0}{\partial v_y} &= -\frac{r\tau}{\eta} \sin\phi_0 & \frac{\partial z_0}{\partial v_z} &= 1 \\
\frac{\partial z_0}{\partial p_x} &= \frac{r\tau}{p_T} (\sin\phi - \frac{r}{\eta} \sin\phi_0) & \frac{\partial z_0}{\partial p_y} &= -\frac{r\tau}{p_T} (\cos\phi - \frac{r}{\eta} \cos\phi_0) & \frac{\partial z_0}{\partial p_z} &= -\frac{r(\phi - \phi_0)}{p_T}
\end{aligned} \tag{6.4}$$

Defining now the matrices

$$F_v = \begin{pmatrix} \frac{\partial t_1}{\partial v_1} & \frac{\partial t_1}{\partial v_2} & \frac{\partial t_1}{\partial v_3} \\ \frac{\partial t_2}{\partial v_1} & \frac{\partial t_2}{\partial v_2} & \frac{\partial t_2}{\partial v_3} \\ \frac{\partial t_3}{\partial v_1} & \frac{\partial t_3}{\partial v_2} & \frac{\partial t_3}{\partial v_3} \\ \frac{\partial t_4}{\partial v_1} & \frac{\partial t_4}{\partial v_2} & \frac{\partial t_4}{\partial v_3} \\ \frac{\partial t_5}{\partial v_1} & \frac{\partial t_5}{\partial v_2} & \frac{\partial t_5}{\partial v_3} \end{pmatrix} = \begin{pmatrix} \frac{\partial p}{\partial v_x} & \frac{\partial p}{\partial v_y} & \frac{\partial p}{\partial v_z} \\ \frac{\partial \tau}{\partial v_x} & \frac{\partial \tau}{\partial v_y} & \frac{\partial \tau}{\partial v_z} \\ \frac{\partial \phi_0}{\partial v_x} & \frac{\partial \phi_0}{\partial v_y} & \frac{\partial \phi_0}{\partial v_z} \\ \frac{\partial D_0}{\partial v_x} & \frac{\partial D_0}{\partial v_y} & \frac{\partial D_0}{\partial v_z} \\ \frac{\partial z_0}{\partial v_x} & \frac{\partial z_0}{\partial v_y} & \frac{\partial z_0}{\partial v_z} \end{pmatrix} \quad \text{for neutrals} \quad (6.5)$$

$$F_p = \begin{pmatrix} \frac{\partial 1/r}{\partial v_x} & \frac{\partial 1/r}{\partial v_y} & \frac{\partial 1/r}{\partial v_z} \\ \frac{\partial \tau}{\partial v_x} & \frac{\partial \tau}{\partial v_y} & \frac{\partial \tau}{\partial v_z} \\ \frac{\partial \phi_0}{\partial v_x} & \frac{\partial \phi_0}{\partial v_y} & \frac{\partial \phi_0}{\partial v_z} \\ \frac{\partial D_0}{\partial v_x} & \frac{\partial D_0}{\partial v_y} & \frac{\partial D_0}{\partial v_z} \\ \frac{\partial z_0}{\partial v_x} & \frac{\partial z_0}{\partial v_y} & \frac{\partial z_0}{\partial v_z} \end{pmatrix} \quad \text{for helices} \quad (6.6)$$

one may find the error covariance matrix for the track parameters as

$$T = F_v^T \cdot V \cdot F_v + F_p^T \cdot T \cdot F_p + 2F_v^T \cdot C_{v,p} \cdot F_p \quad (6.7)$$

7. Vertex topology reconstruction

Interest in vertex topology construction has centered on the field of heavy flavour analysis. In this type of physics several closely spaced vertices – partially from cascade decays – should be recognized and separated from each other. Due to the limited measuring precision this in many cases will not be possible and for a large fraction of tracks usually there will not exist a unique assignment to a single vertex from topological criteria alone. Therefore one usually will have to apply additional criteria originating from physics prejudice, as e.g. assigning to a secondary vertex tracks identified as leptons or to sorting tracks according to some jet algorithm.

The software based on topological criteria only has been written in the spirit of allowing easy introduction of these additional physics criteria while keeping all remaining ambiguities. In order to remain efficient in this task of considering a large number of combinations an elaborate system of flag handling tools has been introduced. Each track has assigned to it three markers describing

- The possible crossing with all other tracks
- The compatibility with possible particle assignment
- The identity of a possible parent particle

These flags are extensively used in most tasks some of which are sketched below.

7.1. PARTICLE IDENTITY ASSIGNMENT

Particle identification usually involves a combination of information coming from different parts of the detector. In general only probability statements can be made and ambiguities will remain. In order to be able to handle these ambiguities, a marker word has been assigned to each track. For each possible particle identity assignment with a probability above a set minimum value one bit in the marker word is being set.

7.2. TRACK SELECTION

In order to reduce the combinatoric task it is useful to restrict the tracks to be considered by rejecting right from the start uninteresting tracks. These may be e.g. badly measured tracks, sections of a spiralling track or, depending on the physics interest, low momentum tracks.

The track indices of a subset of all reconstructed tracks are determined in this task.

7.3. RECONSTRUCTION OF PHOTON CONVERSIONS

- tighter requirements on the electron probability All pairs of oppositely charged tracks with track markers having the electron bit set are investigated for
- formation of a vertex. The fast vertex fitting procedure is used for this purpose
- the vertex being outside the beam vacuum
- the invariant mass of the pair is required to be below a set maximum value. For this purpose the complete vertex fitting procedure is done again, providing also the mass and the incident photon track.
- the incident photon track is required to be compatible with the primary vertex (respectively the beam crossing point in the collider). This check is done by doing a fast vertex fit using one vertex (beam crossing) and one neutral track (photon) as input.
- rejection of vertices with too many additional tracks. This check is done in order to reject secondary interactions. A loop over all other tracks is performed, calling the fast fitting routine with one input vertex and one additional track.

If any of the checks is unsuccessful the specific track pair is abandoned from further consideration. If all checks are successful the photon is added to the available neutral tracks and the bit corresponding to a photon is set in the particle origin marker of the two electrons.

7.4. RECONSTRUCTION OF K_0 'S AND Λ 'S

All pairs of oppositely charged tracks with track markers compatible with $\pi^+\pi^-$, $p\pi^-$ or $\pi^+\bar{p}$ assignment are investigated for

- formation of a vertex. The fast vertex fitting procedure is used for this purpose
- the vertex being separated from the primary vertex or beam crossing. The fast vertex fitting procedure with two input vertices is used for this purpose.
- the invariant mass of the pair is required to be within a given set of standard deviations from the nominal value. Only mass assignments corresponding to K_0 , Λ and $\bar{\Lambda}$, compatible with the decay particle identification markers, are tested. For this purpose the complete vertex fitting procedure is done again, providing also the invariant mass values and the incident V_0 track.
- the incident V_0 track is required to be not too far off the primary vertex (respectively the beam crossing point in the collider). This check is done by trying a fast vertex fit using one vertex (beam crossing) and one neutral track as input.
- rejection of vertices with too many additional tracks. This check is done in order to reject secondary interactions. A loop over all other tracks is performed, calling the fast fitting routine with one input vertex and one additional track.

If all invariant mass checks or any of the other checks is unsuccessful the specific track pair is abandoned from further consideration. If at least one invariant mass value is accepted and all other checks are successful the incoming track is added to the available neutral tracks and bits corresponding to all successful assignments are set in the particle origin marker of the two decay tracks.

7.5. GENERAL VERTEX SEARCH

In the case of photon conversion and V_0 reconstruction the multiplicity of the vertex has been known beforehand. Thus one was able to apply rather straightforward the vertex fitting procedures described earlier in the text. In the more general case this is not so easy and a procedure has been developed with the aim of finding all track combinations which may be coming from a common vertex.

The strategy of this procedure is as follows:

- for the given set of tracks a vertex fit for all pairs of tracks is tried and a marker is generated for each track which contains for all successful fits a bit corresponding to the second track. Simultaneously the number of "crossings" with other tracks is saved for each track
- all tracks which have no crossing (respectively have too few crossings if a minimum multiplicity was specified) are removed from the set and the markers are modified correspondingly
- try to find a vertex with the highest multiplicity. The multiplicity starting value is determined from the highest "crossing" multiplicity found in sufficient number of tracks (and from the maximum multiplicity if specified by the user). All combinations of tracks with the required multiplicity are investigated. One may obtain more than one good vertex perhaps even with some tracks in common

- repeat the procedure with multiplicity reduced by one. Track combinations which are contained already in a higher multiplicity vertex are skipped over. This step is repeated until multiplicity two (or a minimum specified by the user) is reached.

Provisions have been taken to allow the introduction of some physics prejudices into the task of general vertex finding. One may require

- the inclusion of specific tracks or a minimum number of tracks of one set (respectively minimum numbers of several sets). This task is performed by generation of markers for the sets and requiring a minimum of coinciding bits with the marker containing the track combination to be tested for formation of a vertex.
- exclusion of specific track combinations

7.6. PRIMARY VERTEX RECONSTRUCTION

The primary vertex can be reconstructed from

- beam crossing as given by the machine parameters of the collider
- reconstructed charged tracks
- neutral tracks for which the track position in the vicinity of the primary vertex is reasonably well determined as is the case for kaons lambdas and converted photons

In the most simple approach a common vertex of all these objects is searched for. This in general will be unsuccessful as there will be badly reconstructed tracks present and also genuinely missing tracks from short range secondary decays.

The first problem can easily be dealt with by using the general fitting procedure described in the last chapter and selecting the highest multiplicity vertex with acceptable χ^2 , respectively in case of ambiguities selecting the lowest χ^2 vertex.

This method however will often lead to distorted results in case of heavy flavour decays as quite frequently the multiplicity of tracks originating from the primary vertex is lower than those of the heavy flavour decays. In addition these tracks often have low momentum and due to multiple scattering give little position information when traced back to the primary vertex.

An alternative method suggested for this case would be a separation of available tracks into jets, identification of tracks which are incompatible with coming from the beam crossing region and trying to produce a vertex out of the tracks belonging to the jet. The incident track of the jet may then be included into the primary vertex search.

7.7. SEARCH FOR SECONDARY VERTICES

This topic is the most interesting and simultaneously most difficult one. The strategy one has to apply depends very strongly on the physics interest (aim).

If one wants to select a single or few decays of some heavy flavour particle as e.g. for a $D_s \rightarrow \Phi \pi^\pm$ decay a suitable strategy would be:

- loop over all pairs of oppositely charged tracks which are compatible with kaon signature. Require small vertex χ^2 and Φ mass.
- Combine incident Φ track with all charged tracks which are compatible with

pion assignement. Require small vertex χ^2 , separation of vertex from primary vertex and look at invariant mass.

All these tasks can be easily done with the elementary building blocks described in chapters 3, 4 and 5.

If one wants to make a general search for new resonances with a priory undefined decay topology another strategy may be appropriate. One may want to select events which are likely to contain heavy flavour decays and save preliminary event topology information which can be used later on for a fast preselection when looking for specific channels. One strategy which selects events with secondary decays is described below:

- Selection of tracks with momentum above a minimum value which are displaced from the primary vertex by a minimum χ^2 distance
- Search for possible secondary vertices using the set of tracks selected in the previous step. The highest multiplicity vertex is selected. In case of ambiguities the one with lowest χ^2 is chosen. Tracks belonging to the chosen vertex are removed from the sample and the procedure is repeated with the reduced samples until no further vertices can be formed.
- Search for vertices of left over single displaced tracks with other (non displaced) tracks.
- Investigation of ambiguities in track to vertex assignement. One tries to attach each track (above a minimum momentum) to each of the vertices and tables containing the results including the ambiguities are generated.

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Appendix A.

List of topology routines used in the ALEPH program package YTOP

Steering routines

YBDTOP block data
YTIJOB job initialization
YTOIJO initialization subroutine
YTOIRU run initialisation
YTOPOL main steering routine
YTOPNW steering subroutine
YHISTO histogram booking for debugging
YHUSER book user defined histograms

Fitting routines

YFMVTR constraint fit of tracks to common vertex; mass and momentum sum
YFTVTR fast vertex only fit.
YTPAR incident neutral or charged track parameters from output supplied by routine
YFMVTR

Marker manipulation routines

YMKAND generate marker with coinciding bits of two markers set
YMKCLE clear single bit in marker
YMKCNT count number of set bits in marker
YMKCOP copy marker
YMKEXP expand marker bits into array
YMKIDC count identical bits in two markers
YMKIDT test if markers are identical
YMKMIS test for missing bits in marker with respect to reference marker
YMKORR or bits of two markers
YMKSET set single bit in marker
YMKSHR shrink marker by taking out one bit
YMKTST test if specific bit is set in marker
YMKZER clear all bits in marker

Reconstruction routines

YRUSER user supplied analysis routine
YTCNV gamma conversion reconstruction
YTOSVT secondary vertex search (not yet released)

YTPVTX primary vertex reconstruction
 YPRIVX subroutine for primary vertex reconstruction
 YTRV0S v_{zero} (K , Λ and $\bar{\Lambda}$) reconstruction
 YTSTRK selection of charged tracks for analysis
 YVPOSS find all track combinations out of a given set of tracks (and input vertices if required) that form a common vertex
 YVXBLD build vertex from initial vertex and given set of tracks

Miscellaneous routines

YDEFRRF get bank indices for charged tracks
 YMS3IN 3x3 symmetric matrix inversion
 YPIDAS set particle identity flag for reconstructed charged tracks
 YINVMX inversion of symmetric matrix stored in triangular form
 YITCOF find its coordinates on a track
 YVDCOF find vertex detector coordinates on track
 YSVVTX save vertex and related quantities in banks