DESIGNING SUPERCONDUCTING CAVITIES FOR ACCELERATORS

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Abstract
Rapid advances in the performance of superconducting cavities have made RF superconductivity a key technology for accelerators that fulfil a variety of physics needs: high energy particle physics, nuclear physics, neutron spallation sources, and free electron lasers. New applications are forthcoming for frontier high energy physics accelerators, radioactive beams for nuclear astrophysics, next generation light sources, intense proton accelerators for neutron, neutrino and muon sources.

1. INTRODUCTION
The goal of this paper is to discuss design choices for superconducting cavities for various accelerator applications. Two classes of considerations govern structure design. The particular accelerator application forms one class, and superconducting surface properties the other. Designing a superconducting cavity is a strong interplay between these two classes. Typical accelerator driving aspects are the desired voltage, the duty factor of accelerator operation, beam current or beam power. Other properties of the beam, such as the bunch length, also play a role in cavity design. Typical superconducting properties are the microwave surface resistance and the tolerable surface electric and magnetic fields. These properties, which are also discussed in different lectures at the Erice School [1,2,3], set the operating field levels and the power requirements, both RF power as well as AC operating power, together with the operating temperature.

Accordingly, the plan of this article is to briefly describe a number of distinct accelerator applications and the structures which emerged. To understand the evolution we first discuss the key electromagnetic properties of accelerating structures leading to an analysis of the power requirements of superconducting accelerators. The behavior of superconducting surfaces exposed to high surface electric and magnetic fields provide a guide to tailoring the cavity shape to achieve desirable values of key properties for accelerator performance. Both accelerator and surface issues govern the choice of the cavity shape, beam aperture, number of cell per structure as well as the choice of the RF frequency. Mechanical properties also play a role in the design aspects. Finally, input and output power coupling issues interact with cavity design, but these are covered in a different lecture[4].

The discussion here is an overall summary and review of design aspects. We refer the reader to the reference text [5] and review article [6] for a more thorough discussion of many of the design topics and their intimate relationships to the physics of RF superconductivity.

Figure 1 shows a variety of superconducting accelerating cavities, ranging in frequency from 200 MHz to 3000 MHz and ranging in number of cells from one to nine. Most are cavities fabricated from pure sheet niobium and some, especially at frequencies below 500 MHz, are made of copper sputtered with a micron thin layer of niobium. Cavity fabrication issues are discussed in other lectures at the Erice School [3]. All the cavities of Figure 1 are intended for accelerating particles moving at nearly the velocity of light, i.e. v/c = β ≈ 1. Accordingly, the period of a long structure (or the accelerating gap) is λ/2, where λ is the RF wavelength. Particles moving at v ≈ c will cross the gap in exactly a half RF period to receive maximum acceleration.
2. ACCELERATOR REQUIREMENTS AND EXAMPLE SYSTEMS

Superconducting cavities have found successful application in a variety of accelerators spanning a wide range of accelerator requirements. High current storage rings for synchrotron light sources or for high luminosity, high energy physics with energies of a few GeV call for acceleration voltages of less than 10 MV, and carry high CW beam currents up to one amp. Figure 2 [7] shows the accelerating structure based on a 500 MHz, single cell cavity that evolved for the Cornell storage ring CESR/CHESS. The cavity was fabricated from pure sheet niobium. Four such systems provide the needed voltage of 7 MV and beam power of more than one MW. Similar systems are under construction to upgrade the beam current of the existing Taiwan Light Source (SRRC), and for the new Canadian Light Source (CLS). The accelerating gradient choice for all these cases is 7 MV/m or less.

Near the energy frontier, LEP-II at CERN called for an accelerating voltage for nearly 3 GV to upgrade the beam energy from 50 to 100 GeV per beam, with a beam current of a few mA. With a frequency choice of 350 MHz, dominated by higher order mode (HOM) power loss and beam stability considerations, a 4-cell structure emerged[8]. To build 300 such units there was considerable savings in material cost by fabricating the cavity out of copper and coating it with niobium by sputtering. The LEP-II cavities (Figure 3) operated successfully at an average gradient of 6 MV/m.

A one GeV CW linac forms the basis for CEBAF, a 5-pass recirculating accelerator providing 5-6 GeV CW beam for nuclear physics[9]. The total circulating beam current is a few mA. Developed at Cornell, the 5-cell, 1500 MHz cavities (Figure 4) are also fabricated from solid sheet niobium. CEBAF cavities operate at an average accelerating field of 6 MV/m.
All the above accelerators run CW at 100% duty factor. The first pulsed superconducting linac will be for the Spallation Neutron Source (SNS) at Oak Ridge. 6-cell niobium cavities at 804 MHz will accelerate a high intensity ($\approx 10$ mA) proton beam from 200 MeV to 1000 MeV. Figure 5 shows the medium $\beta = 0.64$ cavity that resembles a $\beta = 1$ cavity that is squashed [10]. The duty factor for SNS is 6% and the RF pulse length is one ms. With recent improvement in cavity gradients the anticipated gradient is near 15 MV/m. Besides spallation neutron sources SNS technology could become suitable for high intensity proton linacs for various applications, such as transmutation of nuclear waste or generation of intense muon beams.

The dream machine for the future will be a 500 GeV energy frontier linac colliding electrons and positrons, upgradable to one TeV. As we will see, refrigerator power considerations drive the duty factor of operation to one percent. The average beam current is about 10 $\mu$A. A 9-cell niobium cavity design (Figure 6) has emerged from the TESLA collaboration[11]. With gradients improving steadily over the last decade, the choice of 25 MV/m will lead to 20 km of cavities for the 500 GeV machine. TESLA technology is likely to become the basis for the free electron lasers providing high brightness beams with wavelengths from the infra-red to ultraviolet and ultimately x-rays.

For the far future, acceleration of muons will also benefit from superconducting cavities[12]. A neutrino factory providing an intense neutrino beam from decaying muons may be the first step towards a muon collider that will penetrate the multi-TeV energy scale. At low energies (< a few GeV), where the muons have a large energy spread, the RF frequency has to be very low, e.g 200 MHz, leading to gigantic structures. Once again economics will favor thin film Nb-Cu cavities over
sheet Nb cavities. For comparison, a single cell Nb-Cu cavity at 200 MHz (Figure 1) dominates the size of superconducting cavities for the variety of accelerator applications discussed.

Fig. 5  b = 0.6, 6-cell cavity for SNS, frequency 804 MHz

Fig. 6  1300 MHz 9-cell cavity for TESLA

3. BASICS OF ACCELERATING STRUCTURES

3.1 Accelerating field

Only simple structures can be calculated analytically, such as a cylinder with no beam holes (Figure 7), referred to as the “pill-box cavity.” For our purposes, the analytic calculations of a simple cylindrical cavity are convenient to define the important performance parameters of superconducting cavities. For a cylinder of length \( d \) and radius \( R \) using cylindrical co-ordinates \((\rho, \phi, z)\), the electric \( (E_z) \) and magnetic \( (H_{\phi}) \) fields for the TM\(_{010}\) mode are given by:

\[
E_z = E_0 J_0 \left( \frac{2.405 \rho}{R} \right) e^{-i\omega t}, \quad H_{\phi} = -i \varepsilon_0 \mu_0 E_0 J_1 \left( \frac{2.405 \rho}{R} \right) e^{-i\omega t}
\]

where all other field components are zero. \( J_0 \) and \( J_1 \) are Bessel functions. The angular resonant frequency is given by:

\[
\omega_{010} = \frac{2.405 c}{R}
\]

which is independent of the cavity length.

First we determine the accelerating field, \( E_{acc} \). Assume an electron travelling nearly at the speed of light (c). It enters the cavity at time \( t = 0 \) and leaves at a time \( t = d/c \). To receive the maximum kick from the cavity, the time it takes the particle to traverse the cavity is to equal 1/2 an RF period \( (T_{RF}) \), i.e.
In this case, the electron always sees a field pointing in the same direction. The accelerating voltage (\(V_{\text{acc}}\)) for a cavity is

\[
V_{\text{acc}} = \left| \int_{-\epsilon}^{\epsilon} E_z \, dz \right|
\]

For an electron accelerator with energy >10 MeV, it is sufficiently accurate to use \(v = c\), so that \(t(z) = z/c\). Thus

\[
V_{\text{acc}} = E_0 \left| \int_{-\epsilon}^{\epsilon} e^{i\omega z/c} \, dz \right| = d E_0 \frac{\sin \left( \frac{\omega d}{c} \right)}{\frac{\omega d}{c}} = d E_0 \frac{\omega d}{c}
\]

At 1.5 GHz RF frequency, \(d = c/\lambda = 10 \text{ cm}\), \(V_{\text{acc}}\) simplifies to

\[
V_{\text{acc}} = (2/\pi) \, d \, E_0
\]

The average accelerating electric field (\(E_{\text{acc}}\)) that the electron sees during transit is given by

\[
E_{\text{acc}} = \frac{V_{\text{acc}}}{d} = \frac{2E_0}{\pi}
\]

### 3.2 Peak Fields

To maximize the accelerating field, it is important to minimize the ratios of the peak fields to the accelerating field by selecting a suitable cavity geometry. For the TM\(_{010}\) accelerating mode in a pill-box cavity

\[
E_{pk} = E_0, \quad H_{pk} = \sqrt{\frac{\varepsilon_0}{\mu_0}} j(0.84)E_0 = \frac{E_0}{647 \Omega}
\]

Thus we obtain the following ratios:

\[
\frac{E_{pk}}{E_{\text{acc}}} = \frac{\pi}{2} = 1.6, \quad \frac{H_{pk}}{E_{\text{acc}}} = 2430 \, \text{A/m} \, \text{MV/m} = 30.5 \, \text{oersted MV/m}
\]

Figure 8 shows the electric and magnetic field profiles for a real cavity shape. The peak field ratios for a realistic structure are much larger than for the pill-box. For example, for the TESLA cavity, \(E_{pk}/E_{\text{acc}} = 2.0\) and \(H_{pk}/E_{\text{acc}} = 42\) Oe per MV/m.
Fig. 8 Electric and magnetic field profiles for single cell cavity.

### 3.3 Power Losses and $Q_0$

In order to support the fields in the cavity, currents flow within a thin surface layer of the cavity walls. If the surface resistance is $R_s$, the power dissipated per unit area ($P_a$) due to Joule heating is

$$P_a = \frac{1}{2} R_s H^2$$

The two most salient characteristics of an accelerating cavity are its average accelerating field, $E_{acc}$, and the intrinsic quality factor $Q_0$. We just discussed $E_{acc}$. The quality ($Q_0$), is related to the power dissipation by the definition of $Q_0$

$$Q = \frac{\omega \text{Energy Stored}}{\text{Power dissipated}} = \frac{\omega U}{P_c}$$

where $U$ is the stored energy and $P_c$ is the dissipated power. Since the energy stored in the electric field is equal to that stored in the magnetic field, the total energy in the cavity is given by the power dissipated as,

$$U = \frac{1}{2} \mu_0 \int |H|^2 \, dv \quad P_c = \frac{1}{2} R_s \int |H|^2 \, ds$$

where the first integral is taken over the volume of the cavity, and the second over the surface. Thus

$$Q = \frac{\omega \mu_0 \int |H|^2 \, dv}{R_s \int |H|^2 \, ds} \quad \frac{Q}{R_s} = G = \frac{\omega \mu_0 \int |H|^2 \, dv}{\int |H|^2 \, ds}$$

Here $G$ is called the “geometry factor” for the cavity shape. For a pill box in the TM010 mode, $G = 257$. Scaling arguments show that the ratio of the integrals $V |H|^2 \, dV / S |H|^2 \, ds$ must scale linearly with $a$, or alternatively, inversely with the mode frequency. Therefore the geometry constant, $G$ only depends on the cavity shape and not its size. A typical observed surface resistance for a well prepared superconducting Nb cavity is $R_s = 20 \, \text{n}$.$\Omega$. Thus we have a $Q_0$ value of
For a typical cavity length of \( d = 10 \text{ cm} \) and RF frequency of \( 1.5 \text{ GHz} \), the cavity radius is \( R = 7.65 \text{ cm} \). For an accelerating voltage of \( 1 \text{ MV} \), the following values result for the important features of a superconducting cavity:

\[
E_{\text{acc}} = \frac{V_{\text{acc}}}{d} = 10 \text{ MV/m}
\]

\[E_{\text{pk}} = E_0 = \frac{\pi}{2} E_{\text{acc}} = 15.7 \text{ MV/m}
\]

\[H_{\text{pk}} = 2430 \text{ kA/m} = 305 \text{ Oersted}
\]

\[U = \frac{\pi \epsilon_0 E_0^2}{2} J_1^2 \left( \frac{405}{d} \right) d R^2 = 0.54 \text{ J}
\]

\[P_c = \frac{\omega U}{Q} = 0.4 \text{ W}
\]

The performance of a superconducting cavity is evaluated by measuring the \( Q_0 \) as a function of the cavity field level. These curves bear tell-tale signs of the activities inside the cavity. Figure 9 shows \( Q_0 \) vs \( E \) curves for some high performance TESLA multi-cell structures.

### 3.4 Shunt Impedance

Two important quantities for cavity design are the Shunt Impedance \( R_a \) and the geometric shunt impedance \( R_a/Q \). \( R_a \) is defined in analogy to Ohm’s law from the losses in a cavity at a given accelerating voltage:

\[R_a = \frac{V_{\text{acc}}^2}{P_c}
\]

Ideally we want the shunt impedance to be large for the accelerating mode so that the dissipated power is small. This is particularly important for copper cavities, where the wall power dissipation is a major issue and we wish to have as large an accelerating field as possible. For the TM\(_{010}\) mode we have:

\[R_a = \frac{4 \mu_0 a^2}{\pi^3 \epsilon_0 J_1 \left( 4.05 \right) R \left( R + d \right)} = 2.5 \times 10^{12} \Omega
\]

From the definition of \( Q \) the ratio \( R_a/Q \) turns out to be:
\[ \frac{R_a}{Q} = \frac{V_{acc}^2}{\omega U} \]

The geometric shunt impedance is independent of the surface resistance. The geometric shunt impedance is crucial for determining the beam-cavity interaction in the fundamental and HOMs. Since the ratio \( V_{acc}/U \) scales inversely with the cavity's linear dimensions, \( R_a/Q \) is independent of cavity frequency, and only depends on the cavity geometry. For the pill-box TM_{010} mode we have:

\[ \frac{R_a}{Q} = 150 \frac{dL}{R} = 196 \Omega \]

Beam holes reduce the shunt impedance and enhance the peak surface fields relative to the pill-box case so that for a realistic cavity shape, \( R_a/Q \) drops by a factor of 2. A typical number for the \( R/Q \) of a cavity cell is 100 \( \Omega \).

Finally we define the shunt impedance per unit length, \( r/Q \), which gives the power dissipated per unit length at a given accelerating field \( E_{acc} \). \( r/Q \) increases linearly with frequency.

\[ \frac{r}{Q} = \frac{R/Q}{L} \]

\[ \frac{P}{L} = \frac{E_{acc}^2}{(r/Q)^2 Q_0} \]

For real structures with contoured shapes, beam apertures and beam pipes, it is necessary to use field computation codes, such as MAFIA and Microwave Studio. Figure 10 shows the electric and magnetic fields computed by Microwave Studio for the accelerating mode of a pillbox cavity with a beam hole, and for a round wall cavity. Such codes are also necessary for computing the fields in the higher order modes of a cavity that can have an adverse effect on beam quality or cause instabilities. Figure 11 shows the electric and magnetic fields of the first monopole HOM. Beam induced voltages are also proportional to the \( R/Q \) of HOMs.

Fig.10 (Left) Electric and (Right) Magnetic fields for a round cavity with beam holes.
3.5 Multicell Cavities

A multicell cavity is a structure with several cells coupled together. As with any set of coupled oscillators there are multiple modes of excitation for the full structure for every given mode of a single cell. The frequencies ($f$) of these modes can be cast via Equation 1 in terms of the single cell resonant frequency ($f_0$) and a cell-to-cell coupling strength ($k$) via a coupled LC oscillator model (Figure 12). A measurement of the highest and lowest frequencies directly yields the value of the cell-to-cell coupling (Equation 2). Out of the N modes in the TM010 pass band of a N-cell structure, the accelerating mode is the one where the fields in the neighboring cells are $\pi$ radians out of phase with each other so that each cell provides the same acceleration kick to a velocity-of-light particle that crosses each gap in one-half RF period (Figure 12). Having equal fields in each cell maximizes the overall accelerating voltage and minimizes the peak fields in each cell. A flat field profile is only achieved when the cells are properly tuned relative to each other. As Equations 1 & 2 show, a large number (N) of cells or a small the cell-to-cell coupling ($k$) reduces the spacing between the accelerating mode and its nearest neighbor, making tuning more difficult, and making the field profile more sensitive to any cell-to-cell frequency differences that arise from manufacturing tolerances. Therefore, in an application that demands high total voltage (e.g TESLA, $\approx$ TeV), a high number of cells is desirable, and it is important to increase the aperture and decrease the cell-to-cell distance to increase the cell-to-cell coupling coefficient.
Traveling Wave vs Standing Wave

Superconducting structures operate in the standing wave mode for which the power required to establish the fields is commensurate with wall losses. Hence the RF power that must be supplied is usually comparable to the beam power. Since the peak fields to accelerating field ratios are high in the standing wave, it is worth examining how much gain in peak fields can be expected by operating in the traveling wave mode. Figure 13 shows the reduction in peak surface magnetic field for traveling wave mode operation the $2\pi/3$, $\pi/4$ and $\pi/3$ modes. Although the 25% magnetic field reduction seems attractive at first glance, it comes with the price of smaller gap cells and thus more cells per meter. Consequently the cost of such structures would increase substantially over the standing wave structure, resulting in little overall gain in the total capital cost of a traveling wave superconducting accelerator. Moreover the traveling wave power required to establish the fields would need to be dumped at the end of the structure or recirculated. The first option is wasteful, defeating the main advantage of superconducting cavities, while the second option would increase structure complexity and cost.

\[
\left(\frac{f_n}{f_0}\right)^2 = 1 + 2k \left[1 - \cos\left(\frac{m\pi}{N}\right)\right]
\]

\[
k = \frac{1}{2} \left[ (f^{(N)})^2 - (f^{(1)})^2 \right] \frac{1}{(f^{(1)})^2 - (f^{(N)})^2 [1 - \cos(\pi/N)]}
\]

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Fig. 13 Structure geometry for travelling wave and accompanying decrease of $H_{pk}$.
4. RF SUPERCONDUCTIVITY BASICS: SURFACE RESISTANCE

As explained in the reference text [5], the remarkable properties of superconductivity are attributed to the condensation of charge carriers into Cooper pairs, which then move without friction; hence the zero resistance. At \( T = 0 \) K, all charge carriers condense. At higher temperatures, pairs break up. The fraction of unpaired carriers increases exponentially with temperature, as \( e^{-\Delta/kT} \), until none of the carriers are paired above \( T_c \) – the normal conducting state. Here \( 2\Delta \) is the energy gap of the superconductor, i.e., the energy needed to break up the pairs. In this simplified picture, known as the London two-fluid model, when a DC field is turned on, the pairs carry all the current, shielding the applied field from the normal electrons. Electrical resistance vanishes since Cooper pairs move without friction.

In the case of RF currents, however, dissipation does occur for all \( T > 0 \) K, albeit very small compared to the normal conducting state. While the Cooper pairs move without friction, they do have inertial mass. For high frequency currents to flow, forces must be applied to bring about alternating directions of flow. Hence an AC electric field will be present in the skin layer, and it will continually accelerate and decelerate the normal carriers, leading to dissipation, proportional to the square of the RF frequency. A simplified form of the temperature dependence of Nb for \( T_c/T > 2 \), and for frequencies much smaller than \( 2\Delta/h \approx 10^{12} \) Hz is:

\[
R_s = A \frac{f^2}{T} \exp \left( -\frac{\Delta(T)}{kT} \right) + R_0
\]

Here \( A \) is a constant that depends on material parameters, discussed in another lecture at the Erice School[1]. Based on the very successful BCS theory, expressions for the superconducting surface impedance have been worked out in terms of material parameters. The operating temperature of a superconducting cavity is usually chosen so that the temperature dependent part of the surface resistance is reduced to an economically tolerable value. \( R_0 \), referred to as the residual residual resistance, is influenced by several factors such as the ambient DC magnetic field environment of the cavity or the overall hydrogen gas content of the Nb material.

5. EXAMPLE 1, POWER CONSIDERATIONS FOR STORAGE RINGS

For a few GeV electron storage ring as in a light source or a B-factory, consider a 500 MHz single cell cavity of the geometry in Figure 14 (left), with half wavelength gap about 0.3m, \( R/Q = 89 \) Ohm, and \( G = 270 \) Ohm. Start with a modest CW voltage of one MV from the cavity, at a gradient about 3 MV/m. For a copper cavity, the \( Q_0 = 45,000 \) from the surface resistance of copper (Equation 3), the shunt impedance \( R_a = 4 \) M\( \Omega \), giving a dissipated power 250 kW. This would result in overheating of the copper cell. Water-cooled copper cavities at this frequency can safely dissipate about 40 kW. To bring dissipation to tolerable level it is essential to either lower the gradient or to raise the \( R_a/Q \). Figure 14 (right) shows a normal conducting cell shape with \( R_a/Q = 265 \) \( \Omega \), for which the dissipation drops to 80 kW/cell. For a typical klystron efficiency of 0.5, the AC wall power is 160 kW per cell. But with the small beam hole, HOMs cannot propagate down the beam pipe and many protrusions have to be added to the cell to remove HOMs which make harmful interactions with the beam (Figure 15). While the small beam hole helps to raise the \( R/Q \) of the fundamental mode as desired, it also raises the \( R/Q \) of the HOMs, increasing the danger of beam instabilities. Figure 16 compares the \( R/Q \) of HOMs for the large and small beam hole cases.

\[
R_{n} = \sqrt{\pi f \mu_0 \rho} = 6m\Omega
\]

(3)

For a superconducting Nb cavity of the shape of Figure 14, the BCS surface resistance at 4.2 K leads to a \( Q \) of \( 2 \times 10^9 \), \( R = 0.5 \) tera\( \Omega \), and dissipated power in the wall of about 2 watt. Taking into account refrigerator efficiency (at 4.2 K) of 1/350, the AC wall plug power is 0.7 kW. In this case the
static heat leak and cryogen transfer lines will dominate the dynamic heat load. Even if these contributions total 45 W, the AC wall power due to the refrigerator will be remain a factor of 10 smaller than for the copper cavity. The power economy of superconductivity opens the possibility of raising the gradient to say 10 MV/m, and reducing the number of cells by a factor of 3. Together with the small HOM impedance of the large beam hole the reduced number of cells improves beam quality, avoids beam instabilities and allows higher currents.

To complete the picture, electrons in a 5 GeV storage ring have a synchrotron radiation energy loss of about one MV per turn. For one amp beam current, the beam power that must be supplied through the cavities is one MW. The corresponding AC wall-plug power is 2 MW due to klystron efficiency. In CESR, the over-voltage factor to overcome quantum fluctuations from synchrotron radiation is about 7, so that the total voltage requirement is not 1 MV, but 7 MV. This can be met by four single cell units (Figure 1), each providing about 2 MV at a gradient of 7 MV/m. Each unit has a dynamic heat load of 50 watt and a static heat plus transfer line heat load of 50 W. The total cryogenic heat load is 400 watt, corresponding to an AC power demand of 140 kW. The copper cavity solution (as for PEP-II at SLAC) would call for 9 units (as in Figure 15), each dissipating 80 kW. Taking into account the klystron efficiency (0.5), the structure associated AC power is 1.44 MW, the same factor of 10 higher than the superconducting case, as discussed. But on adding the AC power associated with the beam power (1 MW) to both solutions, one obtains AC power equal to 3.44 MW for the normal-conducting case versus 2.14 MW for the superconducting case. The large beam power reduces the superconducting advantage from a factor of 10 to just 60%. The impedance reduction remains the dominant advantage.

![Fig. 14 A comparison of typical shapes used for (Left) superconducting and (Right) normal conducting cavities.](image1)

![Fig. 15 (Left) Inside view of a copper cell showing reentrant iris and apertures for HOM couplers. (Right) Full view of a copper cell as for PEP-II (SLAC) with HOM couplers[13].](image2)
RF SUPERCONDUCTIVITY BASICS: HIGH FIELD BEHAVIOR

As mentioned, the accelerating field, $E_{\text{acc}}$, is proportional to the peak electric ($E_{\text{pk}}$) as well as the magnetic field ($H_{\text{pk}}$) on the surface of the cavity. Therefore important fundamental aspects of superconducting cavities are the maximum surface fields that can be tolerated without increasing the microwave surface resistance substantially, or without causing a catastrophic breakdown of superconductivity. The ultimate limit to the accelerating field is the RF critical magnetic field, above which the superconducting phase can no longer exist. The RF critical field is related to the thermodynamic critical field. In the process of a phase transition to the normal conducting state, a phase boundary must be nucleated. Because of the rapidly changing RF fields (ns time scale), it is possible for the Meissner state to persist above the thermodynamic critical field ($H_c$) for Type I superconductors, and above the lower critical field ($H_{c1}$) for Type II superconductors. Such a metastable situation can be expected up to a superheating critical field, $H_{sh} > H_c$ (Type I) > $H_{c1}$ (for type II). It is important to note that the RF critical field does not depend on $H_{c2}$. Therefore high field magnet materials, such as Nb-Ti, do not offer correspondingly higher operating fields for superconducting cavities. Indeed for RF superconductivity, it is essential to always operate in the Meissner state. For the most popular superconductor, niobium, $H_{sh}$ is about 0.23 T. These surface fields translate to a maximum accelerating field of 55 MV/m for a typical niobium. The exact values depend on the detailed structure geometry.

Typical cavity performance is significantly below the theoretically expected surface field limit. One important phenomenon that limits the achievable RF magnetic field is “thermal breakdown” of superconductivity, originating at sub millimeter-size regions of high RF loss, called defects. When the temperature outside the defect exceeds the superconducting transition temperature, $T_c$, the losses increase substantially, as large regions become normal conducting (see Figure 16). Measures to overcome thermal breakdown are to improve the thermal conductivity of niobium by purification or to use thin films of niobium on a copper substrate cavity.

The $Q_0$ vs $E_{\text{acc}}$ curve (Figure 9) only gives information on the average behavior of the RF surface. To resolve the local distribution of RF losses and identify various mechanisms, temperature mapping proves to be a powerful diagnostic technique. A chain of rotating carbon thermometers, or an array of fixed thermometers, samples the temperature of the outer wall of the cavity. The temperature map of Figure 16 shows a hot spot that leads to thermal breakdown, and the SEM micrograph reveals a 50 µm culprit copper particle.

In contrast to the magnetic field limit, we know of no theoretical limit to the tolerable surface electric field. Fields up to 220MV/m have been imposed on a superconducting niobium cavity without any catastrophic effects [14]. However, at high electric fields, an important limitation to the
performance of superconducting cavities arises from the emission of electrons from local spots in the high electric field regions of the cavity. This is a problem endemic to all high voltage devices. Power is absorbed by the electrons and deposited as heat upon impact with the cavity walls. Copius x-rays are emitted due to bremsstrahlung. At high fields the exponential drop in $Q_0$ with field suggests that field emission is the dominant limiting mechanism, provided x-rays are also observed (Figure 9).

Fig. 16  (Left) Temperature map at 400 Oe of a 1.5 GHz, single cell cavity showing heating at a defect site, labelled #1 and field emission sites labelled #2, 3, and 4. (b) SEM micrograph of the RF surface taken at site #1 showing a copper particle [5].

Fig. 17  Calculated electron trajectories in a 3-cell 1.5 GHz cavity operating at $E_{pk} = 50$ MV/m. Here the emitter is located in an end cell, just below the iris, where the surface electric field is 44 MV/m. Note that a significant number of electrons emitted during the early part of the RF cycle bend back and strike the wall near the emitter. (Middle) Temperature map from the heating of impacting electrons. (Right) A micron size foreign particle found at an emission site.

Figure 17 shows electron trajectories in one RF period and the heating due to their impact with the cavity wall. A typical emitter is a microparticle contaminant. When emission grows intense at high electric fields it can even initiate thermal breakdown. In many cases, intense field emission eventually leads to momentary voltage breakdown of the vacuum in the cavity. This has mostly a beneficial effect for superconducting cavities, known as conditioning. After a voltage breakdown event, it is usually possible to raise the electric field until field emission grows intense once again at another spot on the cavity surface. We have learned much about the nature of field emission sites and made progress in techniques to avoid them as well as to destroy them by conditioning with high voltage breakdown[5].

In the early stages of the development of superconducting cavities, a major performance limitation was “multipacting”. This is a resonant process in which an electron avalanche builds up within a small region of the cavity surface due to a confluence of several circumstances. With the invention of the round wall cavity shape, multipacting is no longer a significant problem for velocity-
of-light structures. The essential idea to avoid multipacting is to gradually curve the outer wall of the cavity – hence the round wall profile.

If the shape is not rounded, one-surface multipacting will severely limit the cavity performance (Fig. 18). An electron emitted from one region of the surface (usually the outer cylindrical wall) travels in a cyclotron orbit in the RF magnetic field, and returns to near its point of origin. Upon impact it generates a secondary electron which mimics the trajectory of the primary. An exponential build up occurs if the round-trip travel time of each electron is an integer multiple of an RF period, i.e., the electron returns in the same phase of the RF period when it is generated. For the build up to persist, the secondary emission coefficient must be greater than one. This is true for a niobium surface when the electron energy is between 50 eV and 1000 eV [5]. During their excursion into the RF fields, the electrons must gain enough energy from the electric field to generate secondaries on impact. When these conditions are met, an electron avalanche occurs to absorb the RF power, making it impossible to raise the fields by increasing the incident RF power. The electrons impact the cavity walls, which leads to a large temperature rise, thermal breakdown, and in some cases a momentary gas discharge. When the cell shape is rounded (Fig. 18) the electrons are forced to the equator region where the electric field is too low for the electrons to gain sufficient energy to regenerate. The avalanche is arrested.

7. **CHOICE OF CAVITY SHAPE**

There are many factors which influence cavity shape. Multipacting is a key factor that governs the overall rounded contour of the cavity profile. Beam dynamics considerations control the size of the aperture. To lower the peak electric field it is necessary to round the iris region with circular or elliptical arcs. Peak magnetic field considerations influence the shape of the cavity in the large diameter (equator) region where the magnetic field is strongest. Elliptical arc segments increase the strength of the cavity against atmospheric load (see mechanical considerations below) and also provide a slope for efficient rinsing of liquids during surface etching and cleaning.
Mechanical considerations also influence the cavity shape. To prevent the cavity from deforming excessively or even collapsing under atmospheric load it is important to avoid flat regions in the cavity profile (Figure 19). Elliptical segments tend to be strongest. For pulsed operation at high gradients the Lorentz force may be strong enough to cause cell deformation demanding larger thickness walls or stiffening rings near the iris (see Figure 19). Typical detuning coefficients are in the range of a few Hz/(MV/m$^2$). Additional stiffening of multicell structures may be necessary to raise the frequency of mechanical vibration modes. As an extreme example figure shows exaggerated structure deformations for the lowest frequency vibration modes for a 200 MHz 4-cell cavity.

Fig. 19  (Left) Flat versus (Middle) elliptical cavity profile. (Right) Stiffening rings for the TESLA cavity.

Fig. 20  Mechanical resonant modes of a 4-cell, 200 MHz cavity with 8 mm wall thickness. The low resonant frequencies spell trouble in the form of microphonics. Reducing the number of cells or stiffening is essential.

8.  SUPERCONDUCTOR CHOICE

For a material to be useful for accelerators, the primary requirements are a high transition temperature, $T_c$, and a high RF critical magnetic field, $H_{sh}$. Among the elemental superconductors, niobium has the highest $T_c$ and the highest RF critical field. Accordingly, it is a most attractive choice for accelerator cavities. Successful cavities have been made from sheet Nb, or by sputtering Nb onto a copper cavity. The realm of superconducting compounds has been much less explored because of technical complexities that govern compound formation. In looking at compound candidates, it is important to select a material for which the desired compound phase is stable over a broad composition range so that formation of the compound is more tolerant to variations in experimental conditions, making it possible to achieve the desired single phase over a large surface area. Nb$_3$Sn is a promising material. The $T_c$ is 18 K and the RF critical field is 0.4T, twice as high as for Nb. On fundamental grounds the higher field opens up the possibility of accelerating gradients higher than allowed for niobium cavities.
However, the performance for Nb$_3$Sn cavities to date is far lower than for niobium cavities. The new HTS are even further from the microwave performance level desired for application to accelerators.

A strong motivation for using thin films of Nb on copper cavities is to provide increased stability against thermal breakdown of superconductivity. The thermal conductivity of copper at 4.2 K is between 300 to 2000 W/m-K, depending on the purity and annealing conditions, as compared to the thermal conductivity of 300 RRR niobium, which is 75 W/m-K at 4.2 K. The cost saving of niobium material is another potential advantage, significant for large-size, e.g. 350 MHz cavities, as for LEP-II, or for future projects, which aspire to make 200 MHz cavities, where each cell is more than one meter in diameter.

9. EXAMPLE LINEAR COLLIDERS, GRADIENT AND POWER ISSUES

Consider a one TeV cm energy linear collider, the ambition of many international accelerator collaborations. Based on the latest progress in TESLA gradients (Figure 21) we can confidently select $E_{acc} = 30$ MV/m[15], resulting in a 33 km active length linac. TESLA cryomodules have achieved a filling factor of 0.75, so that the real length of superconducting linac would be 45 km. Because of a higher gradient choice (50 MV/m, loaded gradient), a normal conducting linear collider would be shorter, about 25 km.) At a $Q_0$ of $10^{10}$ we determine a dynamic heat load of 90 watt/m to yield a preposterous, total dynamic heat load of 3 MW at 2 K. The capital cost for such a titanic refrigerator would exceed 10 billion dollars, and the AC power to run it would exceed 3 GW, comparable to a nuclear power plant. Hence a superconducting TeV energy linac must run in the pulsed mode with a duty factor of about one per cent, cutting the dynamic load to just 1 W/m, and the total dynamic cryogenic load to 33 kW.

Or we must look toward improvements that could lead to $Q$ values of $10^{11}$. Such $Q$ values have in fact been reached [16] in single cells at 1.6 K (see Figure 22), but remain to be demonstrated in full scale structures inside accelerator cryomodules.

Continuing with the one per cent duty factor scenario, other important heat loads are the static heat and a fraction of the HOM power deposited at low temperature. Assuming another one watt/m for these contributions, and taking into account the entire accelerator length of 44 km, the grand total cryogenic heat load is $33kW + 44 kW = 77 kW$. At 2 K the refrigerator efficiency is $1/750$, leading to a AC wall plug power load of 58 MW.

As in the case of the storage ring example above, we now include the beam power. For a superconducting linear collider the typical value for the total beam power is 30 MW[11]. At a klystron efficiency of 70% the RF installation then calls for an AC power of 43 MW. The grand total AC power becomes 100 MW. An important figure of merit, the efficiency of AC power to beam power conversion, is an attractive 30%. Typical efficiencies of normal-conducting linac options range around 10% due to the large RF power needed to fill copper structures to high gradients [17]. Beam powers are also kept low to manage HOMs. Hence a superconducting linac opens the route to high luminosity via high beam powers, rather than squeezing the spot size at the collision point to nano-meter dimensions as for normal conducting linacs.

The peak beam power for the superconducting linac is 30 MW divided by the duty factor, which comes out to 3 GW, or roughly 100 kW/m. Here is another advantage of the superconducting option. Because of the long filling time allowed by the low cavity wall losses, cavities can be filled slowly (ms) reducing the peak RF power requirement compared to a normal conducting linear collider. Here filling times must be of the order of ns, and peak power in the order of hundred MW per meter to achieved high gradients (70 MV/m). The peak total RF power falls in the multi-Terawatt regime.
10. CONCLUSIONS

When designing a superconducting cavity, many trade-offs must be made between accelerator requirements and cavity performance issues. Reviewing the choice of RF frequency, a high frequency is better because structures are smaller, easier to handle and the per meter structure cost is lower. Small surface area means fewer defects that can cause thermal breakdown and fewer emitters that can cause field emission. Higher r/Q (ohm per meter) means reduced dynamic heat load to liquid helium for the same operating gradient and the same length of accelerator. Smaller volume, high frequency structures contains less stored energy at a given field and lead to reduced AC wall plug power. Finally, a topic we have not discussed here, higher frequency structures have less capture of field emitted electrons in beam pipe, reducing the possibility of polluting the beam. Power economy considerations make it important to keep superconducting cavities clean enough that dark currents are very low.

A low frequency choice is better because the superconducting state (BCS) surface resistance ($\alpha f^2$) is lower resulting in higher Q at fixed operating temperature. The r/Q (per meter) of HOMs are lower, which is better for beam stability and HOM power loss. Since each cell is longer there are fewer cells per meter. For the same number of cells per structure the end associated costs are less.

Reviewing considerations that govern aperture choice, a large aperture is better because of larger beam clearance and greater cell-to-cell energy coupling which reduces field non-uniformity caused by errors in cell shape, allowing a larger number of cells per cavity unit. A large aperture also improves coupling of power from input coupler to cells in the presence of beam loading. For higher mode effects, a large aperture reduces wakefields both long- and short-range resulting in better beam quality and allowing higher beam current. On the other hand, a small aperture is better because it reduces
Epk/Eacc and Hpk/Eac. The resulting higher R/Q in accelerating mode also means lower RF heat load into liquid helium.

Turning to number of cells, a small number of cells per structure unit makes it easier to obtain a flat field profile and leads to less power for each window/coupler to deliver. Again, a smaller cavity area means fewer defects that can lead to thermal breakdown and fewer emitters that cause field emission. There is less chance of trapped HOMs and it is easier to remove HOM from couplers on the beam pipe which intercept fields in the end cells only. On the other hand, a large number of cells is better because it minimizes system costs by reducing wasted space between cells and minimizes the effects of fringing field (which lower R/Q).

To recap the salient results of the storage ring and linear collider examples, superconducting cavities excel in applications requiring continuous wave (CW) or long-pulse accelerating fields above a few million volts per meter (MV/m. Since the ohmic power loss in the walls of a cavity increases as the square of the accelerating voltage, copper cavities become uneconomical when the demand for high CW voltage grows with particle energy. A similar situation prevails in applications that demand long RF pulse length, or high RF duty factor. Here superconductivity brings immense benefits. The surface resistance of a superconducting cavity is many orders of magnitude less than that of copper. After accounting for the refrigerator power needed to provide the liquid helium operating temperature, a large net gain factor remains to provide many advantages.

Copper cavities are limited to gradients well below a few MV/m in CW and long-pulse operation because the capital cost of the RF power and the AC-power operating cost become prohibitive. For example, several MW/m of RF power are required to operate a copper cavity at 5 MV/m. There are also practical limits to dissipating high power in the walls of a copper cavity. The surface temperature becomes excessive causing vacuum degradation, stresses and metal fatigue due to thermal expansion. On the other hand, copper cavities offer higher accelerating fields (≈ 50 MV/m) for short pulse (µs) and low duty factor (< 0.1%) applications. For such applications it is important to provide abundant peak RF power (e.g. 100 MW/m in order to reach the high fields.

There is another important advantage that SRF cavities bring to accelerators. The presence of accelerating structures has a disruptive effect on the beam, limiting the quality of the beam in aspects such as energy spread, beam halo, or even the maximum current. Because of their capability to provide higher voltage, SRF systems can be shorter, and thereby impose less disruption. Due to their high ohmic losses, the geometry of copper cavities must be optimized to provide a high electric field on axis for a given wall dissipation. This requirement tends to push the beam aperture to small values, which disrupts beam quality. By virtue of low wall losses, it is affordable to design an SRF cavity to have a large beam hole, reduce beam disruption and provide high quality beams for physics research.

There has been much progress in understanding the gradient and Q₀ limitations in superconducting cavities. Through better understanding, new techniques have been developed to overcome the limitations. Producing high gradients and high Q₀ with Nb cavities demands excellent control of material properties and surface cleanliness. As a result of the improved understanding and the invention of new treatments, there has been much progress in reducing the spread in gradients that arises from the random occurrence of defects and emitters. Prescreening the starting material by eddy current scanning reduces the number of defects that can cause thermal breakdown. High RRR, high thermal conductivity Nb reduces the impact of any remaining defects. It will be important to aim for higher RRR in large area cavities, where there is a high chance of defects and contamination. High pressure rinsing greatly reduces the number of field emitters. High pulsed power processing destroys accidental field emitter contaminants. This technique will continue to be necessary in order to realize - in accelerators - the high intrinsic gradient potential of SRF cavities. There is now excellent prognosis for reaching 25 – 35 MV/m for future colliders. The road to 40 MV/m is opening up. Although most successful cavities are based on Nb, some exploratory work has been carried out on other materials.
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REFERENCES

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