Center Vortex Model for the Infrared Sector of $SU(3)$ Yang-Mills Theory

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Abstract. In this talk, we review some recent results of the center vortex model for the infrared sector of $SU(3)$ Yang-Mills theory. Particular emphasis is put on the order of the finite-temperature deconfining phase transition and the geometrical structure of vortex branchings. We also present preliminary data for the ’t Hooft loop operator and the dual string tension near the phase transition.

Introduction

The vortex picture of the Yang-Mills vacuum, initially proposed as a possible mechanism of colour confinement, has recently attracted a renewed attention. This is mainly due to the advent of new gauge fixing techniques which permit the detection of center vortex structures directly within lattice Yang-Mills configurations. Numerical studies have revealed that the center projection vortices detected in this way do locate true physical objects (rather than lattice artifacts) [1], and there is by now ample evidence that the infrared properties of Yang-Mills theory can be accounted for in terms of vortices [2].

Based on these ideas, a random vortex world-surface model was introduced as an effective low-energy description of $SU(2)$ Yang-Mills theory [3]; it has recently been extended to the gauge group $SU(3)$ [4]. The fundamental assumption is that the long-range structure of Yang-Mills theory is dominated by extended tubes of center flux tracing out closed surfaces in space-time. Consequently, we realise our model on a space-time lattice in which the fixed spacing $a$ represents the transverse thickness of vortices. The random surfaces created on this lattice are weighted by a model action containing a Nambu-Goto and a curvature term, with dimensionless coupling constants $\epsilon$ and $c$, respectively. Physically, this means that vortices have a certain surface tension and they tend to be stiff. For further details on our model and the determination of the parameters $\epsilon$ and $c$ (as well as the vortex extension $a = 0.39\, \text{fm}$), the reader is referred to [4].

Finite Temperature Phase Transition and Vortex Branching

Fig. 1 shows histograms of the action densities measured on $30^3 \times 2$ lattices at the critical points for the two gauge groups $G = SU(3)$ (left panel) and $G = SU(2)$ (right panel). As can be clearly seen, the $SU(3)$ transition exhibits the shallow double-peak structure characteristic for a weak first order transition, while the $SU(2)$ transition is...
continuous (second order). This qualitative behaviour is in agreement with findings from lattice gauge theory.

Since triality is only conserved mod 3, an arbitrary number $\nu = 0, \ldots, 6$ of vortex surfaces can meet at each link. The odd values $\nu = 3, 5$ are not allowed in $SU(2)$ and represent genuine $SU(3)$ vortex branchings. This phenomenon is best studied in 3D slices of the lattice, whence possible branching links are projected onto points of type $\nu$. From fig. 2 we conclude that the largest volume fraction in the confined phase corresponds to non-branching vortex matter ($\nu = 2$), with a considerable probability of both branchings ($\nu = 3, 5$) and self-intersections ($\nu = 4, 6$). Only 15% of the volume is not occupied by vortices ($\nu = 0$). In the deconfined phase ($T > T_c$), the situation is qualitatively unchanged for time-slices, while space slices show virtually zero branchings above $T_c$. This can be understood if the vortices undergo a (de)percolation phase transition above $T_c$ and most vortex clusters wind directly around the short time direction [3], [4].

‘t Hooft Loop

The ‘t Hooft loop can be viewed as a vortex creation operator [5] that implements twisted boundary conditions when extended over an entire lattice plane [6]. It has been shown to be an alternative (dis)-order parameter for the deconfinement phase transition.
whose behaviour is dual to the Wilson loop [6].

This expectation is confirmed in our model: The left panel of fig. 3 exhibits a linear rise of the free energy with the area of the ’t Hooft loop, which permits to define a dual string tension in the deconfined phase. As we approach the phase transition from above, the dual string tension quickly vanishes (cf. right panel of fig. 3). Precise measurements close to the transition reveal a small discontinuity \( \Delta \tilde{\sigma} \approx (34 \text{ MeV})^2 \), which should be compared to the ordinary zero-temperature string tension \( \sigma_0 = (440 \text{ MeV})^2 \) setting the overall scale. This demonstrates the weakness of the first order transition for \( G = SU(3) \).

Conclusions

In this talk, the physical foundation of the center vortex model for the infrared sector of \( SU(3) \) Yang-Mills theory has been outlined. Only a selection of the results obtained so far could be presented. Among the effects discussed were the order of the deconfinement phase transition, the structure of branching points and the exact determination of the discontinuity in the free energy of the ’t Hooft loop. Interesting questions for future investigations are the study of deconfinement in higher colour groups, in particular the influence of complex branching patterns, as well as the coupling to quarks and the generation of a chiral condensate for \( G = SU(3) \).

REFERENCES