Flux tube counting or Casimir scaling

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Abstract. QCD confines quarks in all representations. From lattice calculations and fat center vortices model, we discuss that the coefficient of the linear term in the potential is proportional to both casimir scaling and the number of fundamental strings.

Explaining the confinement of quarks in QCD is still a challenging problem. Even though perturbative techniques describe very well the behavior of quarks at short distances, the mechanism of confinement which prevents quarks and gluons to be found free, is still a puzzle. Many lattice calculations confirm the confinement of quarks in the fundamental and higher representations at intermediate distances, but phenomenological models have not been fully successful in describing the infrared behavior of quarks and gluons. One of the features of the confinement is that the potential between quarks increases by distance. Recent numerical calculations [1, 2] show that the string tension, the coefficient of the linear term in the potential, is representation dependent and proportional to the eigenvalue of the quadratic casimir operator of the representation. This proportionality is called “Casimir scaling”. The proportionality of the potential with casimir operator is expected for short distances where the force between quarks can be described by one gluon exchange and the coupling is proportional to the quadratic casimir operator. But this behavior is still not understood for intermediate distances. On the other hand, some people believe the string tension between two quarks of a representation can be obtained by multiplying the fundamental string tension times the number of fundamental flux tubes embedded into that representation. The fundamental (elementary) string is a string which connects a fundamental heavy quark with an anti-quark. In this paper, we take a closer look at lattice results and explain that string tensions may also agree with the flux tube counting idea. We also discuss results obtained for potentials between quarks from the fat center vortices model for SU(2), SU(3) and especially our recent calculations for SU(4) and show that string tensions are in agreement with both casimir scaling and flux tube counting but they agree better with flux tube counting especially for larger gauge groups.

Although lattice calculations show a qualitative agreement between string tensions and casimir operators, looking more carefully at lattice results, one can interpret string ratios with the idea of flux tube counting. Cross signs on the plot on the left hand side of figure 1 show ratios of string tensions of SU(3) sources of various representations to that of the fundamental representation obtained from lattice calculations [1]. In this paper, we sometimes call sources in higher representations as “quarks”. Circles indicate casimir ratios and diamonds show the number of fundamental strings in each representation. As claimed by lattice people, ratios of string tensions are proportional to casimir operators.
On the other hand, the plot shows that there is a rough agreement with the number of fundamental tubes as well. A. Armoni et al. [3] have explained why ratios from the lattice are larger than the number of fundamental fluxes. If we define the potential between two quarks as the potential between fundamental strings, then, when strings are very far from each other, they do not have any interaction and the string tension is equal to the string tension of quarks in the fundamental representation times the number of strings. On the other hand, if we put quarks in an appropriate distance, the elementary strings attract each other and this attraction reduces the string tension to some number less than the potential of elementary strings. If quarks get even closer, an overlap between strings happens. Therefore, they repel each other and the string tension between two sources decreases such that it gets larger than the potential between fundamental tubes. As Armoni et al. have discussed, the typical length/thickness ratio of the fundamental string of lattice calculations is not large enough. Thus, an overlap between fundamental strings may exist which leads to a repulsion and therefore makes string ratios larger than the number of fundamental fluxes.

Now, we discuss results obtained from the fat center vortices model for SU(2), SU(3) and SU(4) gauge groups. The center vortices model is a phenomenological model which claims that the QCD vacuum is a condensate of color magnetic vortices. These vortices are responsible for confining color charges. For intermediate distances, center vortex model has predicted the confinement of quarks in the fundamental representation. M. Faber et al. [4] have observed the confinement of quarks for all representations by making the vortices thick enough. The plot on the right hand side of figure 1 shows ratios of string tensions for SU(2) sources [4]. Potentials for quark charges in the $j = 1/2$, 1, 3/2 are calculated and potentials ratios of sources with the $j = 1$ (adjoint) and $j = 3/2$ to that of the fundamental quark ($j = 1/2$) are plotted. As indicated in the figure, ratios start up at of casimir ratios which are 8/3 and 5 for $j = 1$ and $j = 3/2$, respectively. Figure
2 shows potential ratios of quarks in various representations to that of the fundamental one, for SU(3) [5] and SU(4) [6] gauge groups. Again, ratios start up roughly at ratios of corresponding casimirs but change so that at some region, which is different for each representation, get close to the number of fundamental strings of that representation. The agreement with flux tube counting in the most linear part of the potential is better for SU(4) than SU(3) [7]. This is in agreement with reference [8] which claims that by increasing the number of gauge groups, the interaction between fundamental strings decreases and the total string tension would be the fundamental string tension times the number of strings.

![FIGURE 2. Ratios of potentials between quarks of higher representations to that of the fundamental one, for SU(3) and SU(4) gauge groups. Casimir ratios and the number of fundamental strings are shown in the first and second parentheses, respectively. Ratios start at corresponding casimirs but get close to the number of fundamental strings.](image)

We conclude that both lattice calculations and the fat center vortices model predict a linear regime for the potential between quarks of the fundamental and higher representations. The string tension in that region is proportional to both casimir scaling and the number of fundamental tubes. The proportionality with the number of fundamental tubes seems to be better from fat center vortices model especially for SU(4) gauge group.

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REFERENCES