Recent Progress in Intersecting D-brane Models

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Abstract: The aim of this article is to review some recent progress in the field of intersecting D-brane models. This includes the construction of chiral, semi-realistic flux compactifications, the systematic study of Gepner model orientifolds, the computation of various terms in the low energy effective action and the investigation of the statistics of solutions to the tadpole cancellation conditions.

1 Introduction

Since its discovery in the year 2000 [1], the field of intersecting D-brane models has developed and matured considerably and is by now a well established string theoretic framework for (semi-)realistic string compactifications (for reviews see [2]). While in the beginning most of the effort went into developing new model building techniques and studying simple examples, during the last 2 years one of the main questions was the computation of the low energy effective action and its physical consequences. At least for flat string backgrounds now techniques are available to compute the tree-level Kähler potential [3], the Yukawa couplings [4,5], gauge threshold corrections [6] and the susy breaking soft terms on D3 and D7-branes [8,9,10].

Independently we have learned that compactifications with non-vanishing background fluxes give rise to a scalar potential, which allows to freeze some of the moduli notoriously present in supersymmetric string compactifications [11]. These observations not only led to a proposal for realizing metastable de-Sitter vacua in string theory [12] but also have drastically influenced the way we think about the landscape of string vacua. In fact, the number of vacua appearing in such flux compactifications is so enormous that without any further guidance it is quite questionable whether there is any chance to ever find the realistic string vacuum. Alternatively, a statistical approach to the string vacuum problem was proposed in [13] and methodologically pioneered in [14].

In this article I would like to briefly summarize some of the recent developments in the field of intersecting D-brane models. Please note that the selection of topics reflects the author’s preferences and is not meant to disgrace other important contributions in this field. In section 2, I will review recent attempts to combine flux compactifications with intersecting branes by constructing semi-realistic chiral flux compactifications in a treatable setting. Section 3 is devoted to summarize recent studies on a large set of intersecting D-brane models on highly curved backgrounds, namely orientifolds of Gepner
models. In section 4, I will try to briefly sketch what has been achieved in determining the low energy effective action for intersecting D-brane models. Finally, section 5 surveys a very recent investigation of the statistics of solutions to the tadpole cancellation conditions. This article is based on a review talk the author gave at the 37th Symposium Ahrensboop, 23-27 August 2004, and as such not only contains results published by the author himself but also by many others.

2 Semi-realistic flux compactifications

During the last years string compactifications with non-trivial background fluxes were studied in many variations. It turned out that these models can solve some of the problems purely geometric string compactification notoriously had. In particular, certain fluxes induce an effective potential that still possesses supersymmetric minima, which allows to freeze (some of) the moduli generically appearing in string theory. It was also possible to break supersymmetry in a controlled way by turning on additional internal flux components. Finally, by taking also some non-perturbative effects into account, for the first time strong evidence was given that non-supersymmetric meta-stable de-Sitter vacua do exist in string theory [12].

However, the original framework contained only parallel D3-branes on which no semi-realistic gauge theory can arise. To reconcile this, it was proposed to combine ideas from flux compactifications with ideas from intersecting respectively magnetised branes to build chiral semi-realistic string models with fluxes and partly frozen moduli [15]. Concretely, the Type IIB closed string background was chosen to be the orbifold $M = T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with Hodge numbers $(h_{21}, h_{11}) = (51, 3)$ (the T-dual of the Type IIA orientifold studied in [16]). In addition one performs the orientifold projection $\Omega R(-1)^F$, where $R$ reflects all six internal directions. Flux compactifications on the mirror symmetric Calabi-Yau given by the orbifold with discrete torsion were also considered and will be discussed in more detail in [17].

Turning on 3-form fluxes $H_3$ and $F_3$, the Chern-Simons term in the Type IIB effective action induces a 4-form tadpole given by

$$N_{\text{flux}} = \frac{1}{(4\pi^2\alpha')^2} \int_M H_3 \wedge F_3. \quad (1)$$

The fluxes obey the Bianchi identity and take values in $H^3(M, \mathbb{Z})$, i.e.

$$\frac{1}{(2\pi)^2\alpha'} \int_M H_3 \in N_{\text{min}} \mathbb{Z}, \quad \frac{1}{(2\pi)^2\alpha'} \int_M F_3 \in N_{\text{min}} \mathbb{Z}, \quad (2)$$

where $N_{\text{min}}$ is an integer guaranteeing that in orbifold models only untwisted 3-form fluxes are turned on, for which we can trust the supergravity approximation. Taking also the orientifold projection into account for the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold one gets $N_{\text{min}} = 8$. Defining $G_3 = \tau H_3 + F_3$ the kinetic term for the G-flux

$$S_G = -\frac{1}{4\kappa_{10}^2 3(\tau)} \int_M G \wedge \star_6 G, \quad (3)$$

generates a scalar potential which can be derived from the GVW-superpotential [18, 19]

$$W = \int_M \Omega_3 \wedge G. \quad (4)$$
As is apparent the scalar potential only depends on the complex structure moduli and the
dilaton. It is of no-scale type and vanishes for imaginary self-dual fluxes (ISD) \(*_6 G = i G\),
e.g. for \(G\) of type \((2,1)\) or \((0,3)\). One can show that the minimum is supersymmetric if
\(G\) is solely of type \((2,1)\).

In order to cancel the resulting tadpoles, one introduces in the usual way magnetised
D9-branes, which are T-dual to the intersecting D6-branes studied in many papers. Such
a magnetised brane is characterised by three pairs of integers \((n^I_a, m^I_a)\) which satisfy
\[
\frac{m^I_a}{2\pi} \int_{T^2_I} F^I_a = n^I_a, \tag{5}
\]
where the \(m^I_a\) denote the wrapping number of the D9-brane around the torus \(T^2_I\) and \(n^I_a\) is
the magnetic flux. The orientifold projection acts as follows on these quantum numbers
\[
\Omega \mathcal{R}(-1)^{F_L} : (n^I_a, m^I_a) \rightarrow (n^I_a, -m^I_a). \tag{6}
\]
In order for each brane to preserve the same supersymmetry as the orientifold planes,
they have to satisfy
\[
\sum_I N_a n^I_a n^2_a n^3_a = 8 - \frac{N_{\text{flux}}}{4} \tag{6}
\]
\[
\sum_I N_a n^I_a m^J_a m^K_a = -8 \text{ for } I \neq J \neq K \neq I. \tag{7}
\]
In order for each brane to preserve the same supersymmetry as the orientifold planes,
they have to satisfy
\[
\sum_I \arctan \left( \frac{m^I_a K^I_a}{n^I_a} \right) = 0, \tag{8}
\]
where \(K^I_a\) denotes the volume of the I-th torus \(T^2\) in units of \(\alpha'\). The number of chiral
fermions between two different magnetised branes is given by the index
\[
I_{ab} = \prod_I (n^I_a m^I_b - m^I_a n^I_b) \tag{9}
\]
and can lead to matter in bifundamental, symmetric or anti-symmetric representations of
the gauge group.

Taking the flux quantisation with \(N_{\min} = 8\) into account, the contribution of the flux
to the D3-brane tadpole is given by \(N_{\text{flux}}/4 \in 16 \mathbb{Z}\). Therefore, for non-trivial flux the
right hand side of the D3-brane tadpole cancellation condition \([4]\) is always negative. In
\([15]\) this led to the conclusion that no globally supersymmetric solutions to the tadpole
cancellation conditions do exist. However this was too naive, namely in \([20]\) it was shown
that there exist supersymmetric branes which give the ”wrong” sign in one of the four
tadpole cancellation conditions. Consider for instance the magnetised brane \((n^I_a, m^I_a) =
(-2,1)(-3,1)(-4,1)\), which is supersymmetric for
\[
\arctan(A_1/2) + \arctan(A_2/3) + \arctan(A_3/4) = \pi \tag{10}
\]
and contributes as \((-24,-4,-2,-3)\) to the four tadpole conditions. Precisely branes of
this type were used in \([20, 21]\) to construct globally supersymmetric, chiral, MSSM like
flux compactifications. For illustrative purposes, let me present here only one of their
examples.

Choosing the 3-form flux as
\[
G_3 = \frac{8}{\sqrt{3}} e^{-\frac{2\pi}{3}} (d\bar{z}_1 dz_2 dz_3 + d\bar{z}_1 d\bar{z}_2 dz_3 + d\bar{z}_1 dz_2 d\bar{z}_3), \tag{11}
\]
Table 1: Wrapping numbers for semi-realistic model.

<table>
<thead>
<tr>
<th>$N_a$</th>
<th>$(a_1, m_1)$</th>
<th>$(a_2, m_2)$</th>
<th>$(a_3, m_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_a = 3$</td>
<td>(1, 0)</td>
<td>(1, 1)</td>
<td>(1, -1)</td>
</tr>
<tr>
<td>$N_b = 1$</td>
<td>(0, 1)</td>
<td>(1, 0)</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>$N_c = 1$</td>
<td>(0, 1)</td>
<td>(0, -1)</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>$N_d = 1$</td>
<td>(1, 0)</td>
<td>(1, 1)</td>
<td>(1, -1)</td>
</tr>
<tr>
<td>$N_{h_1} = 1$</td>
<td>(-2, 1)</td>
<td>(-3, 1)</td>
<td>(-4, 1)</td>
</tr>
<tr>
<td>$N_{h_2} = 1$</td>
<td>(-2, 1)</td>
<td>(-4, 1)</td>
<td>(-3, 1)</td>
</tr>
<tr>
<td>$N_f = 4$</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
</tr>
</tbody>
</table>

yields a contribution $N_{\text{flux}}/4 = 48$ to the tadpole condition and freezes the moduli at $U^I = \tau = e^{2\pi i/3}$. Introducing the supersymmetric branes shown in Table 1 cancels all the tadpoles and gives rise to a one-generation MSSM-like model with gauge group

$$G = SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times [U(1)^{1} \times USp(8)].$$

Supersymmetry enforces the additional constraint $A_2 = A_3$. For more technical and phenomenological details of such models please consult the original literature. Note that the branes $b, c$ can be placed directly on top the corresponding $O7$-planes yielding a gauge group $SU(2) \times SU(2)$. This example shows that it is indeed possible to construct supersymmetric semi-realistic string models with fluxes and partly frozen moduli. This is an encouraging observation, but of course much more work is needed to really establish an entire class of such models. A first step towards generalizations of these kinds of models has been carried out in [22]. A different approach allowing more general gauge fluxes on $T^6$ has been proposed in [23].

3 Gepner Model orientifolds

After some earlier attempts [24], there was quite a revival of interest in the construction of Gepner model orientifolds during the last year [25, 26, 27, 28]. Since Gepner models are known to describe exactly solvable non-linear sigma models on certain Calabi-Yau manifolds at radii of string scale size, the study of such models tells us something about a regime which is not accessible via perturbative supergravity methods. I would like to emphasize that while the construction of supersymmetric intersecting brane models in the geometric regime of the Calabi-Yau moduli space is hampered by our ignorance about concrete examples of special Lagrangian 3-cycles, at small radii conformal field theory techniques applied to Gepner models allow us to construct a plethora of concrete intersecting D-brane models.

One starts with Type IIB string theory on a Calabi-Yau threefold, where the internal part is described by a GSO projected $\mathcal{N} = 2$ superconformal field theory (SCFT) with central charge $c = 9$. Gepner proposed to use tensor products of unitary representations of the $\mathcal{N} = 2$ super Virasoro algebra for the internal SCFT, which has to be equipped with a GSO projection onto states of integer $U(1)$ charge. There are 168 tensor products of this sort, which can be identified with certain Calabi-Yau spaces given by hypersurfaces in weighted projective spaces. However, by additional orbifold respectively simple current extensions, this set of modular invariant SCFTs can be extended to $O(1000)$ different models.
Next one defines an orientifold projection $\Omega \sigma$, where $\Omega$ denotes the word-sheet parity transformation and $\sigma$ an internal symmetry of the SCFT, like phase symmetries or extra quantum symmetries acting differently on various twisted sectors. There have been different approaches to proceed with the orientifold construction.

First, following the earlier attempts in [24], one can implement the orientifold projection and compute directly the loop-channel Klein-bottle amplitude [25, 26]. This allows one to fix the crosscap state up to some sign factors, which have to be determined later. From the crosscap states one can read off the various non-vanishing tadpoles, which are to be cancelled by additional sources given by rational boundary states (D-branes) of the Gepner model. Note that these highly symmetric rational boundary states à la Cardy are only a small subset of all possible supersymmetric boundary states. It would be interesting to construct more general boundary states and use them for model building. Requiring consistency between the boundary and crosscap states allows one to fix the signs in the crosscap states. It only remains to solve the tadpole cancellation conditions, which for a complicated model can become quite involved by hand.

In the second approach one invokes known results from the study of the crosscap states in general rational conformal field theory [29, 30]. Therefore one starts with the crosscap and boundary states and from there determines the rest of the model [27, 28]. Clearly both approaches are equivalent and historically both have been used.

The main advantage of this SCFT approach is that everything is algebraic and the resulting expressions for the tadpole conditions, the annulus and Möbius strip amplitudes can be written in general form admitting the implementation in a computer code. In fact in [28] a systematic computer search for three-generation semi-realistic models was carried out. This impressive search is by far the most ambitious attempt carried out ever to search for Standard-like intersecting D-brane models. It revealed that there exist $O(10^5)$ of models resembling the Standard Model at least in their topological quantities. The authors also investigated other phenomenological features like the number of Higgs fields, the number of adjoint scalars, the tree-level gauge couplings and the number of hidden sector gauge groups. For details I would like to refer the readers to the original work [28].

It would be interesting to see how many of these Standard-like models survive, if one also requires more refined data to agree with the Standard Model. As an example one should develop techniques to compute Yukawa couplings, i.e. the three point function of three boundary changing operators. Since all radii are fixed at the order of the string scale, it is far from clear how a hierarchy between the Yukawa’s could be generated. It could be like proposed in [11] that only the third generation is massive at string tree level, and that the lighter two generations get their mass via loop corrections.

In view of the statistical approach to the string vacuum problem, to be discussed in section 5, the set of Gepner model orientifolds can clearly provide a nice testing ground. It would be interesting to develop statistical methods to determine the various distributions of gauge theoretic observables in the ensemble of Gepner model orientifolds and test the results against a Monte-Carlo based frequency computation.

A question which appears to be much harder to answer is what happens when one turns on fluxes in the Gepner model. Since Gepner models live at string size radii, the supergravity techniques one usually invokes to study flux compactifications are not applicable. Moreover, one cannot simply take the Gepner models and continuously deform them to large radii for they are lying on lines of marginal stability and supersymmetry is generally broken by the deformation. Therefore, before even thinking about fluxes, one
has to study what happens for deformations away from the Gepner point.

4 The low energy effective action

In this section I would like to briefly summarize the status of knowledge about the low energy effective action arising from supersymmetric intersecting D-brane models. Only a very tiny selection of the relevant material is given and I would like to refer the reader for more details to the still growing original literature.

It is clear that in order to make contact between stringy constructions and low energy particle physics, the determination of the low energy effective action is absolutely essential. Since the precise form of the action depends sensitively on the details of the string model, it is first important to work out the necessary general technical tools and apply them to sufficiently simple toy models. This describes precisely the situation at the moment, where we are learning how such computations are performed for relatively simple examples.

4.1 The supersymmetric effective action

The effective $\mathcal{N} = 1$ supersymmetric action depends on the holomorphic gauge kinetic function $f(Z)$, the holomorphic superpotential $W(Z)$ and the non-holomorphic Kähler potential $K(Z, \phi Z)$.

- Gauge couplings: Each stack of D6-branes comes with its own gauge coupling. For a supersymmetric brane wrapping a 3-cycle $\pi_a$, the tree level result for the holomorphic gauge kinetic function reads [31, 32]

$$f(U_i) = \frac{M_s^3}{(2\pi)^4} \left[ e^{-\varphi} \int_{\pi_a} \Re(\Omega_3) + 2i \int_{\pi_a} C_3 \right]$$

where $C_3$ denotes the R-R three-form. Apparently, the gauge coupling at tree level only depends on the complex structure moduli. It is known that the gauge couplings receive corrections at the one-loop level, the so-called gauge threshold correction. These are very hard to compute for a generic model, as one has to know the massive string spectrum. For intersecting branes on a toroidal (orbifold) background these threshold corrections have been computed in [6]. The main result is that the one-loop correction for open string sectors preserving $\mathcal{N} = 4$ supersymmetry vanish. For $\mathcal{N} = 2$ sectors they both depend on the complex and Kähler moduli, whereas for $\mathcal{N} = 1$ they are given by the nice expression for the gauge coupling of $SU(N_a)$

$$\Delta_{ab} = -b_{ab} \ln \frac{\Gamma(1 - \theta_{ba}^1)\Gamma(1 - \theta_{ba}^2)\Gamma(1 + \theta_{ba}^1 + \theta_{ba}^2)}{\Gamma(1 + \theta_{ba}^1)\Gamma(1 + \theta_{ba}^2)\Gamma(1 - \theta_{ba}^1 - \theta_{ba}^2)},$$

where $b_{ab} = N_b I_{ab} \text{Tr}(Q_a^2)$ and $\theta_{ba}^I = \frac{1}{2} \Phi_{ba}^I$ with $\Phi_{ba}^I$ denoting the difference of the intersection angles on the three $T^2$ factors. Implicitly, this expression only depends on the complex structure moduli.

- Kähler potential: The Kähler potential for the various massless chiral multiplets arising in intersecting D-brane models has been determined in [3] by a string scattering computation. Again for concreteness it was assumed that one is dealing with a toroidal (orbifold) background. Here I would like to present only one of the many
nice results contained in [3], namely the Kähler potential for the charged chiral superfields. These fields arise from open strings stretched between two intersecting D-branes and the Kähler potential reads

\[ G_{ab} = \kappa_4^{-2} \prod_{l=1}^{3} (T^l - \overline{T}^l)^{-\theta^l_{ab}} \sqrt{\frac{\Gamma(\theta^l_{ab})}{\Gamma(1 - \theta^l_{ab})}} \] (15)

depending on the Kähler moduli and implicitly also on the complex structure moduli.

- The superpotential: In section 2, we have already encountered one possible contribution to the superpotential. Namely considering the T-dual Type IIB set-up with magnetised D-branes, turning on 3-form flux \( G_3 \) induces the GVW-type superpotential (4). Another contribution contains the Yukawa couplings for three massless chiral multiplets. The resulting physical Yukawa couplings, i.e. those for canonical normalized fields are given by [7]

\[ Y_{ijk} = e^{K/2} (K_{ab}K_{bc}K_{ca})^{-\frac{1}{2}} W_{ijk} \]

\[ = 2\pi \prod_{l=1}^{3} \left[ 16\pi^2 \frac{\Gamma(1 - \theta^l)\Gamma(1 - \nu^l)\Gamma(\theta^l + \nu^l)}{\Gamma(\theta^l)\Gamma(\nu^l)\Gamma(1 - \theta^l - \nu^l)} \right]^{\frac{1}{4}} \sum_m \exp \left( -\frac{A_I(m)}{2\pi\alpha'} \right) \] (17)

with \( \theta^l = \frac{1}{\pi} \Phi^l_{ba} \) and \( \nu^l = \frac{1}{\pi} \Phi^l_{cb} \). The sum is over all world-sheet instantons with the boundary given by the three intersecting D-branes. These sums have been analysed in [4], where it was shown that they can be expressed in terms of theta-functions. It was shown in [5] that in the mirror symmetric model with magnetised branes the superpotential part \( W_{ijk} \) of the Yukawa couplings can be obtained by a tree-level (in \( \alpha' \)) computation.

### 4.2 Soft susy breaking terms

For phenomenological reasons it is clearly not sufficient to determine the supersymmetric effective action. Since supersymmetry must be broken at a certain scale, it is desirable to also have control over a supersymmetry breaking mechanism and compute the resulting soft supersymmetry breaking terms. For supersymmetry breaking via fluxes in Type IIB string theory with magnetised D-branes such a program has been carried out recently [8, 9, 10]. Here two different approaches have been pursued: One can either include the effect of fluxes in the Dirac-Born-Infeld action for the D3 and D7 branes and expand the resulting action to lowest order in the transverse coordinates and read off the resulting soft terms. Alternatively, one can parameterize the susy breaking by the VEVs of the various auxiliary fields of the closed string moduli superfields and use the general supergravity formalism for determining the resulting soft-terms. These two approaches are expected to be equivalent. Again for more detailed results the reader should consult the original literature. Here I would like to just state some general observations:

- The soft terms vanish for D3-branes in ISD flux backgrounds. However, for \( 0D3 \) branes the soft terms are non-vanishing and in particular, masses are induced for the brane position moduli.

- The susy breaking soft terms on D7 branes vanish for (2,1) fluxes, but are non-vanishing for a susy breaking (0,3) component of the flux. For (2,1) fluxes also supersymmetric \( \mu \)-terms for the brane moduli are generated, which is in accord with the F-theory expectation [33].
• The induced susy breaking scale on the D7-branes is roughly of the order $\frac{M^2}{M_{pl}}$ and therefore for TeV scale susy breaking favours a string scale in the intermediate regime.

5 Statistics of intersecting D-branes

After the realization that flux compactifications give rise to a densely populated landscape of string vacua, M.R. Douglas was the first to propose that, given these $10^{500}$ different vacua, one should better try a statistical approach to the string vacuum problem [13]. It was pointed out that such an approach might even turn out to be predictive in the sense that it leads for instance to strong statistical correlations having the potential to falsify string theory.

In [14, 34] very powerful statistical methods were developed to determine distribution of flux vacua over the complex structure moduli space. These were extended and refined in [35] to also count the number of Minkowskian backgrounds. Of course for phenomenological reasons it is very important to also include the brane sector into the statistics. Some general claims were made already in the original work [13], while more refined methods were presented in [36] in the context of intersecting D-branes respectively magnetized D-branes.

More concretely in [36], for the ensemble of intersecting branes on certain toroidal orientifolds, the statistical distribution of various gauge theoretic quantities was studied, like the rank of the gauge group, the number of models with an $SU(M)$ gauge factor and the number of generations. In [36] the examples of intersecting branes on the $T^2$, $T^4/\mathbb{Z}_2$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds were discussed.

Let us briefly summarize the main computational technique used to determine these distributions. The first step is to determine all or at least a large, preferably representative subset of supersymmetric branes. After solving the supersymmetry constraints, in all the examples discussed in [36], this was given by a subset $S$ of the naively allowed wrapping numbers $X_I$. As a constraint one faces the various tadpole cancellation conditions

$$\sum_{a=1}^{k} N_a X_{a,I} = L_I$$

with $I = 1, \ldots, b_3/2$ and $L_I$ denoting the contribution from the orientifold planes and the fluxes. Realizing that the problem of counting the number of solutions to these equations is similar in spirit to the counting of unordered partitions of an integer, the method used for determining the approximate expansion in the latter case can be generalized to our problem. It turns out that the number of such solutions is given by the expression

$$\mathcal{N}(\vec{L}) \approx \frac{1}{(2\pi i)^{\frac{b_3}{2}}} \int \prod_I dq_I \frac{dL_I}{q_I^{L_I+1}} \exp \left( \sum_{X_I \in S} \frac{\prod_I q_I^{X_I}}{1 - \prod_I q_I^{X_I}} \right),$$

which can be evaluated at leading order by a saddle point approximation with

$$f(\vec{q}) = \sum_{X_I \in S} \frac{\prod_I q_I^{X_I}}{1 - \prod_I q_I^{X_I}} - \sum_I (L_I + 1) \log q_I.$$  

The saddle point is determined by the condition $\nabla f(\vec{q})|_{\vec{q}_0} = 0$, and the second order
saddle point approximation reads

\[
\mathcal{N}^{(2)}(\vec{L}) = \frac{1}{\sqrt{2\pi}} \frac{e^{f(\vec{q}_0)}}{\sqrt{\det \left[ \left( \frac{\partial^2 f}{\partial q_i \partial q_j} \right) \right]_{\vec{q}_0}}}. 
\]

(21)

For the more complicated statistical distributions, one gets similar results, which all can be estimated via the saddle-point approximation.

After having checked that the number of solutions to the tadpole cancellation conditions for the 8D, 6D and 4D examples are finite, various gauge theoretic distributions were computed and compared to a brute force computer classification. Cutting a long story short, the following qualitative results we obtained

- The probability to find an \(SU(M)\) gauge factor scales like

\[
P(M) \simeq \exp \left( -\sqrt{\frac{\log L}{L}} M \right)
\]

(22)

and for \(\sum_{i=1}^k M_i \ll L\) satisfies mutual independence, i.e. \(P(M_1 \ldots M_k) = \prod_i P(M_i)\).

- The rank distribution yields approximately a Gauss curve with the maximum depending on the complex structure moduli and whether one allows multiple wrapping or not.

- Defining a measure for the chirality of a solution by \(\chi = \pi' \circ \pi\), in the 6D case a scaling like

\[
P(\chi) \simeq \exp (-\kappa \sqrt{\chi})
\]

(23)

was found with \(\kappa\) denoting some constant depending presumably on the \(L_I\).

- A strong statistical correlation between the rank of the gauge group and the chirality (number of families) was found, which can be traced back to the tadpole cancellation conditions. The higher the chirality is, the lower becomes the average rank of the gauge group.

It is interesting to study what happens when one combines the flux statistics with the D-brane statistics. The question is whether by averaging over more parameters maybe the various distributions become essentially uniform. To get a first glimpse, the T-dual Type IIB model with fluxes and magnetized D9-branes was considered. For the rank distribution for instance one gets the following expression

\[
\overline{P}(r) = \frac{1}{N_{\text{norm}}} \sum_{N_{\text{flux}}=0}^{N_{\text{flux}}^{\text{max}}} (N_{\text{flux}} + 1)^K \mathcal{N}(r; L_0 - N_{\text{flux}}, L_1, L_2, L_3),
\]

(24)

where \(\mathcal{N}(r, L_I)\) is just the unnormalised part of the distribution \(19\). Varying the number of 3-cycles, \(K\), gives the distribution shown in Figure 1. One realizes that the distribution is far from being uniform and that for large values of 3-cycles new maxima appear, which for instance contain models of the sort discussed in section 2.

Clearly, we are just beginning to approach the problem of unravelling the statistics on the landscape of string theory. The final aim would be to perform the statistics over as
Figure 1: The rank distribution after averaging over flux vacua for $L_0 = L_1 = L_2 = L_3 = 8$, $U_1 = 1$ and $N_{\text{flux}}^{\text{max}} = 11$.

many parameters as possible to really get a realistic picture of what overall statistical averages can tell us about the distribution of various physical quantities. The methods shown above might play an important role whenever one encounters string theoretic constraints similar to the tadpole cancellation conditions. More modestly, as a next step it would be interesting to study the distributions of heterotic string vacua and to see whether, as expected from string dualities, they feature similar patterns as the orientifolds. As I have mentioned already, Gepner model orientifolds might provide a nice testing ground for comparing and possibly refining the technical statistical tools. In principle, having agreed upon a good statistical ensemble one would like to address questions concerned directly with the Standard Model, like

• What is the percentage of models having the right gauge group, matter and number of families?

• How drastically is this number reduced by requiring more detailed constraints, like the right gauge and Yukawa couplings, the right Higgs couplings, absence of exotic matter?

• Having installed all phenomenological constraints, how does the distribution of the susy breaking scale and the cosmological constant look like?

The answers to these questions will strongly depend on possible statistical correlations among the various quantities, the realization of which I consider as the most interesting aspect of this endeavour.

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