Structure formation constraints on the Jordan–Brans–Dicke theory

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We use cosmic microwave background data from WMAP, ACBAR, VSA and CBI, and galaxy power spectrum data from 2dF, to constrain flat cosmologies based on the Jordan–Brans–Dicke theory, using a Markov Chain Monte Carlo approach. From these data we obtain a conservative bound on the Brans–Dicke parameter \( \omega \) to be greater than 100.

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I. INTRODUCTION

Jordan–Brans–Dicke (JBD) theory \[1,2\] is the simplest extended theory of gravity, depending on one additional parameter, the Brans–Dicke coupling \( \omega \), as compared to General Relativity. As Einstein’s theory is recovered in the limit \( \omega \to \infty \), there will always be viable JBD theories as long as General Relativity remains so too. As such, it acts as a laboratory for quantifying how accurately the predictions of General Relativity stand up against observational tests. The most stringent limits are derived from radar timing experiments within our Solar System, with measurements using the Cassini probe \[3\] now giving a two-sigma lower limit \( \omega > 40,000 \) (improving pre-existing limits \[4\] by an order of magnitude).

With precision cosmological data now available, particularly on cosmic microwave background (CMB) anisotropies from the Wilkinson Microwave Anisotropy Probe (WMAP) \[5\], it is becoming feasible to obtain complementary constraints from the effect of modified gravity on the structure formation process, as suggested in Ref. \[6\]. That paper focussed on the way that \( \omega \) alters the Hubble scale at matter–radiation equality, which is a scale imprinted on the matter power spectrum, in an attempt to identify how large an effect can be expected. Subsequently, the expected microwave anisotropy spectra in the JBD theory were computed \[7\].

In this paper we make a comprehensive comparison of predictions of the JBD theory to current observational data, using WMAP and other CMB data plus the galaxy power spectrum as measured by the two-degree field (2dF) galaxy redshift survey. We define JBD models in terms of eight parameters, which are allowed to vary simultaneously. Our paper is closest in spirit to work by Nagata et al. \[8\], who considered a more general model, the harmonic attractor model, which includes JBD as a special case. However their dataset compilation was restricted to the WMAP temperature power spectrum, and, moreover, our work is the first study of this kind based on a Markov Chain Monte Carlo technique.

The constraint we will obtain is not competitive with the very stringent solar system bound given above, but is complementary in that it applies on a completely different length and time scale. Such constraints can therefore still be of interest in general scalar–tensor theories where \( \omega \) is allowed to vary. In that regard, our result is most comparable to cosmological constraints imposed on \( \omega \) from nucleosynthesis, which give only a weak lower limit of \( \omega > 32 \) \[9\].

II. FORMALISM

A. Background cosmology

The Lagrangian for the JBD cosmology is

\[
\mathcal{L} = \frac{m^2_{\text{Pl}}}{16\pi} \left( \Phi R - \frac{\omega}{\Phi} \partial_{\mu} \Phi \partial^{\mu} \Phi \right) + \mathcal{L}_{\text{matter}},
\]

where the Brans–Dicke coupling \( \omega \) is a constant, and \( \Phi(t) \) is the Brans–Dicke (BD) field whose present value must give the observed gravitational coupling. We have included factors of \( m_{\text{Pl}} \) to define \( \Phi \) as dimensionless.

The equations for a spatially-flat Friedmann universe are \[1,2,10\]

\[
\left( \frac{\dot{a}}{a} \right)^2 + \frac{\dot{\Phi}}{a} \frac{\ddot{\Phi}}{\dot{\Phi}} = \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\dot{\Phi}} \right)^2 + \frac{8\pi}{3m^2_{\text{Pl}}} \rho; \]

\[
\ddot{\Phi} + 3\left( \frac{\dot{a}}{a} \right) \dot{\Phi} = \frac{8\pi}{(2\omega + 3)m^2_{\text{Pl}}} (\rho - 3p),
\]

where \( a(t) \) is the cosmological scale factor, and \( \rho \) and \( p \) are the energy density and pressure summed over all types of material in the Universe.

The Universe is assumed to contain the same ingredients as the WMAP concordance model \[5\], namely dark energy, dark matter, baryons, photons and neutrinos. We make the simplifying assumptions of spatial flatness, dark energy in the form of a pure cosmological constant, and effectively massless neutrinos whose density is related to that of photons by the usual thermal argument. The
cosmological parameters are required for the evolution of $\Phi$ to increase as $\Lambda$ domination sets in. The cosmological parameters are $\omega = 200$, $h = 0.72$, and $\rho_{m,0} = 0.3$ in units of the standard cosmology critical density.

The present value of $\Phi$ must correctly reproduce the strength of gravity seen in Cavendish-like experiments, which requires

$$\Phi_0 = \frac{2\omega + 4}{2\omega + 3},$$

where here and throughout a subscript ‘0’ indicates present value. We will assume that the value of $\Phi_0$ in our Solar System is representative of the Universe as a whole, though this may not be absolutely accurate [11]. We also assume that the initial perturbations are given by a power-law adiabatic perturbation spectrum.

When the Universe is dominated by a single fluid there are a variety of analytic solutions known [12], where $\Phi$ is typically constant during a radiation era, slowly increasing during a matter era, and then more swiftly evolving as dark energy domination sets in. However we need solutions spanning all three eras and so will solve the equations numerically, for which we use the integration schemes used by [17].

The architecture of defast is based on the version 4.0 of cmbfast, although there has been a progressive code fork in the subsequent versions. defast takes as input the parameter set described in the previous subsection, and returns the microwave anisotropy spectra (for temperature and polarization) and the matter power spectrum. A dynamical and fluctuating scalar field, playing the role of the dark energy and/or the Brans–Dicke field, is included into the analysis together with the other cosmological components, following the existing general scheme [17].

In order to bring the model description into the formalism used by defast, we redefine the Brans–Dicke field and coupling according to

$$\phi^2 = \omega \frac{m_{\text{Pl}}^2}{2\pi} \xi; \ \xi = \frac{1}{4\omega},$$

which brings the Lagrangian into the form

$$\mathcal{L} = \frac{1}{2} \xi \phi^2 R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + \mathcal{L}_{\text{matter}},$$

where $\phi$ is now a canonical scalar field non-minimally coupled to gravity. We implement the cosmological constant in the code by giving $\phi$ a constant potential energy.

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**FIG. 1:** Evolution of the Brans–Dicke field from early in radiation domination to the present. It is just possible to see the evolution of $\Phi$ increase as $\Lambda$ domination sets in. The cosmological parameters are $\omega = 200$, $h = 0.72$, and $\rho_{m,0} = 0.3$ in units of the standard cosmology critical density.
Our calculations include the effect of perturbations, with the initial perturbations in $\phi$ fixed by the requirement of adiabaticity. The correction to the background expansion rate from the dynamics of $\phi$ is the most relevant effect on the CMB power spectrum, appearing as a projection plus a correction to the Integrated Sachs–Wolfe, as discussed in detail in Ref. \[10\].

C. Data analysis

The data we use are taken from VSA \[18\], CBI \[19\], ACBAR \[20\], WMAP \[21\] and the 2dF galaxy redshift survey \[22\]. In order to incorporate the 2dF data, the galaxy bias parameter $b$ is taken to be a free parameter for which the analytic marginalization scheme of Ref. \[23\] can be applied.

Our present analysis does not include supernovae data. Inclusion of the modification to the luminosity distance from $\omega$ would be straightforward. However the variation of the gravitational coupling $G$ means that supernovae can no longer be assumed to be standard candles, and Ref. \[24\] suggests that the effect from varying $G$ dominates. Further, inclusion of supernovae data may be particularly susceptible to the possibility that the local value of $\Phi$ in the vicinity of the supernova may not match the global cosmological value \[11\]. Nevertheless, it would be interesting to investigate robust methods for including such data.

We carry out the data analysis using the now-standard Markov Chain Monte Carlo posterior sampling technique, by modifying the June 2004 version of the CosmoMC program \[25\] to call DEFAST to obtain the spectra. CosmoMC computes the likelihood of the returned model and assembles a set of samples from the posterior distribution. We take full advantage of CosmoMC’s MPI capabilities by running the code across 30 Sun V60x Xeon 2.8GHz processors. The Metropolis–Hastings algorithm is run at a temperature of 2.0 in order to better sample the non-Gaussian direction of our posterior distribution which results from the degeneracy between $H_0$ and $\ln \xi$, both of which have a strong effect on the angular diameter distance. The final chains are then cooled and importance sampled \[25\]. It can be noted that for the purposes of posterior sampling, we have parametrized the Brans–Dicke cosmology using $\ln \xi \equiv -\ln 4\omega$ simply because it is more straightforward to obtain the samples we need, while simultaneously suppressing the possibility of jumping to regions with $\omega < 1$.

The results that we present are based on around 200,000 raw posterior samples, and while the basic constraints can be derived with significantly fewer samples, this large number assures more robust constraints on the derived parameter $\omega$ when we use importance sampling in order to adjust for the change in prior density \[25\].

III. OBSERVATIONAL CONSTRAINTS

Turning first to the constraints on the basic parameter set, from Figure 2 we note the overall consistency of our results with the current observational picture (see for example Ref. \[2\] and a work by two of the current authors Ref. \[26\]), finding the regions of highest posterior density to be

\[
\begin{align*}
0.021 < \Omega_B h^2 &< 0.025, \\
0.097 < \Omega_c h^2 &< 0.13, \\
62 < H_0 &< 77, \\
0.68 < Z &< 0.95, \\
0.90 < n_s &< 1.01, \\
17.7 < A_s &< 27.3.
\end{align*}
\]

The primary focus of our study has been to derive constraints on the Brans–Dicke parameter for which, from the outset, we have expected only to find a one-sided bound; the situation can only become more interesting when both the angular diameter distance and the recombination history become much better probed by the CMB. This expectation is indeed confirmed by the data, as shown in Figure 3 in which we display the region of highest posterior density. From this plot we read off a conservative bound and the main result of this paper to be

\[
\ln \xi < -6.0.
\]

This corresponds to the constraint on the Brans–Dicke parameter

\[
\omega > 100,
\]

FIG. 2: Marginalized 1D posterior distributions (solid lines) on the base parameters as listed in Section II. Also displayed is the mean likelihood of the binned posterior samples (dotted lines).
FIG. 3: Marginalized 1D posterior distributions (solid lines) on the Brans–Dicke parameter ln ξ (upper panel). Also displayed are the derived importance sampled constraints (correcting for the change in prior density) on the more familiar ω (lower panel, no smoothing). We read off a conservative bound of ln ξ > −6, corresponding to a bound on the Brans–Dicke parameter ω > 100.

which has been obtained by importance sampling to correct for the change in prior density when changing parameters from ln ξ to ω (we note that the mean likelihood of the binned posterior obtained from sampling ln ξ performs well for putting a bound on ω, demonstrating less sensitivity to the details of the prior density). This bound is nicely consistent with the expectation for WMAP given by the Fisher matrix analysis of Ref. [7].

We present in Figure 4 the 2D posterior constraints in the ln ξ–H₀ plane, in order to demonstrate the degeneracy and covariance between these two parameters. In a more refined analysis, one could replace H₀ with the dimensionless parameter rₛ/Dₐ more appropriate to the study of the CMB, where rₛ is the sound horizon at recombination and Dₐ is the angular diameter distance to the last-scattering surface [27]. This geometrical degeneracy is further illustrated in Figure 5 where we display two models, a ΛCDM model with parameters θ≡{Ω_bh², Ω_ch², H₀, z_s, n_S, 10¹⁰ A_s, ω}={0.025, 0.12, 70, 0.84, 1.0, 23, ∞}, and a Brans–Dicke model with parameters θ={0.023, 0.12, 77, 0.80, 0.98, 22, 100}.

We note in passing that Figure 5 shows a shift in the location of the baryon oscillations in the matter power spectrum as compared to the ΛCDM model, and hence high precision measurements of those may assist in constraining ω.

Our ultimate constraint ω > 100 can be compared with that of Nagata et al. [8], who quote results corresponding to ω > 1000 at two-sigma and ω > 50 at four-sigma. The former constraint is much stronger than projected in Ref. [8], and stronger than one would expect from a naïve assessment that the corrections to observables should be of order 1/ω. If we plotted a model with ω = 1000 in our Figure 5, it would lie practically on top of the ΛCDM model. However their latter constraint is in reasonable agreement with ours, and they do highlight that it is this constraint which corresponds to a sharp ridge of deteriorating chi-squared in their analysis, indicating that their constraint should conservatively be taken as ω > 50.

IV. CONCLUSIONS

We have derived a constraint on Jordan–Brans–Dicke gravity from current cosmological observations, including cosmic microwave background (CMB) anisotropy data and the galaxy power spectrum data. Our main result is to obtain a conservative bound on the Brans–Dicke parameter ω > 100. This result is complementary to the very strong Solar System limit provided by Cassini, ω > 40000, as it probes entirely different length and timescales. Our analysis is based on a Markov Chain Monte Carlo technique varying the basic cosmological
FIG. 5: A comparison between a ΛCDM model (solid line) and a Brans–Dicke ΛCDM model with $\omega = 100$ (dashed line). Detailed parameters are given in Section III.

At the present precision level, the greatest part of the constraining power comes from the shape of the CMB acoustic peaks, in particular from the first-year observations of WMAP. Therefore, assuming an extension to four years of the WMAP observations, we expect an improvement by a factor of a few on the limit on $\omega$ from cosmology. Further help is also expected from other structure formation data, as they improve quality and precision in coming years.

A leap forward in this and other contexts is expected from the observations of the Planck Surveyor probe, to be launched in 2007. Those observations are expected to be cosmic variance limited for the whole spectrum of CMB temperature anisotropy down to the damping tail at the arcminute scale; an analogous result is expected for the gradient mode of CMB polarization anisotropies. In light of our result and of these expectations, it is conceivable that the constraints on the gravity theory from cosmology may get close to those from the solar system by the end of this decade.

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