Scalar Gravity and Higgs Mechanism

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The role of the auxiliary scalar field $\phi$ in Brans-Dicke theory is discussed. If a constant vacuum energy is assumed to be the origin of dark energy, then the corresponding density parameter would be a quantity varying with $\phi$; and almost all of the fundamental components of our universe can be unified into the dynamical equation for $\phi$. As a generalization of Brans-Dicke theory, we propose a new gravity theory with a complex scalar field $\varphi$ which is coupled to the cosmological curvature scalar. Through such a coupling the Higgs mechanism is naturally incorporated into the evolution of the universe, and a running mass scale is obtained in which the gauge symmetry breaks spontaneously.

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I. INTRODUCTION

Recent observational data, in particular the Hubble diagram of type I supernova \cite{1}(1998) and the fit of cosmological parameters to the Wilkinson Microwave Anisotropy Probe (WMAP) data\cite{2}, have given support to a novel scenario for our universe. The observable universe, which may contain three density components, could in fact have serious departures from the previously assumed standard cosmological model. In this novel scenario, a dark energy dominates the universe today and drives its acceleration. This energy must be distributed smoothly on large scales, and be of negligible effect during early epochs. However, the amount of dark energy may in fact be of the same order of magnitude as the matter during a long period of cosmological history. This problem can be resolved by making modifications to the right-hand-side of Einstein’s field equation, but may require a fine-tuning of the different density components of the universe. For example, an additional scalar field of matter may demand a tracking behavior in playing the role of dark energy or dark matter. Therefore we prefer to make modifications to the left-hand-side of Einstein’s equations by adding geometry terms.

A number of models for the dark energy have been suggested. The model based on general relativity with a constant vacuum energy is by far the simplest one. This is effectively the same as the cosmological constant in standard general relativistic cosmology. However, the presently predicted values from theory are much greater than those inferred from observations, so much so that we may have to appeal to the Anthropic Principle\cite{4, 5}. The quintessence model, which invokes a very light and slowly evolving scalar field, requires its potential to be so flat that it is difficult to explain how the tiny mass of this field can stay safe from quantum corrections. Other models include a network of topological defects, and the call for extra dimensions. But all these have conceptual problems which need to be further clarified\cite{3, 6, 7, 8}.

In contrast, all searches for the signs of new physics have only confirmed the remarkable success of the particle standard model. These confirmations have been attributed to the success of the Higgs mechanism, but the corresponding Higgs particle has not been found. Maybe we should change the concept. As we know, the Higgs mechanism requires that the Higgs scalar is coupled to all fundamental particles and provides their mass. Therefore it should be a universal coupling, and possibly only gravitational interactions could do this. The coupling between Higgs’ complex scalar and electromagnetic gauge field is of particular note, and is also required in the standard process of Higgs mechanism. As far as it is known, there are only two kinds of interactions at play on the photon field: the electromagnetic interaction and the gravitational interaction. Therefore if it is assumed that Higgs’ complex scalar is without charge of electricity, then the Higgs scalar can only be interpreted as gravitational. The transfer of the gravitational interaction may be realized through just such a coupling. Furthermore, the particle standard model is still not able to give a full interpretation of the origin of the hierarchy between the weak scale and the unification scale. It may also imply that this problem is connected with the energy scale of universe.

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II. COSMOLOGICAL PROPERTIES OF GRAVITATIONAL SCALAR FIELD

Brans-Dicke theory is an alternative relativistic theory of gravity \([9, 10]\). Compared with general relativity, as well as the metric tensor of space-time which describes the geometry there is also an auxiliary scalar field \(\phi\) which describes the gravity. The testing of Brans-Dicke theory using stellar distances and the CMB temperature and polarization anisotropy have been discussed in \([11, 12]\).

Before applying the Brans-Dicke theory to cosmology, we start by writing the RW line element as

\[
\text{ds}^2 = -dt^2 + a^2(t)\hat{g}_{ij}dx^i dx^j.
\]

(1)

Where \(i, j\) run from 1 to 3, \(a(t)\) is the scale of the non-compact 3-dimensional space with constant curvature \(K\). The action of Brans-Dicke theory with non-vanishing vacuum energy reads

\[
I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( \phi \cdot R + \omega g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right)
+ \int d^4x \sqrt{-\hat{g}} \cdot (\mathcal{L}_{\text{matter}} - \Lambda),
\]

(2)

where \(R\) is the space-time curvature scalar, \(\phi\) is an auxiliary gravitational scalar field, \(\omega\) is a parameter of Brans-Dicke theory, and the vacuum energy density from spontaneous symmetry breaking in quantum field theory is denoted as \(\Lambda\). Here, \(\Lambda > 0\). Then the corresponding field equations are

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{\phi} T_{\mu\nu} - \frac{1}{\phi} (g_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi - \phi_{\mu\nu})
- \frac{\omega}{\phi^2} \phi_{\mu\nu} + \frac{1}{2} \frac{\omega}{\phi^2} g_{\mu\nu} \nabla_\sigma \phi \nabla^\sigma \phi
- \frac{8\pi}{\phi} \cdot \Lambda g_{\mu\nu},
\]

(3)

and the field equation for \(\phi\) reads

\[
\Box^2 \phi = \phi_{\mu\mu} = \frac{8\pi}{-3 + 2\omega} (T_{\text{matter}} + 4\Lambda).
\]

(4)

The matter stress-energy-momentum tensor may be written as \(T_{\mu\nu} = (\rho_m + p_m) u_\mu u_\nu + p_m g_{\mu\nu}\), and then the classically conserved perfect fluid energy momentum tensor is

\[
\frac{\partial \rho_m}{\partial t} = -3\frac{\dot{a}}{a} (\rho_m + p_m).
\]

(5)

After a straightforward calculation using equation (1), we also obtain the non-zero components of the Ricci tensor

\[
3+1R_{00} = -3\frac{\ddot{a}}{a};
\]

(6)

\[
3+1R_{ij} = \left( \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{K}{a^2} \right) g_{ij};
\]

(7)

Therefore, the fundamental equations of Brans-Dicke cosmology are

\[
1 + \frac{K}{a^2} = \frac{8\pi}{3H^2} \left[ \frac{\rho}{\phi} - \frac{\omega}{16\pi} \dot{\phi}^2 \left( \frac{3H}{8\pi} \dot{\phi} + \frac{\Lambda}{\phi} \right) \right];
\]

(8)

\[
\frac{\dot{a}}{a} = -\frac{4\pi}{3} \left[ \frac{2}{-3 + 2\omega} \left( \frac{\rho}{\phi} - 4\frac{\omega}{16\pi} \dot{\phi}^2 \left( \frac{3H}{8\pi} \dot{\phi} + \frac{\Lambda}{\phi} \right) \right) - 2\left( \frac{3H}{8\pi} \dot{\phi} + \frac{\Lambda}{\phi} \right) \right].
\]

(9)

As was discussed above, it is assumed here that the vacuum energy plays the role of dark energy in the present universe, therefore

\[
\Omega_{\text{vacuum}} = \frac{8\pi}{3H^2} \left( \frac{\Lambda}{\phi} \right),
\]

(10)
which is no longer a constant for a fixed Hubble parameter $H$. There would be an obvious depression in $\Omega_{\text{vacuum}}$ if the scalar field $\phi$ of Brans-Dicke theory rolls to a large number. The dynamical equations of $\phi$ can be derived from equation (4), and are

$$\frac{\ddot{\phi}}{\phi} = -8\pi(1 - 3r_m) \cdot \left( \frac{\rho}{\phi} \right) + 8\pi \cdot \left( -\frac{3H \dot{\phi}}{8\pi \phi} \right) - \frac{32\pi}{3 - 2\omega} \cdot \left( \frac{\Lambda}{\phi} \right).$$  \tag{11}

As a remarkable character, the equation (11) contains almost all of cosmological density components demonstrated by Friedman’s equations (8-9).

### III. SCALAR GRAVITY

As a generalization of Brans-Dicke theory, we consider a new theory of gravity with a complex scalar field $\varphi$ which is coupled to the curvature scalar. It is natural to construct the action as

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ \kappa \varphi \varphi^* \cdot R + \omega g^{\mu\nu} D_\mu \varphi (D_\nu \varphi)^* - \lambda (\varphi \varphi^*)^2 \right] + \int d^4x \sqrt{-g} \cdot L_{\text{matter}}. \tag{12}

Here the coupling constants $\kappa$, $\omega$ and $\lambda$ are all dimensionless, which is consistent with the requirement of renormalizability. In addition, we also restrict them to be positive.

### IV. HIGGS MECHANISM

From the RW metric given in equation (1), the curvature scalar reads

$$R = 6 \frac{\ddot{a}}{a} + 6 \dot{a}^2 + 6 \frac{K}{a^2}. \tag{13}

The sign of the curvature scalar would change with the evolution of our universe. On the other hand, the potential of complex scalar $\varphi$ can be written as

$$V(\varphi) = -\kappa R \varphi \varphi^* + \lambda (\varphi \varphi^*)^2. \tag{14}

For a homogeneous isotropic curvature scalar $R$, when $R$ evolves into a positive quantity, there exist non-trivial minimums. The value of these minimums are distributed on the circle

$$|\varphi| = \sqrt{\frac{\kappa R}{2\lambda}} := \frac{v}{\sqrt{2}}. \tag{15}

Hence the gauge symmetry breaks spontaneously. For convenience, we only consider the $U(1)$ gauge symmetry in this Letter. The Higgs mechanism requires the special gauge transformation

$$\varphi(x) \longrightarrow \varphi'(x) = \eta(x) + \frac{v}{\sqrt{2}}; \tag{16}

A_\mu \longrightarrow B_\mu = A_\mu + \frac{1}{e} \nabla_\mu \xi(x); \tag{17}

D_\mu = \nabla_\mu + ieA_\mu \longrightarrow D'_\mu = \nabla_\mu + ieB_\mu. \tag{18}

and therefore

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ \omega \nabla_\mu \eta + \sqrt{\frac{\kappa R}{2\lambda}} \nabla^\mu (\eta + \sqrt{\frac{\kappa R}{2\lambda}}) + \eta^2 \kappa R - 3\eta^2 \kappa R 

- 2\sqrt{2\kappa R} \eta^3 - \lambda \eta^4 + \omega e^2 B_\mu B^\mu (\eta + \sqrt{\frac{2\kappa R}{\lambda}}) \eta + \omega \frac{\kappa R}{2\lambda} e^2 B_\mu B^\mu + \frac{\kappa^2 R^2}{4\lambda} \right] + \int d^4x \sqrt{-g} \cdot L_{\text{matter}}. \tag{19}$$
In our framework, the running of the vacuum energy density with the evolution of the universe is realized. We have

$$\Lambda = \frac{1}{16\pi} \cdot \frac{\kappa^2 R^2}{4\lambda}. \quad (20)$$

Here the Higgs field is a real gravitational scalar $\eta$, which is generated by spontaneous symmetry breaking of the complex field $\varphi$. Therefore, the dynamical mass of the Higgs field is $m_\eta = \sqrt{\frac{2\kappa R}{\omega}}$ and the dynamical equation for the Higgs field is

$$\Box^2 (\eta + \sqrt{\frac{\kappa R}{2\lambda}}) = \frac{1}{2\omega} [-4\kappa R\eta - 6\sqrt{2\kappa R\lambda} \eta^2 - 4\lambda \eta^3 + 2\omega e^2 B_\mu B^\mu \eta + \omega e^2 B_\mu B^\mu \sqrt{\frac{2\kappa R}{\lambda}}]. \quad (21)$$

In fact, it is not only the gauge boson obtains mass (see equation 19 for this boson mass). If it is assumed that the coupling between fermions and this gravitational scalar $\varphi$ exists, the fermions can also obtain mass in this picture. Here we consider the simple Higgs-lepton coupling

$$G_e \cdot [\bar{e}_R \varphi^+ (\nu_e)_L + (\bar{e}_e \nu_e) L \varphi e_R]. \quad (22)$$

After the gauge symmetry breaks, the electron obtains mass

$$m_e = G_e \sqrt{\frac{\kappa R}{2\lambda}}. \quad (23)$$

Therefore, the mass of fundamental particles in this scenario depend on the cosmological curvature scalar at spontaneous symmetry breaking, and are not uniquely fixed quantities. However, as we discussed above, whether the gauge symmetry breaks or not is determined by the sign of the curvature scalar, and is also determined by the evolving energy scale of the universe. Thus fundamental particles could not be distributed homogeneously on all physical energy scales in the present time. In addition, experiments have also shown that the mass of fundamental particles are stable at the present time. We think the reason for this apparent contradiction may be that our theory is still a semi-classical one. Mass stability may be obtained by considering the average value of the cosmological scalar curvature $R$, in a similar way in which Newton’s constant $G$ can be related to the average value of the scalar field $\phi$ in Brans-Dicke theory.

V. CONCLUSIONS

In this Letter, we have proposed a new form of scalar gravity which naturally provides a candidate Higgs field for Higgs mechanism. We have also investigated the dynamical character of a gravitational scalar in cosmology, which may help to solve a number of problems in the present cosmology. One of the key ideas is that if a spontaneous symmetry breaking of the coupling between the curvature scalar and a gravitational scalar field occurs on the cosmological scale, some additional geometrical terms can be naturally introduced into the field equations. Secondly, we have argued that a gravitational scalar is also qualified to be a candidate for the Higgs field. Our present model is however still simplistic and will be clarified further in future papers.

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