The FLUKA Monte Carlo, non–perturbative QCD and Cosmic Ray cascades

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Abstract

The FLUKA Monte Carlo code, presently used in cosmic ray physics, contains packages to sample soft hadronic processes which are built according to the Dual Parton Model. This is a phenomenological model capable of reproducing many of the features of hadronic collisions in the non perturbative QCD regime. The basic principles of the model are summarized and, as an example, the associated $\Lambda-K$ production is discussed. This is a process which has some relevance for the calculation of atmospheric neutrino fluxes.

1 Introduction

The description of cosmic ray interactions in the Earth’s atmosphere requires a model which is capable of reproducing the dynamics of hadronic interactions in a wide energy range, including phase space regions which have not been experimentally accessed at accelerators. Therefore models which are theoretically founded are the best candidates to cover reliably the whole region of interest. Therefore QCD is the fundamental reference in the development of modern codes for atmospheric showers. Unfortunately the bulk of hadronic interactions belongs to the non perturbative regime of QCD, and therefore we have not exactly calculable models that can be safely applied in cosmic ray physics. In order to address the non perturbative region different attempts have been devised, arriving to the definition of phenomenological schemes with parameters that must be adjusted on the basis of experimental data. The FLUKA Monte Carlo code\cite{1}, an all–purpose code for transport and interaction of particles and nuclei already applied in cosmic ray physics\cite{2}, contains hadronic packages belonging to the above defined class. Here we review the main features of the high energy hadron interaction models of FLUKA. The specific features which bring to the production of strange and charmed particles are then reviewed in more detail in view of their specific importance in some aspects of cosmic ray physics.

2 The high energy hadronic models in FLUKA

FLUKA is based, as far as possible, on the “microscopic” approach, \textit{i.e.} the one starting from the basic hadron constituents and from their known properties. Each step has to be self–consistent and must be based on accepted physical bases. Performances are optimized comparing with particle production data at single interaction level. No tuning whatsoever is performed on “integral” data, such as calorimeter resolutions, thick target yields, etc. Therefore, final predictions are obtained with a minimal set of free parameters, fixed for all energies and target/projectile combinations. Results in complex cases as well as scaling laws and properties come forth naturally from the underlying physical models and the basic conservation laws are fulfilled \textit{a priori}.
A comprehensive understanding of hadron–nucleon (h–N) interactions over a wide energy range is of course a basic ingredient for a sound description of hadron–nucleus ones. Elastic, charge exchange and strangeness exchange reactions are described as far as possible by phase–shift analysis and/or fits of experimental differential data. Standard eikonal approximations are often used at high energies.

At intermediate energies, the inelastic channel with the lowest threshold (single pion production) opens already around 290 MeV in nucleon-nucleon interactions, and becomes important above 700 MeV. In pion-nucleon interactions the production threshold is as low as 170 MeV. Both reactions are normally described in the framework of the isobar model: all reactions proceed through an intermediate state containing at least one resonance. There are two main classes of reactions, those which form a resonant intermediate state (possible in π-nucleon reactions) and those which contain two particles in the intermediate state. The former exhibit bumps in the cross sections corresponding to the energy of the formed resonance. Partial cross sections can be obtained from one–boson exchange theories and/or folding of Breit–Wigner with matrix elements fixed by N–N scattering or experimental data. Resonance energies, widths, cross sections, and branching ratios are extracted from data and conservation laws, whenever possible, making explicit use of spin and isospin relations. They can be also inferred from inclusive cross sections when needed. For a discussion of resonance production, see for example [3, 4, 5].

As soon as the projectile energy exceeds a few GeV, the description in terms of resonance production and decay becomes more and more difficult. The number of resonances which should be considered grows exponentially and their properties are often poorly known. Furthermore, the assumption of one or two resonance creation is unable to reproduce the experimental finding that most of the particle production at high energies occurs neither in the projectile nor in the target fragmentation region, but rather in the central region, for small values of Feynman $x$ variable. Different models, based directly on quark degrees of freedom, must be introduced.

The features of “soft” interactions (low-$p_T$ interactions) cannot be derived from the QCD Lagrangian, because the large value taken by the running coupling constant prevents the use of perturbation theory. Models based on interacting strings have emerged as a powerful tool in understanding QCD at the soft hadronic scale, that is in the non-perturbative regime.

A theory of interacting strings can be managed by means of the Reggeon-Pomeron calculus in the framework of perturbative Reggeon Field Theory [3], an expansion already developed before the establishment of QCD. Regge theory makes use explicitly of the constraints of analyticity and duality. On the basis of these concepts, calculable models can be constructed and one of the most successful attempts in this field is the so called “Dual Parton Model” (DPM), originally developed in Orsay in 1979 [7]. It provides the theoretical framework to describe hadron-nucleon interaction from several GeV onwards. In DPM a hadron is a low-lying excitation of an open string with quarks, antiquarks or diquarks sitting at its ends. In particular mesons are described as strings with their valence quark and antiquark at the ends. (Anti)baryons are treated like open strings with a (anti)quark and a (anti)diquark at the ends, made up with their valence quarks.

At sufficiently high energies, the leading term in high energy scattering corresponds to a “Pomeron” (IP) exchange (a closed string exchange with the quantum numbers of vacuum), which has a cylinder topology. By means of the optical theorem, connecting the forward elastic scattering amplitude to the total inelastic cross section, it can be shown that from the Pomeron topology it follows that two hadronic chains are left as the sources of particle production (unitarity cut of the Pomeron). While the partons (quarks or diquarks) out of which chains are stretched carry a net color, the chains themselves are built in such a way to carry no net color, or to be more exact to constitute color singlets like all naturally occuring hadrons. In practice, as a consequence of color exchange in the interaction, each colliding hadron splits into two colored system, one carrying color charge $c$ and the other $ar{c}$. These two systems carry together the whole momentum of the hadron. The system with color charge $c$ ($ar{c}$) of one hadron combines with the system of complementary color of the other hadron, in such a way to form two color neutral chains. These chains appear as two back-to-back jets in their own centre-of-mass systems. The exact way of building up these chains depends on the nature of the projectile-target combination (baryon-baryon, meson-baryon, antibaryon-baryon, meson-meson). Let us take as example the case of nucleon-nucleon (baryon-baryon) scattering. In this case, indicating with $q_p^v$ the valence quarks of the projectile, and with $q_t^v$ those of the
target, and assuming that the quarks sitting at one end of the baryon strings carry momentum fraction \( x_p^v \) and \( x_t^v \) respectively, the resulting chains are \( q_t^v - q_p^v q_p^v \) and \( q_p^v - q_t^v q_t^v \), as shown in fig. 1.

![Figure 1: Leading two-chain diagram in DPM for \( p - p \) scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities.](image)

Energy and momentum in the centre-of-mass system of the collision, as well as the invariant mass squared of the two chains, can be obtained from:

\[
\begin{align*}
E_{ch1}^* & \approx \frac{\sqrt{s}}{2} (1 - x_p^v + x_t^v) \\
E_{ch2}^* & \approx \frac{\sqrt{s}}{2} (1 - x_t^v + x_p^v) \\
p_{ch1}^* & \approx \frac{\sqrt{s}}{2} (1 - x_p^v - x_t^v) = -p_{ch2}^* \\
s_{ch1} & \approx s (1 - x_p^v) x_t^v \\
s_{ch2} & \approx s (1 - x_t^v) x_p^v
\end{align*}
\]  

The single Pomeron exchange diagram is the dominant contribution, however higher order contributions with multi-Pomeron exchanges become important at energies in excess of 1 TeV in the laboratory. They correspond to more complicated topologies, and DPM provides a way for evaluating the weight of each, keeping into account the unitarity constraint. Every extra Pomeron exchanged gives rise to two extra chains which are built using two \( q\bar{q} \) couples excited from the projectile and target hadron sea respectively. The inclusion of these higher order diagrams is usually referred to as multiple soft collisions.

Two more ingredients are required to completely settle the problem. The former is the momentum distribution for the \( x \) variables of valence and sea quarks. Despite the exact form of the momentum distribution function, \( P(x_1, ..., x_n) \), is not known, general considerations based on Regge arguments allow
to predict the asymptotic behavior of this distribution whenever each of its arguments goes to zero. The behavior turns out to be singular in all cases, but for the diquarks. A reasonable assumption, always made in practice, is therefore to approximate the true unknown distribution function with the product of all these asymptotic behaviors, treating all the rest as a normalization constant.

The latter ingredient is a hadronization model, which must take care of transforming each chain into a sequence of physical hadrons, stable ones or resonances. There are two basic assumptions. One is that of *chain universality*, which assumes that once the chain ends and the invariant mass of the chain are given, the hadronization properties are the same regardless of the physical process which originated the chain. Therefore the knowledge coming from hard processes and $e^+e^-$ collisions about hadronization can be used to fulfill this task. The other is the scale invariance of fragmentation functions, when $\sqrt{s}$ is much larger than the masses. There are many more or less phenomenological models which have been developed to describe hadronization (examples can be found in [8, 9]). In principle hadronization properties too can be derived from Regge formalism [10]. The original FLUKA model makes use of the hadronization model of [9].

It is possible to extend DPM to hadron-nucleus collisions too [7], making use of the so called Glauber-Gribov approach. The Glauber-Gribov model [13, 14, 15] represents the diagram interpretation of the Glauber cascade. The $\nu$ interactions of the projectile originate $2\nu$ chains, out of which 2 chains (valence-valence chains) struck between the projectile and target valence (di)quarks, $2(\nu - 1)$ chains (sea-valence chains) between projectile sea $q - \bar{q}$ and target valence (di)quarks.

A pictorial example of the chain building process is depicted in fig. 2 for $p - A$: similar diagrams apply to $\pi - A$ and $\bar{p} - A$ respectively.

![Figure 2](image)

Figure 2: Leading two-chain diagrams in DPM for $p - A$ Glauber scattering with 4 collisions. The colour and quark combinations shown in the figure are just one of the allowed possibilities.

The distribution of the projectile energy among many chains naturally softens the energy distributions of reaction products and boosts the multiplicity with respect to hadron-hadron interactions. In this way, the model accounts for the major A-dependent features without any degree of freedom, except in the treatment of mass effects at low energies.

The Fermi motion of the target nucleons must be included to obtain the correct kinematics, in particular the smearing of $p_T$ distributions. All nuclear effects on the secondaries are accounted for by the subsequent steps. In FLUKA these are performed in the framework of a generalized intranuclear cascade (GINC) which embeds quantum correction effects like for instance the *formation zone* concept, a sort of
“materialization” time. For further details see [16].

At very high energies, those of interest for high energy cosmic ray studies (10–10^5 TeV in the lab), hard processes cannot be longer ignored. They are calculable by means of perturbative QCD and can be included in DPM through proper unitarization schemes which consistently treat soft and hard processes together. This is for instance what is implemented in the framework of the “2-component Dual Parton Model” (i.e. soft + hard collisions) in the DPMJET model [17]. Here the hard collisions are those which are described by the tree–level QCD diagrams.

The latest releases of FLUKA are now interfaced to DPMJET-II.53, so to allow the sampling of high energy nucleus-nucleus collisions for E_{lab} from 5-10 GeV/n up to 10^9-10^{11} GeV/n).

The original interface to the DPMJET-II.53 version has recently been upgraded to comply with the DPMJET-III version. Beyond the full description of nucleus–nucleus collisions, the interface to DPMJET allows to extend the FLUKA energy limits for hadronic simulations in general.

3 Strangeness production in high energy collisions in FLUKA

The production of particles containing s or heavier quarks in nucleon–nucleon collisions requires to pick up such quarks from the sea. This, in the framework of Dual Parton Model may occur for soft processes in two cases:

1. in the process of chain formation when valence and sea (di)quarks combine together. This occurs in multi-Pomeron exchange (relevant only at high energy) and in the multiple chains deriving from the Glauber expansion in the case of nucleons in a nucleus.

2. in the process of chain hadronization, when the color string tension materializes in q − ¯q or qq − ¯q¯q pairs. Numerically this is the most important contribution.

In particular, the hadronization of valence diquark-quark chains is the fundamental mechanism to produce K mesons and strange baryons. A reliable description of this production can be relevant in cosmic ray applications ons. In particular it has been pointed out how, for kinematical reasons, kaons have a fundamental role in the generation of atmospheric neutrinos at high energy [19], and the uncertainties on kaon production are one of the important contributions to the systematic error in the calculations of fluxes, especially above 100 GeV.

Table 3 shows the average multiplicity of some particles originating from the hadronization of chains with invariant mass of 10 GeV occurring in p–nucleon scattering as obtained from FLUKA. This table is calculated after that e.m. and strong decays have already occurred (therefore there are no more η’s, K⋆’s etc.).

The extraction of s − ¯s pairs in the hadronization process may give rise to different configurations. Some examples are reported in Figs 3,4,5 where we consider the string between a valence diquark from the projectile and a valence quark from the target (solid lines). The dashed lines represent the q − ¯q or qq − ¯q¯q pairs materialized in the process. It must be intended that with a proper probability the suggested particles are replaced by the corresponding excited states (resonances) like K⋆’s.

As far as particle production in cosmic ray showers is concerned, an interesting case is the production at high rapidity (or x_F). On average, the valence diquark from the projectile in the laboratory frame carries the highest fraction of momentum of the incident nucleon.

Therefore, on average, the the particles which are produced close in rapidity to the projectile diquark give a major contribution to the yield (see for instance the concept of “spectrum weighted moment” [20]). The case shown in the left panel of Fig 3 is the typical case in which the excitation of a s − ¯s pair leads to the associated Λ K production. The valence diquark can dissociate before hadronization (using the mechanism that has been called “popcorn” in the Lund chain fragmentation model JETSET [21]), as shown in the right panel of Fig 3 and in Fig 4. Sometimes a strange baryon might not be associated to a K meson, but to the corresponding anti–baryon, as shown in the left panel of Fig 4. In other cases, K meson can be produced in pairs without the association to a strange baryon. An example is shown in Fig 4.
Figure 3: Left: associated $\Lambda K$ production. Right: $\Sigma K$ production with “popcorn” effect.

Figure 4: Left: $\Lambda \bar{\Lambda}$ production with double “popcorn” effect. Right: Another example of “popcorn” leading to $\Xi K$ production.

Figure 5: Example where $K^+ K^-$ (or $K^0 \bar{K}^0$ if $d$ quark replace the $u$ one) are produced, without any associated strange baryon.
Table 1: Particle multiplicities from the hadronization of chains with invariant mass of 10 GeV occurring in p–nucleon scattering as obtained from FLUKA

<table>
<thead>
<tr>
<th></th>
<th>uu–u</th>
<th>uu–d</th>
<th>ud–u</th>
<th>ud–d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>0.803</td>
<td>0.777</td>
<td>0.538</td>
<td>0.511</td>
</tr>
<tr>
<td>(\overline{p})</td>
<td>0.073</td>
<td>0.073</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>(n)</td>
<td>0.241</td>
<td>0.265</td>
<td>0.511</td>
<td>0.538</td>
</tr>
<tr>
<td>(\overline{n})</td>
<td>0.075</td>
<td>0.073</td>
<td>0.075</td>
<td>0.074</td>
</tr>
<tr>
<td>(\pi^+)</td>
<td>2.957</td>
<td>2.490</td>
<td>2.530</td>
<td>2.047</td>
</tr>
<tr>
<td>(\pi^-)</td>
<td>2.577</td>
<td>2.580</td>
<td>2.736</td>
<td>2.522</td>
</tr>
<tr>
<td>(\pi^0)</td>
<td>1.788</td>
<td>2.271</td>
<td>2.056</td>
<td>2.739</td>
</tr>
<tr>
<td>(K^+)</td>
<td>0.233</td>
<td>0.213</td>
<td>0.230</td>
<td>0.209</td>
</tr>
<tr>
<td>(K^0)</td>
<td>0.199</td>
<td>0.217</td>
<td>0.205</td>
<td>0.224</td>
</tr>
<tr>
<td>(K^-)</td>
<td>0.165</td>
<td>0.258</td>
<td>0.168</td>
<td>0.162</td>
</tr>
<tr>
<td>(\overline{K}^0)</td>
<td>0.160</td>
<td>0.159</td>
<td>0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>0.064</td>
<td>0.064</td>
<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>(\overline{\Lambda})</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
</tr>
</tbody>
</table>

It can be expected that \(K\) mesons associated to strange baryons might be harder, in average, than those who are not associated. This because the baryons have a higher probability to be produced closer in rapidity to the diquark from the projectile. In the case of \(\Lambda\), which contains an \(s\) valence quark, this feature must be visible for the associated \(K^+\) (and \(K^0\)), containing an \(\bar{s}\) valence quark, and not for \(K^−\).

In order to test this prediction we have generated p–Air interactions at analyzing the distribution of \(X_{lab} = E/E_0\) of the highest rapidity \(K\) meson, distinguishing events in which \(\Lambda\) were produced from the rest. The results for \(E_0 = 100\) GeV are shown in Fig. 6 for \(K^+\), in Fig. 7 for \(K^0\) and in Fig. 8 for \(K^−\). These last, together with \(\overline{K}^0\), are mostly associated to the correspondent anti–meson.

The expectation for a harder spectrum of \(\Lambda\) associated \(K^+\) and \(K^0\) is confirmed. However the fraction of events with strange baryons production is only \(~0.12\) at this energy.

The capability of FLUKA of reproducing experimental data on \(\Lambda\) production, at least for few hundreds of GeV of proton energy, is shown in Fig. 9 where the measured double differential cross section in laboratory momentum and angle for \(\Lambda\) produced in p–p interaction at 300 GeV in the laboratory frame (points) are compared to Monte Carlo predictions (shaded histograms). The typical hard spectrum of forward produced baryons is clearly reproduced in shape and in absolute value.

Analogous mechanisms apply for charmed particle production. However the transverse mass suppression is quite strong in the Lund string fragmentation model, leading to a probability of creating a \(c–\bar{c}\) pair of the order of \(u: \bar{d} = s: \bar{s} = c: \bar{c} = 1: 1: 0.3: 10^{-11}\) [8]. In the BAMJET model [9] this probability is higher but enough to be significant (\(\sim 10^{-5}\)). In case of multiple collisions, as in nucleon–nucleus scattering, or for multi–Pomeron exchange, where color chains are built also with quarks and anti–quarks from the sea sitting at one or both ends, additional heavy quark production is possible. In any case, at very high energies, it has been shown that the dominant contribution in charm production comes from the hard parton–parton collisions as described by perturbative QCD [23]. For these processes in this energy range FLUKA relies on the interface to DPMJET.
Acknowledgments

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References


Figure 6: $X_{lab}$ distributions of $K^+$ produced in p–Air collisions at $E_0 = 100$ GeV. Events where $K$ mesons are associated to $\Lambda$ production are distinguished from the others.
Figure 7: $X_{lab}$ distributions of $K^0$ produced in p–Air collisions at $E_0 = 100$ GeV. Events where $K$ mesons are associated to $\Lambda$ production are distinguished from the others.
Figure 8: $X_{lab}$ distributions of $K^-$ produced in p–Air collisions at $E_0 = 100$ GeV. Events where $K$ mesons are associated to $\Lambda$ production are distinguished from the others.
Figure 9: Double differential cross section in laboratory momentum and angle for Λ produced in p–p interaction at 300 GeV in the laboratory frame. Points are experimental data [22] and shaded histogram is the FLUKA Monte Carlo prediction.