CP violation,
an experimental perspective∗

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I present a review of current and near-future experimental investigations of CP violation. In this review, I cover limits on particle electric dipole moments (EDMs) and CP violation studies in the K and B systems. The wealth of results from the new B factories provide impressive constraints on the CKM quark mixing matrix elements. Current and future measurements are focusing on processes dominated by loop diagrams, which probe physics at high mass scales in low-energy experiments.

Keywords: CP violation, B physics, K physics, electric dipole moments.

1. Introduction
The experimental investigation of CP violation seeks to answer profound questions about nature. One that is often mentioned is, "Why is the universe made entirely of matter?" The answer to this question must include some kind of CP violation. That is, nature can not be symmetric under the combined operation of charge conjugation C and parity inversion P. In the Standard Model, CP violation is due to the irreducible phase contained in the 3-generation CKM quark-mixing matrix. However, the baryon asymmetry in the universe is difficult, if not impossible, to explain with CP violation from the CKM matrix. Neutrino experiments have recently shown that neutrinos are not massless particles, as is assumed in the Standard Model. This opens the possibility of explaining the baryon asymmetry in the universe with CP violation arising from flavor mixing in the lepton sector (the so-called theory of leptogenesis). Another possibility is that the CP violation involved in generating the baryon asymmetry is due to physics beyond the Standard Model, such as supersymmetry.

The study of CP violation addresses the following more general question, "What, if anything, lies beyond the Standard Model of particle physics?" There are several well-motivated reasons for suspecting that the Standard Model is not the final theory of particle physics. In many extensions of the Standard Model, CP violation is not naturally suppressed. One can search for new physics by testing the predictions

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the Standard Model makes for a wide variety of $CP$ violating observables. New $CP$-violating phases from non-Standard Model virtual particles in loop corrections can provide a window to discovering new physics at high mass scales. This approach is most promising when the Standard Model process is naturally suppressed. In many cases, the Standard Model prediction for the $CP$ violating observable is quite precise, in which case a significant discrepancy would be a clear sign of new physics.

2. Particle Electric Dipole Moments

A non-zero electric dipole moment (EDM) of a particle, such as a neutron, electron, or muon, violates both parity ($P$) and time-reversal ($T$) symmetry. This is because the EDM must lie along the direction of the spin vector of the particle. The energy of a particle in an electric field is given by $-d \mathbf{s} \cdot \mathbf{E}$, where $d$ is the EDM. Under parity inversion, the sign of $\mathbf{E}$ changes, while $\mathbf{s}$ remains the same, since angular momentum is an axial vector ($\mathbf{l} = \mathbf{r} \times \mathbf{p}$), thus the term of the Hamiltonian given above changes sign under parity. Similarly, time reversal changes the spin direction while leaving $\mathbf{E}$ the same, changing the sign of the EDM term in the Hamiltonian. The $CPT$ theorem, a fundamental principle of quantum field theory, states that nature is invariant under the combined operation of $C$, $P$, and $T$. This implies that $T$ violation must be compensated by a similar amount of $CP$ violation.

In the Standard Model, particle EDMs are extremely small, since the leading contributions are from 3-loop diagrams. A non-zero particle EDM has never been observed. The current experimental limits are all orders of magnitude above the Standard Model estimates. In some new physics scenarios, particle EDMs are greatly enhanced. For instance, the leading supersymmetric EDM contributions enter at the one-loop level. I present the current and future experimental sensitivity to the muon and neutron EDM in the remainder of this section.

2.1. The EDM of the muon

The $g-2$ experiment at Brookhaven has recently released a new preliminary upper limit on the muon EDM. The primary mission of the $g$-2 experiment was to make the most precise measurement of the anomalous magnetic moment of the muon ($a_\mu \equiv (g-2)/2$). The precession of the muon spin, due to the anomalous magnetic moment, is given by

$$\omega_p = \frac{e}{m_\mu c} a_\mu \vec{B},$$

where $\vec{B}$ is the strength of the uniform magnetic field of the storage ring. The spin precesses in the plane of the storage ring (i.e. $\omega_p$ is parallel to $\vec{B}$). The electrons from muon decay are preferentially emitted along the direction of the muon spin. This allows the average muon spin to be tracked by detecting the position of the decay electrons at several locations around the storage ring.
If the muon has a small, non-zero EDM $d_\mu$, the precession vector in Equation 1 becomes

$$\vec{\omega}_p = \frac{e}{m_\mu c} \left[ a_\mu \vec{B} + \frac{1}{2} f \left( \vec{\beta} \times \vec{B} \right) \right],$$

(2)

where $f$ is proportional to the muon EDM ($d_\mu = f e \hbar / 4 m_\mu c$). The EDM contribution, due to the induced electric field in the muon rest frame, is in the radial direction. The muon spin precession plane is now slightly tilted with respect to the plane of the storage ring. The experimental technique is to detect this small tilt by monitoring the vertical position of the decay electrons. The vertical displacement from $d_\mu$ will be exactly 90° out of phase with respect to the $g-2$ precession and have the frequency $\omega_p$. Significant vertical displacements due to detector misalignment combined with coherent betatron oscillations of the beam must be carefully removed in the data analysis.

The result of the analysis \(^{[7]} \) is $d_\mu = (-0.1 \pm 0.7 \pm 1.2) \times 10^{-19}$ e-cm, consistent with zero. The 95 % C.L. upper limit is $d_\mu < 2.8 \times 10^{-19}$ e-cm, which is about a factor of 4 lower than the previous limit \(^{[8]} \). This is still 16 orders of magnitude above the Standard Model estimate $d_\mu \approx 10^{-35}$ to $10^{-38}$ e-cm. However, there are a few new-physics scenarios \(^{[9]} \) which allow $d_\mu$ to be as large as just 1 to 7 orders of magnitude below the current limit. The next generation experiment \(^{[10]} \) hopes to extend the sensitivity by another 4 orders of magnitude by “freezing” the $g-2$ precession with a strong radial $E$ field, thus removing the largest source of systematic error. In this case, the only spin precession is due to the EDM, which would produce a significant up-down asymmetry in the decay electrons for a $d_\mu$ close to the current limit.

### 2.2. The EDM of the neutron

The current best constraint on the EDM of the neutron comes from the RAL/Sussex experiment at ILL \(^{[11]} \). It uses the Ramsey resonance technique to measure the precession frequency of polarized ultra-cold neutrons in a volume with parallel or antiparallel $E$ and $B$ fields. The precession frequency is given by

$$\omega_p = \frac{1}{\hbar} \left[ 2 \mu_n \cdot \vec{B} + 2 d_n \cdot \vec{E} \right],$$

(3)

where $\mu_n$ and $d_n$ are the neutron magnetic and electric dipole moments respectively. It is easy to see that the difference in $\omega_p$ measured with $E$ parallel and antiparallel to $B$ gives $d_n$ through the relation $\Delta \omega = \frac{1}{\hbar} \left[ 4 d_n \cdot \vec{E} \right]$. It is essential to continuously monitor the strength of the static $B$ field in order to avoid a false EDM signal due to drift of the $B$ field between the $\omega_p$ measurements with $E$ parallel and antiparallel to $B$. This was achieved by simultaneously and continuously measuring the precession frequency of $^{199}$Hg within the same volume as the ultra-cold neutrons. Using $^{199}$Hg as a co-magnetometer removed what was the largest source of systematic uncertainty in the previous round of experiments.
The experiment measured $d_n = (-3.4 \pm 3.9 \pm 3.1) \times 10^{-26}$ e-cm, consistent with zero. The 90% C.L. upper limit is $|d_n| < 6.3$ e-cm, which is 5 orders of magnitude above the Standard Model estimates of $d_n$ which are in the range $d_n \approx 10^{-33}$ to $2 \times 10^{-31}$ e-cm. Even though this impressive experimental result is consistent with zero, it tells us quite a bit about possible new physics scenarios. For example, one-loop contributions to the neutron EDM in SUSY can give values of order $|d_n(SUSY)| \approx \left( \frac{100 \text{ GeV}}{m} \right) \sin \phi_{A,B} \times 10^{-23}$ where $m$ is the supersymmetric mass scale and $\phi_{A,B}$ are model-dependent phases. If the SUSY mass scale is of order 100 GeV, the current $d_n$ limit implies that $\phi_{A,B}$ must be small (of order $10^{-10}$). 

The $d_n$ limit also addresses a long-standing mystery of the Standard Model—the so-called “strong CP problem.” The most general QCD lagrangian contains a term that would give rise to a $d_n$ of order $|d_n(QCD)| \approx 3 \times 10^{-16} \bar{\theta}$. The Standard Model provides no explanation for why $\bar{\theta}$ must be so small (at least $10^{-10}$). Peccei and Quinn proposed a solution, which adds a new symmetry to the Standard Model and predicts the existence of a new particle (the axion), which has yet to be observed.

The proposed next-generation LANSCE neutron EDM experiment hopes to increase the $d_n$ sensitivity by another 2 orders of magnitude.

### 3. CP Violation in the Standard Model

$CP$ violation in the Standard Model is due to the irreducible complex phase within the 3-generation, CKM quark-mixing matrix. The electroweak coupling strength in the reaction $W^+ \rightarrow q_i \bar{q}_j$ is proportional to the CKM matrix element $V_{ij}$ where $q_i = (u,c,t)$ and $\bar{q}_j = (\bar{d},\bar{s},\bar{b})$. For the $CP$-conjugate reaction $W^- \rightarrow \bar{q}_i q_j$, the CKM matrix element is replaced by its complex conjugate $V_{ij}^*$. If $V_{ij}$ has a non-trivial phase (not 0 or $\pi$), this phase violates $CP$. The $CP$ violating phase is only observable through the quantum-mechanical interference of at least two amplitudes which have both non-zero $CP$ conserving and $CP$ violating relative phase differences.

Wolfenstein introduced a very useful parameterization of the CKM matrix in terms of 4 fundamental parameters ($\lambda, A, \rho, \eta$),

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \lambda \bar{\lambda} (\rho - i \eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & \lambda \bar{\lambda}^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 \bar{\lambda}^2 & 1 \end{pmatrix} + O(\lambda^4),$$

which is an expansion in powers of $\lambda \equiv \sin \theta_c \approx 0.22$, where $\theta_c$ is the Cabibbo angle. In this parameterization, two important features are evident: the most off-diagonal elements ($V_{ub}$ and $V_{td}$) 1) are the smallest and 2) contain the non-trivial phase information. The phase of $V_{ub}$ can be experimentally probed in $B$ rare charmless

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*a*Now that we know that neutrinos are not massless, a complex phase within the neutrino mixing matrix provides another mechanism for $CP$ violation. However, only $CP$ violation arising from the CKM matrix is relevant for the rest of this discussion.
decays. The phase of $V_{td}$ enters through loop diagrams, such as the box diagrams describing $B^0$ or $K^0$ mixing or penguin diagrams shown in Figure 1.

4. **CP Violation in the Kaon System**

$CP$ violation was discovered 40 years ago in a famous experiment at the Brookhaven National Laboratory where the decay $K_L \rightarrow \pi^+\pi^-$ was observed for the first time. The $\pi^+\pi^-$ final state is $CP$ even and is the dominant decay of the $K_S$ meson. If $CP$ is conserved, only one of either the $K_S$ or the $K_L$ can decay to $\pi^+\pi^-$, but not both.

This kind of $CP$ violation is called indirect $CP$ violation or $CP$ violation in mixing. It is due to the interference in $K^0 \leftrightarrow \bar{K}^0$ mixing between the amplitudes describing transitions through virtual intermediate states (e.g. the box diagram) and on-shell intermediate states (e.g. $K^0 \rightarrow \pi\pi \rightarrow \bar{K}^0$).

4.1. **Direct $CP$ violation**

The second important kind of $CP$ violation in the kaon system does not depend on $K^0 \leftrightarrow \bar{K}^0$ mixing. If the $CP$ violation is due to at least two decay amplitudes directly interfering, it is called “direct $CP$ violation.” Establishing this kind of $CP$ violation has been the focus of a series of dedicated experiments.

The direct-$CP$ violating observable is the double ratio

$$\frac{\Gamma (K_L \rightarrow \pi^+\pi^-)/\Gamma (K_S \rightarrow \pi^+\pi^-)}{\Gamma (K_L \rightarrow \pi^0\pi^0)/\Gamma (K_S \rightarrow \pi^0\pi^0)} = \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \approx 1 + 6 \text{ Re } (\epsilon'/\epsilon) \quad (5)$$

The NA48 and KTeV recently published their results that removed doubt about whether $\text{Re } (\epsilon'/\epsilon)$ is non-zero. The latest world average is $\text{Re } (\epsilon'/\epsilon) = (16.7 \pm 2.3) \times 10^{-4}$. This result rules out the superweak theory of $CP$ violation, where $CP$ violation is entirely from neutral meson mixing. The measured value of $\text{Re } (\epsilon'/\epsilon)$ is consistent with the Standard Model calculations, however large theoretical uncertainties from the calculation of hadronic matrix elements prevent this from being a precision test.
4.2. The Golden Mode – $K_L \rightarrow \pi^0\nu\bar{\nu}$

A decay mode that is the focus of much current and future experimental effort is the so-called “golden mode” $K_L \rightarrow \pi^0\nu\bar{\nu}$. The decay amplitudes are dominated by electroweak loop diagrams. The hadronic physics can be calibrated with the common $K^+ \rightarrow \pi^0 e^+\nu$ decay. This makes the calculation of the branching fraction extremely reliable. The branching ratio is proportional to $\text{Im} \left( V_{td} V_{ts}^* \right) = \frac{\eta A^2 \lambda^5}{\lambda^5}$, thus giving access to the Wolfenstein parameter $\eta$. Some new physics scenarios can enhance the branching fraction by up to 2 orders of magnitude above the Standard Model prediction of $(3.0 \pm 0.6) \times 10^{-11}$.

Observing this decay is extremely challenging. The $K_L$ can’t be fully reconstructed due to the unobserved neutrinos. The main background is from the decay $K_L \rightarrow \pi^0\pi^0$ has a branching fraction is $10^7$ times larger than $K_L \rightarrow \pi^0\nu\bar{\nu}$. The two additional photons from the extra $\pi^0$ in this mode must be detected with extremely high efficiency in order to veto the $\pi^0\pi^0$ background.

The best direct experimental limit on the branching fraction is from KTeV\cite{KTeV}, where they used $\pi^0 \rightarrow e^+e^-\gamma$. They found $\mathcal{B}(K_L \rightarrow \pi^0\pi^0) < 5.9 \times 10^{-7}$ at 90% C.L., which is well above the predicted value. An indirect limit can be derived\cite{Indirect} using the branching fraction from the related decay $K^+ \rightarrow \pi^+\nu\bar{\nu}$. Using the latest results from the E949 experiment at BNL\cite{E949} gives

$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) < 4.4 \times \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$$
$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) < 1.7 \times 10^{-9} \text{ 90% C.L.},$$

which is only 2 orders of magnitude to the predicted value. The E391a experiment at KEK\cite{E391a} is the first dedicated experiment for measuring the $K_L \rightarrow \pi^0\nu\bar{\nu}$ branching fraction. It’s considered a pilot project, since the estimated sensitivity is a factor of a few above the Standard Model prediction. Using the data that were taken earlier this year, they expect to have a sensitivity of about $4 \times 10^{-10}$. We eagerly await the E391a results, since some new physics models\cite{NewPhysics} allow for an enhancement of the branching fraction which could drive it up to or above the E391a sensitivity.

5. CP Violation in the B System

The $B$ system is an excellent place to test the Standard Model description of CP violation. Unlike the kaon system, the observable CP asymmetries are large (of order 1). The theoretical uncertainty in calculation of the expected CP asymmetry is extremely small in some special cases (about 1% or less), allowing for precision tests of the theory. Two new asymmetric-energy $B$ factories were built specifically for making these measurements – the PEP-II storage ring with the Babar experiment at SLAC and the KEKB storage ring with the Belle experiment at KEK. Each experiment has accumulated data samples of well over 200 million $B\bar{B}$ events over the last 3.5 years. As I will describe below, these enormous datasets have taken our understanding of CP violation in the Standard Model to a new level of precision.
A unitarity constraint from the $1^{\text{st}}$ and $3^{\text{rd}}$ columns of the CKM matrix provides a geometrical construction that’s useful for relating observables in the $B$ system to the fundamental CKM parameters. This is the so-called “Unitarity Triangle”, shown in Figure 2, which graphically represents the constraint $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$. The interior angles$^b$ of the triangles can be measured with CP asymmetries. In the following sections, I will describe the current status of measuring the angles of the triangle.

5.1. Time-dependent CP asymmetries at the asymmetric B factories

For final states that can be reached by both $B^0$ and $\bar{B}^0$ decay, $B^0 \leftrightarrow \bar{B}^0$ mixing provides two amplitudes with different CKM phases that can interfere with each other – a necessary requirement for CP violation to occur. The time-dependent CP asymmetry is defined as

$$A_{CP}(f; t) = \frac{N(B^0(t) \rightarrow f) - N(B^0(t) \rightarrow \bar{f})}{N(B^0(t) \rightarrow f) + N(B^0(t) \rightarrow \bar{f})}$$

where the notation $B^0(t)$ means that the meson was known to be (or “tagged” as) a $B^0$, as opposed to a $\bar{B}^0$, at $t = 0$. The $B^0$ and $\bar{B}^0$ from the decay of the $\Upsilon(4S)$ must remain flavor-antisymmetric, even while undergoing $B^0 \leftrightarrow \bar{B}^0$ mixing, due to Bose-Einstein statistics. This means that the relevant time in Eqn. 6 is the time difference $\Delta t$ between the two $B$ decays, since they must be in a flavor-opposite state at $\Delta t = 0$. Charged decay products of the $B$ that does not decay to $f$ in the event are used to infer the flavor of both $B$ mesons at $\Delta t = 0$.

In general, the $A_{CP}(f; \Delta t)$ takes the form

$$A_{CP}(f; \Delta t) = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2} \sin \Delta m_d \Delta t - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta m_d \Delta t$$

$^b$The Babar collaboration uses the $\beta, \alpha, \gamma$ naming convention for the Unitarity Triangle angles, while the Belle collaboration uses $\phi_1, \phi_2, \phi_3$. I will use the $\beta, \alpha, \gamma$ convention in this note.
\[ S_f \equiv \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}. \] (8)

The parameter \( \lambda_f \), in the Standard Model, is given by \( \lambda_f \equiv e^{-i2\beta} \bar{A_f}/A_f \), where \( A_f \) (\( \bar{A_f} \)) is the amplitude for the \( B^0 \) (\( B^0 \)) to decay to \( f \). This parameter should not be confused with the Wolfenstein parameter \( \lambda \).

The distance between the decay points of the two \( B \) mesons must be observable in order to reconstruct \( \Delta t \) for an event. This is the motivation for boosting the \( \Upsilon(4S) \) frame in the lab by colliding \( e^+ \) and \( e^- \) beams of unequal energies. To a good approximation, \( \Delta t \) is simply proportional to the longitudinal separation between the \( B \) decay vertices along the beam boost direction (\( \Delta t \approx \Delta z/\beta \gamma c \)).

5.2. The first precision test of CP violation – \( \sin 2\beta \) from \( J/\psi K_S \)

The decay \( B^0 \to J/\psi K_S \) is very special for two reasons. First, it is a CP eigenstate with a relatively large branching fraction. Second, and more importantly, only a single CKM phase appears in the leading decay amplitudes. The Standard Model predicts, to within about 1%, \( |A_{J/\psi K_S}/A_{J/\psi K_S}| = 1 \), \( \lambda_{J/\psi K_S} = -e^{-i2\beta} \), and

\[ S_{J/\psi K_S} = \sin 2\beta, \quad C_{J/\psi K_S} = 0. \]

These relations hold for many charmonium \( K_S \) final states and for \( J/\psi K_L \) with \( S_{J/\psi K_L} = -S_{J/\psi K_S} \).

Out of all time-dependent CP asymmetry measurements at the new \( B \) factories, the measurement of \( \sin 2\beta \) from \( J/\psi K_S \) has by far the largest and cleanest sample of signal events. The average of the latest results from Babar and Belle gives \( \sin 2\beta = 0.726 \pm 0.037 \). Indirect constraints on \( \beta \) from, \( |V_{ub}/V_{cb}| \), \( \epsilon_K \), \( \Delta m_d \), and the limit on \( \Delta m_s \) restrict \( \beta \) to be the range of \( [13^\circ, 31^\circ] \) at the 95% C.L. One of the 4 solutions for \( \beta \) from \( \sin 2\beta \) gives \( \beta = (23.3 \pm 1.6)^\circ \). This impressive agreement between the indirect constraints, which do not involve CP violation in the \( B \) system, and the direct measurement led Yosef Nir to conclude “the Kobayashi-Maskawa mechanism of CP violation has successfully passed its first precision test.”

5.3. Looking for New Physics – the \( b \to s \) penguin modes

The \( b \to s \) penguin is the dominant decay amplitude for a set of charmless final states, most notably \( \phi K_s \). The CKM phase in the leading decay amplitudes is the same as for \( J/\psi K_S \), so to first order these modes should also measure \( \sin 2\beta \). That is, \( S_f \approx -\eta_f \sin 2\beta \) and \( C_f \approx 0 \), where \( \eta_f \) is the CP eigenvalue of the final state \( f \). The comparison of \( \sin 2\beta \) measured with these \( b \to s \) penguin modes with the measurement from \( J/\psi K_S \) is an intriguing test of the Standard Model. Some speculate that loop diagrams containing virtual new physics (e.g. SUSY) particles may also be contributing to the \( b \to s \) modes. This would, in general, introduce new CP violating phases which could significantly alter the \( S_f \) and \( C_f \) coefficients.
The first measurements of $S_{\phi K_S}$ were presented at the ICHEP 2002 conference in Amsterdam. Both the Babar and Belle measurements were negative, opposite the expected sign, and averaged to $S_{\phi K_S} = -0.39 \pm 0.41$, which was 2.7 $\sigma$ from $S_{J/\psi K_S}$. This result generated quite a bit of excitement including dozens of phenomenology papers, evaluating $\delta S = S_{\phi K_S} - S_{J/\psi K_S}$ in various new physics scenarios. At ICHEP 2004 in Bejing, updated results were shown from datasets more than 3 times larger than those used in the initial measurements. The current average is now $S_{\phi K_S} = +0.34 \pm 0.21$, only 1.8 $\sigma$ away from $S_{J/\psi K_S}$. However, if one naively averages $-\eta_f S_f$ from all of the $b \to s$ penguin measurements, one gets $\langle -\eta_f S_f \rangle = +0.42 \pm 0.08$, which is 3.6 $\sigma$ from $S_{J/\psi K_S}$. Before concluding that this is evidence for new physics, deviations from $-\eta_f S_f = \sin 2\beta$ from sub-dominant Standard Model contributions must be carefully evaluated.

5.4. On to the next angle – $\alpha$ from charmless decays

The general technique for measuring the angle $\alpha$ of the Unitarity Triangle is to measure the time-dependent asymmetry coefficients $S_f$ and $C_f$ of a CP eigenstate $f$ for which the leading decay amplitude involves a $b \to u$ transition. In the ideal case, where the CKM phase of the decay amplitude is purely that of the $b \to u$ transition, $\lambda_f = \eta_f e^{-i2\beta} e^{-i2\gamma} = \eta_f e^{i2\alpha}, S_f = \eta_f \sin 2\alpha$, and $C_f = 0$. If the tree diagram for $B^0 \to \pi^+ \pi^-$ were the only decay amplitude, the $\pi^+ \pi^-$ would be ideal for measuring $\alpha$. Unfortunately, we know that penguin loop diagrams give non-negligible contributions to the total $\pi^+ \pi^-$ decay amplitude.

When a second decay amplitude with a different CP-violating weak phase, such as a penguin contribution, is included, the $\lambda_f$ parameter is no longer just a pure phase. The $\lambda_f$ becomes

$$\lambda_f = e^{-i2\alpha} \frac{T_f + P_f e^{+i\gamma} e^{i\delta_f}}{T_f + P_f e^{-i\gamma} e^{i\delta_f}}$$

where I have used the unitarity of the CKM matrix to define the $T$ and $P$ decay amplitudes in terms of just two weak phases. The magnitudes of the decay amplitudes $T_f$ and $P_f$ are real numbers and $\delta_f$ is relative strong (CP-conserving) phase between the $T$ and $P$ decay amplitudes. The problem now is that $P_f/T_f$ and $\delta_f$ can not be reliably calculated in a model-independent way, so they must be treated as unknowns that must be determined from experimental data. This is a significant complication. The time-dependent CP asymmetry coefficient $C_f$ can be non-zero, due to direct CP violation, and is proportional to $\sin \delta_f$. The $S_f$ coefficient is $\sqrt{1 - C_f^2} \sin 2\alpha_{\text{eff}}$, where $\alpha_{\text{eff}}$ goes to $\alpha$ as $|P_f/T_f| \to 0$.

5.4.1. The $\pi\pi$ system

Gronau and London pointed out that the penguin contribution can be calculated using isospin relations from the measured decay rates of all of the $\pi\pi$ states ($\pi^+ \pi^-$,
\( \pi^0\pi^0, \text{ and } \pi^\pm\pi^0 \) for \( B^0 \) and \( \overline{B}^0 \) separately. This is quite challenging experimentally and is limited by an 8-fold discrete ambiguity in the extraction of \( \alpha \) in the range 0 to \( \pi \). Grossman and Quinn noted a useful inequality derived from a geometrical analysis of the \( B \) and \( \overline{B} \) isospin triangles:

\[
\sin^2(\alpha - \alpha_{\text{eff}}) \leq \frac{\mathcal{B}(B^0 \to \pi^0\pi^0) + \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)}{\mathcal{B}(B^+ \to \pi^+\pi^0) + \mathcal{B}(\overline{B}^- \to \pi^-\pi^0)},
\]

which says that if the \( \pi^0\pi^0 \) branching fraction is small, the penguin contribution is small and the measured effective value \( \alpha_{\text{eff}} \) is close to the true value of \( \alpha \). Both the Babar and Belle experiments have seen evidence of the \( \pi^0\pi^0 \) mode. The current Grossman-Quinn bound gives \( (\alpha - \alpha_{\text{eff}})_{\pi^+\pi^-} < 35^\circ \) at 90\% C.L. which, unfortunately, isn’t very restrictive. This shortcut can’t be used for estimating \( \alpha \) from \( \alpha_{\text{eff}} \) – the full isospin analysis is required for the \( \pi\pi \) modes.

### 5.4.2. The \( \rho\rho \) system

The \( \rho^+\rho^- \) mode is not necessarily a \( CP \)-eigenstate, since a vector-vector final state can have \( CP \)-even \((L = 0, 2)\) and \( CP \)-odd \((L = 1)\) angular momentum configurations. The good news is that the angular analysis of the \( \rho^+\rho^- \) mode is consistent with full longitudinal polarization, thus it’s effectively a \( CP \)-even final state, just like \( \pi^+\pi^- \). The even better news is that the \( \rho^0\rho^0 \) mode has not been seen, implying that the penguin contribution is small, unlike the \( \pi\pi \) system. The Grossman-Quinn bound gives \( (\alpha - \alpha_{\text{eff}})_{\rho^+\rho^-} < 11^\circ \) at 90\% C.L. implying that \( S_{\rho^+\rho^-} \approx \sin 2\alpha \). The Babar collaboration has performed the time-dependent \( CP \) analysis of \( \rho^+\rho^- \) and find:

\[
C_L(\rho^+\rho^-) = -0.23 \pm 0.24 \text{ (stat)} \pm 0.14 \text{ (syst)} \\
S_L(\rho^+\rho^-) = -0.19 \pm 0.33 \text{ (stat)} \pm 0.11 \text{ (syst)}
\]

for the longitudinally polarized component. The solution, of the two available in the range 0 to \( \pi \), closest to the value consistent with other CKM constraints gives \( \alpha = [96 \pm 10 \text{ (stat)} \pm 4 \text{ (syst)} \pm 11 \text{ (peng)}]^\circ \).

### 5.4.3. The \( \rho\pi \) system

The \( \rho\pi \) system offers a unique way to resolve tree and penguin contributions in a charmless decay. Snyder and Quinn noted that a time-dependent \( CP \) analysis of the \( \pi^+\pi^-\pi^0 \) Dalitz plane could, in principle, determine \( \alpha \) without discrete ambiguities. The regions where the different \( \rho\pi \) states overlap in the Dalitz plane provide the key information needed to disentangle the tree and penguin contributions.

Thus far, only the Babar experiment has attempted the time-dependent Dalitz analysis, however both Babar and Belle have analyzed the \( \rho^\pm\pi^\mp \) mode as a quasi-two-body final state, by only selecting events near the \( \rho^\pm \) resonance and...
removing events in the regions of the Dalitz plot where the $\rho\pi$ states interfere. One interesting outcome of the quasi-two-body analysis of $\rho^{\pm}\pi^{\mp}$ is the following direct $CP$ asymmetry

$$A_{\rho\pi}^{-+} = \frac{N(B^0 \to \rho^+\pi^-) - N(B^0 \to \rho^-\pi^+)}{N(B^0 \to \rho^+\pi^-) + N(B^0 \to \rho^-\pi^+)}$$

(11)

which is the direct $CP$ asymmetry for the diagrams where the $\pi$ is from the virtual $W$. The two experiments measure

Belle: $A_{\rho\pi}^{-+} = -0.53 \pm 0.29$ (stat) $^{+0.09}_{-0.04}$ (syst)

Babar: $A_{\rho\pi}^{-+} = -0.47^{+0.14}_{-0.15}$ (stat) $\pm 0.06$ (syst),

The Babar value is derived from the full Dalitz analysis. A non-zero value of $A_{\rho\pi}^{-+}$ is evidence of direct $CP$ violation due to the penguin and tree amplitudes interfering. The results of the Babar full Dalitz analysis give $\alpha = [113^{+27}_{-17} (\text{stat}) \pm 6 (\text{syst})]$° with no discrete ambiguities in the range 0 to $\pi$.

5.4.4. Combining the modes – $\alpha$ from charmless $B$ decays

None of the three systems ($\pi\pi$, $\rho\rho$, or $\rho\pi$) gives a precise determination of $\alpha$ on its own, although that may change in the future. Discrete ambiguities, or false solutions, are a problem, especially for $\pi\pi$. However, only the true solution will be the same for all three systems – any overlap amongst the false solutions is accidental. Figure 3 shows the individual constraints on $\alpha$ from the three systems and the combined constraint. Combining all three modes gives $\alpha = [103 \pm 11]$°, which is in excellent agreement with the allowed value from other CKM constraints $\alpha_{\text{CKM}} = [98 \pm 16]$°. This is another victory for the Standard Model.

5.5. Direct $CP$ violation again – $A(K^{\pm}\pi^{\mp})$ is non-zero

The $K^{\pm}\pi^{\mp}$ mode is the main background from $B$ decays for $\pi^{\pm}\pi^{-}$, but it’s very interesting in its own right. The CKM factors for the top quark penguin amplitude are much larger than the tree amplitude: $|P/T|_{CKM} = |V_{tb}V_{ts}/V_{ub}V_{us}| \approx 1/(0.4\lambda^2) \approx 50$. However, the penguin loop diagram is suppressed with respect to the tree, so this somewhat reduces the factor of 50. Since this mode is “self-tagging” ($K^{+}\pi^{-}$ only comes from a $B^0$ and $K^{-}\pi^{+}$ only comes from a $\bar{B}$) the only kind of Standard Model $CP$ violation it can exhibit is through interfering decay amplitudes, or direct $CP$ violation. The current measurements of the asymmetry

$$A_{K\pi} = \frac{N(K^-\pi^+) - N(K^+\pi^-)}{N(K^-\pi^+) + N(K^+\pi^-)}$$

(12)

are

Babar: $A_{K\pi} = -0.133 \pm 0.030$ (stat) $\pm 0.009$ (syst)

Belle: $A_{K\pi} = -0.101 \pm 0.025$ (stat) $\pm 0.005$ (syst)
with an average of \( A_{K\pi} = -0.114 \pm 0.020 \), which is 5.7 \( \sigma \) from zero. Like \( \text{Re}(\epsilon'/\epsilon) \) in the kaon system, this establishes the phenomenon of direct \( CP \) violation in the \( B \) system.

5.6. **Mission impossible? – \( \gamma \) from \( B^{\pm} \rightarrow DK^{\pm} \)**

The angle \( \gamma \) of the unitarity triangle, which is the relative phase of the \( b \rightarrow u \) transition with respect to the \( b \rightarrow c \) transition, is by far the most difficult to measure. A fairly straightforward and theoretically clean method was proposed by Gronau, London, and Weyler.\(^47\) The decay \( B^{-} \rightarrow D^{0}K^{-} \) proceed via a \( b \rightarrow c \) tree diagram, while the decay \( B^{-} \rightarrow D^{0}K^{-} \) goes through a color-suppressed \( b \rightarrow u \) diagram. For final states that both the \( D^{0} \) and the \( D^{0} \) can decay to, these two paths will interfere, thus enabling the determination of the relative weak phase \( \gamma \).

A crucial parameter in this method is the relative size of the \( b \rightarrow u \) and \( b \rightarrow c \) amplitudes, which is defined \( r_{B} = |A(b \rightarrow u)/A(b \rightarrow c)| \). Unfortunately, \( r_{B} \) can’t be reliably calculated. A rough estimate is \( r_{B} \approx 0.4 F_{cs} \), where the 0.4 is from the ratio of CKM elements and \( F_{cs} \) is an unknown color suppression factor, which is expected to be in the range of \([0.2,0.5]\). This gives an expected range for \( r_{B} \) of \([0.1,0.2]\).

The original \( B \rightarrow DK \) proposal\(^14\) was to use \( D \) decays to \( CP \) eigenstates (e.g. \( \pi^{+}\pi^{-} \)) thereby forcing equal \( D^{0} \) and \( D^{0} \) decay amplitudes by construction. The problem with this technique is that the interference terms are of order \( r_{B} \), which may be small. An alternative \( B \rightarrow DK \) method\(^15\) uses \( D \) decays to flavor-specific final states (e.g. \( K^{-}\pi^{+} \)). If the dominant \( B \) decay \( (B^{-} \rightarrow D^{0}K^{-}) \) is combined with the suppressed \( D \) decay \( (D^{0} \rightarrow K^{+}\pi^{-}) \) the overall amplitudes of the \( b \rightarrow u \) and \( b \rightarrow c \) paths become comparable, thus maximizing the interference effects at the cost

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**Fig. 3.** Individual constraints on \( \alpha \) from the analysis of the \( \pi\pi \), \( \rho\rho \), and \( \rho\pi \) systems. The solid line is the combined constraint. The hatched region is the constraint from the CKM fit excluding the results shown in the figure.
of a lower overall rate. Finally, a hybrid $B \to DK$ approach was recently proposed, where one performs a direct CP violation analysis in the Dalitz plane of a 3-body $D$ decay, such as $D \to K_s\pi^+\pi^-$. The resonance structures of the Dalitz plane can be externally determined from copious $c\bar{c}$ samples. Overlapping resonances from the $D^0$ and $\bar{D}^0$ produce “hot spots” of relatively large interference with a known strong phase variation, which in principle allows for an ambiguity-free determination of $\gamma$ in the range $0$ to $\pi$.

Many measurements have been made of $B \to D^{(*)}K^{(*)}$ decay rates using $D$ decays to $CP$ eigenstates, however the statistical errors are still substantial. Both the Babar and Belle collaborations have investigated $[K^\pm\pi^-]DK^-$ and its charge conjugate. They both see only hints of a signal, although the hint is larger for Belle. The upper limits on the $[K^\pm\pi^-]DK^-$ rate can be translated into the following $90\%$ upper limits on $r_B$: $r_B < 0.28$ (Belle), $r_B < 0.23$ and $r_B^* < 0.21$ (Babar) where the last ($r_B^*$) is for the $D^*K$ mode. These results favor a small value of $r_B$.

Belle and Babar have also performed the $DK$ Dalitz analysis using the decay $D \to K_s\pi^+\pi^-$ using both $DK$ and $D^*K$ decays. Choosing the (single) solution for $\gamma$ in the range $0$ to $\pi$, the results are

$$\text{Belle} \quad \gamma = [77 \pm 17_{\text{stat}} \pm 13_{\text{syst}} \pm 11_{\text{model}}]^{\circ},$$

$$\text{Babar} \quad \gamma = [88 \pm 41_{\text{stat}} \pm 19_{\text{syst}} \pm 10_{\text{model}}]^{\circ},$$

where the Belle analysis used the Feldman-Cousins technique, while the Babar analysis interpreted their measurement using a Bayesian approach. The last uncertainty is from the resonance parameters in the Dalitz model. The large difference in statistical errors is partly due to the fact that the Belle (Babar) data favor a large (small) value of $r_B$.

All of the $B \to DK$ techniques mentioned in this section depend on the same set of three unknowns: $r_B$, $\gamma$, and $\delta_B$, where $\delta_B$ is the strong phase difference between the $b \to c$ and the $b \to u$ decay amplitudes. For this reason, it makes sense to combine all of the measurements in a global analysis. For example, the UTfit group has combined all $B \to DK$ data available using a Bayesian analysis. They find $r_B = 0.10 \pm 0.04$, and $\gamma = [68 \pm 19]^{\circ}$, which is consistent with the value from other CKM constraints.

Other approaches to measuring $\gamma$ include the time-dependent analysis of $B^0 \to D^{(*)}\pi$ and $B^0 \to D^{(*)}K^{(*)}$, where the asymmetry is proportional to $\sin(2\beta + \gamma)$, and studying the $B \to K\pi$ mode.

5.7. $B$ system summary

The new $B$ factories KEKB and PEP-II and their associated experiments, Belle and Babar, have contributed greatly to our understanding of CP violation. The measurement of $\sin 2\beta$ from charmonium decays was the first precision test of the Standard Model description of CP violation – a test it passed with flying colors. The $b \to s$ penguin decay modes show an interesting discrepancy with $\sin 2\beta$ from
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charmonium decays, which may be a hint of new physics. Resolving this discrepancy is a high priority for current and future experiments. Much progress has been made in determining the angle $\alpha$ of the unitarity triangle with charmless decays, most notably $\rho^+\rho^-$ and $\rho\pi$. Direct CP violation has been established in the $B$ system with the measurement of $A_{K\pi}$. Finally, determining the angle $\gamma$ of the unitarity triangle looks like it’s going to be difficult, as expected – $r_B$ is not large.

6. Concluding Remarks

The Standard Model description of CP violation is remarkably successful. There are many new and improved experimental constraints and no significant discrepancy has been found, although there are some interesting anomalies. Current and future experiments are now focusing on studying processes dominated by loop diagrams in the standard model: particle EDMs, $K_L \to \pi^0\nu\bar{\nu}$, and CP asymmetries in penguin-dominated $B$ decays. Loop diagrams bring in sensitivity to high virtual mass scales where new physics contributions may be significant. With experimental advances expected on all fronts, the study of CP violation in the next decade promises to be very exciting.

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43. Figure courtesy of Andreas Höcker and the CKM fitter group.
44. CKM fitter group (http://ckmfitter.in2p3.fr).
54. UTfit group (http://www.utfit.org).