Modified Dihadron Fragmentation Functions in Hot and Nuclear Matter

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Medium modification of dihadron fragmentation functions due to gluon bremsstrahlung induced by multiple scattering is studied in both deeply inelastic scattering (DIS) off large nuclei and high-energy heavy-ion collisions. The modified fragmentation functions for dihadrons are found to follow closely that of single hadrons leading to a weak nuclear suppression of their ratios as measured by HERMES in DIS experiments. Meanwhile, a moderate medium enhancement of the near-side correlation of two high $p_T$ hadrons is found in central heavy-ion collisions, partially due to trigger bias caused by the competition between parton energy loss and the initial Cronin effect.

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Medium modification of the jet structure has emerged as a new diagnostic tool for the study of partonic properties of the dense matter. The modification goes beyond a mere suppression of inclusive spectra of leading hadrons and can be extended to include the modification of many particle observables, the simplest of which are two-hadron correlations within the jet cone. Such two-hadron correlations have been measured both in DIS and high-energy heavy-ion collisions. While the two-hadron correlation is found slightly suppressed in DIS off a nucleus versus a nucleon target, it is moderately enhanced in central Au + Au collisions relative to that in $p + p$. This is in sharp contrast to the observed strong suppression of single inclusive spectra in both DIS and central $A + A$ collisions.

Theoretically, multi-particle correlations from jet fragmentation and their medium modification can be studied through $n$-hadron fragmentation functions which can be defined as the overlapping matrices of partonic field operators and $n$-hadron states. These $n$-hadron fragmentation functions are non-perturbative and involve long-distance processes. However, they may be factorized from the hard perturbative processes and their evolution with momentum scale may be systematically studied in perturbative QCD (pQCD), similarly as the usual single inclusive hadron fragmentation functions.

In this Letter, we report our first study of the medium modification of dihadron fragmentation functions in both cold nuclei and hot quark-gluon matter. The medium modification of dihadron fragmentation functions in DIS off nuclei will be derived within the framework of generalized factorization and twist expansion. The results are then extended to the case of parton propagation in heavy-ion collisions. With exactly the same parameters determined from the modified single hadron fragmentation functions, the medium modification of two-hadron correlations in both DIS off nuclei and heavy-ion collisions are predicted and compared to the experimental data.

Similar to single hadron fragmentation functions, dihadron fragmentation functions of a quark can be defined as the overlapping matrices of the quark fields and two-hadron final states,

$$D_{q_1,q_2}^{h_1,h_2}(z_1,z_2) = \frac{z^4}{4z_1z_2} \int \frac{d^2q_\perp}{4(2\pi)^2} \int \frac{dp^4}{(2\pi)^4} \prod_{S=2} \delta \left( z - \frac{p_h^+}{p^+} \right) \left[ \gamma^+ \int d^4xe^{ip_\perp x} \sum_{S-2} \langle 0|\bar{\psi}_q(x)|p_1,p_2,S-2\rangle\langle p_1,p_2,S-2|\bar{\psi}_q(0)|0 \rangle \right], \quad (1)$$

and can be factorized from the hard processes, where $p_h^+ \equiv p_1^+ + p_2^+$ is the total momentum of the two hadrons with flavors $h_1$ and $h_2$, $z \equiv z_1 + z_2$ is the corresponding total momentum fraction and $q_\perp = p_1\perp - p_2\perp$ is the relative transverse momentum. They also satisfy the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations as have been derived in Ref. 6. One unique feature of the DGLAP equations for dihadron fragmentation functions is the contribution from independent fragmentation of two partons after the parton splitting, which involves the product of two single fragmentation functions. These DGLAP equations for dihadron fragmentation functions have been solved numerically and the $Q^2$ evolution of the dihadron fragmentation functions agrees very well with results from JETSET Monte Carlo simulations of $e^+ + e^- \rightarrow h_1 + h_2 + X$ processes. Although both the single and dihadron fragmentation functions evolve rapidly with $Q^2$, their ratio has a very weak $Q^2$ dependence. Since there are no experimental data available for dihadron fragmentation functions, JETSET Monte Carlo results will be used as the initial condition for the vacuum dihadron fragmentation functions in this study. For single hadron fragmentation functions, the BKK parameterization will be used; this also agrees well with JETSET results.

Applying factorization to dihadron production in single jet events in DIS off a nucleus, $e(L_1) + A(p) \rightarrow$
$e(L_2) + h_1(p_1) + h_2(p_2) + X$, one can obtain dihadron semi-inclusive cross section,

$$E_{L_2} \frac{d^2 \sigma^{h_1 h_2}_{\text{DIS}}}{d^2 L_2 dz_1 dz_2} = \frac{\alpha_s^2}{2\pi s} Q^2 \frac{dW^{\mu \nu}}{dz_1 dz_2},$$

(2)
in terms of the semi-inclusive tensor at leading twist,

$$\frac{dW^{\mu \nu}}{dz_1 dz_2} = \sum_q \int dx f_q^A(x,Q^2)H^{\mu \nu}(x,p,q) \times D_{q}^{h_1 h_2}(z_1, z_2, Q^2).$$

(3)

In the above, $D_{q}^{h_1 h_2}(z_1, z_2)$ is the dihadron fragmentation function, $H^{\mu \nu} = (1/2) \text{Tr}(L_1\gamma_{\mu}L_2\gamma_{\nu})$, the factor $H^{\mu \nu}$ represents the hard part of quark scattering with a virtual photon which carries a four-momentum $q = [-Q^2/2q^-, q^-, 0, 0]_\perp$ and $f_q^A(x,Q^2)$ is the quark distribution in the nucleus which has a total momentum $A[p^+,0,0]_\perp$. The hadron momentum fractions, $z_1 = p_1^2/q^-$ and $z_2 = p_2^2/q^-$, are defined with respect to the initial momentum $q^-$ of the fragmenting quark.

At next-to-leading twist, the dihadron semi-inclusive tensor receives contributions from multiple scattering of the struck quark off soft gluons inside the nucleus with induced gluon radiation. One can reorganize the total contribution (leading and next-to-leading twist) into a product of effective quark distribution in a nucleus, the contribution (leading and next-to-leading twist) into a induced gluon radiation. One can reorganize the total contribution (leading and next-to-leading twist) into a product of effective quark distribution in a nucleus, the contribution (leading and next-to-leading twist) into a induced gluon radiation.

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$$D_{q}^{h_1 h_2}(z_1, z_2) = \int Q^2 \frac{d^2 \sigma^{h_1 h_2}}{d^2 p_1} \frac{\alpha_s}{2\pi} \times \left[ \int_{z_1+z_2}^{1} \frac{dy}{y} \left( \Delta P_{q\rightarrow qg}(y,x_B,x_L,L_\perp) \right) D_{q}^{h_1 h_2} \left( \frac{z_1}{y}, \frac{z_2}{y} \right) + \Delta P_{q\rightarrow qg}(y,x_B,x_L,L_\perp) D_{q}^{h_1 h_2} \left( \frac{z_1}{y}, \frac{z_2}{y} \right) \right] \times D_{q}^{h_1}(\frac{z_1}{y}) D_{q}^{h_2}(\frac{z_2}{1-y} + (h_1 \rightarrow h_2)).$$

(4)

In the above, $x_B = -Q^2/(2p^+q^-)$, $x_L = l_\perp^2/(2p^+q^-)(1-y)$, $l_\perp$ is the transverse momentum of the radiated gluon, $\Delta P_{q\rightarrow qg}$ and $\Delta P_{q\rightarrow qg}$ are the modified splitting functions whose forms are identical to that in the modified single hadron fragmentation functions. The switch $(h_1 \rightarrow h_2)$ is only meant for the last term, which represents independent fragmentation of the quark and gluon after the induced bremsstrahlung. The corresponding modified splitting function,

$$\Delta P_{q\rightarrow qg} = \frac{1 + y^2}{1-y} \frac{C_A 2\pi\alpha_s T_{qg}^A(x_B,x_L)}{(l_\perp^2 + \langle k^2_L \rangle) N_c f_q^A(x_B,Q^2)},$$

(5)
is similar to $\Delta P_{q\rightarrow qg}$ but does not contain contributions from virtual corrections. In the above, $C_A = 3$, $N_c = 3$, and $\langle k^2_L \rangle$ is the average intrinsic parton transverse momentum inside the nucleus.

Note that both modified splitting functions depend on the quark-gluon correlation function $T_{qg}^A$ in the nucleus that also determines the modification of single hadron fragmentation functions. For a Gaussian nuclear distribution, it can be estimated as

$$T_{qg}^A(x_B,x_L) = \tilde{C}(Q^2) m_N R_A f_q^A(x_B)(1 - e^{-x_L^2/s_A^2}),$$

(6)

where, $x_A = 1/m_N R_A$, $m_N$ is the nucleon mass and $R_A = 1.12 A^{1/3}$ is the nuclear radius. The overall constant $\tilde{C} \propto \int x_L G(x_L)$ is only parameter in the modified dihadron fragmentation function which might depend on the kinematics of the DIS process but is identical to the parameter in the modified single fragmentation functions. In the phenomenological study of the single hadron fragmentation functions in DIS off nuclei, $\tilde{C} = 0.006$ GeV$^2$ is determined within the kinematics of the HERMES experimental data. With the determined parameter $\tilde{C}$, one can calculate the effective fractional energy loss of the quark,

$$\langle \Delta z \rangle = \int_0^{Q^2} \frac{d\ell_T^2}{\ell_T^2} \int_0^1 dz \frac{1 + (1 - z)^2}{\ell_T^2 + \langle k_T^2 \rangle} \times \frac{C_A 2\pi\alpha_s T_{qg}^A(x_B,x_L)}{N_c f_q^A(x_B)},$$

(7)

With no additional parameters in Eq. (6), one can predict the nuclear modification of dihadron fragmentation functions within the same kinematics. Since dihadron fragmentation functions are connected to single hadron fragmentation functions via sum rules, it is more illustrative to study the modification of the conditional distribution for the second rank hadrons,

$$N_2(z_1, z_2) = D_{q}^{h_1 h_2}(z_1, z_2)/D_{q}^{h_1}(z_1),$$

(8)

where $z_1$ and $z_2 < z_1$ are the momentum fractions of the triggered (leading) and associated (secondary) hadrons, respectively. Shown in Fig. 1 is the predicted ratio of the associated hadron distribution in DIS off a nitrogen $(A = 14)$ target to that off a proton $(A = 1)$, as compared to the HERMES experimental data. As in the experiment, we integrate over $z_1 > 0.5$ in both the single and dihadron fragmentation functions. The agreement between the prediction and the data is remarkable given that no free parameters are used. The suppression of the associated hadron distribution $N_2(z_1, z_2)$ at large $z_2$ due to multiple scattering and induced gluon bremsstrahlung in a nucleus is quite small.
compared to the suppression of the single fragmentation functions \[.\] Since \(N_2(z_1, z_2)\) is the ratio of double and single hadron fragmentation functions, the effect of induced gluon radiation or quark energy loss is mainly borne by the single spectra of the leading hadrons. This is similar to the evolution of the dihadron fragmentation function with momentum scale in the vacuum \[.\] At small values of \(z_2\), the modified dihadron fragmentation function rises above its vacuum counterpart more than the single fragmentation functions. This is due to the new contribution where each of the detected hadrons emanates from the independent fragmentation of the quark and the radiated gluon.

\[
\langle \Delta z \rangle = \frac{\langle E_T \rangle_{\text{loss}} - \langle E_T \rangle_{\text{no-loss}}}{\langle E_T \rangle_{\text{loss}}} \text{, which is completely determined from the suppression of single inclusive hadron spectra and away-side correlation. Since the high } p_T^{\text{trig}} \text{ biases the jet production position towards the surface of the overlapped region, } \langle \Delta z \rangle \text{ associated with a triggered hadron is always smaller than the average energy loss of both the away-side jet and jets in non-triggered events.}
\]

Using \(\langle \Delta z \rangle = \langle E_T \rangle_{\text{loss}} / \langle E_T \rangle\) we first determine the parameter \(C\) in Eq. \(\text{[\ref{eq:1}]}\) and in turn calculate both the modified single and dihadron distributions. The ratio of such associated hadron distributions in \(Au + Au\) versus \(p + p\) collisions, referred to as \(I_{AA} / I_{pp}\), is plotted as the solid line in Fig. \(\text{[\ref{fig:2}]}\) together with the STAR data \[.\] as a function of the number of participant nucleons which represents the centrality of heavy-ion collisions. In contrast to the slight nuclear suppression of the associated hadron distribution at large \(z_2\) in DIS off a nucleus (Fig. \(\text{[\ref{fig:3}]}\), we see some enhancement in central \(Au + Au\) collisions, which is mainly caused by trigger bias toward a larger initial jet energy and therefore smaller \(z_1\) and \(z_2\). The enhancement apparently increases with \(N_{\text{part}}\) because of increased initial gluon density and system size of the produced dense matter which leads to an increased total parton energy loss. In the most peripheral collisions, the effect of smaller parton energy loss is countered by the Cronin effect due to initial state multiple scattering that biases toward smaller \(\langle E_T \rangle\) relative to \(p + p\) collisions. As a result, the associated hadron distribution is slightly

![FIG. 1: Results of the medium modification of the associated hadron distribution \(N_2(z_1, z_2)\), see Eq. \(\text{[\ref{eq:5}]}\) in a cold nuclear medium versus its momentum fraction. The momentum fraction of the leading hadron \(z_1\) is integrated over all allowed values above 0.5. The ratio of this quantity in Nitrogen (N) with that in deuterium (D) are the HERMES data points.](image-url)
suppressed. The same phenomenon was seen in $d + Au$ collisions [14]. For higher $p_T^{\text{trig}} = 10 \text{ GeV}$, the suppression will diminish because the Cronin effect disappears. On the other hand, the enhancement in central collisions becomes larger because of the increased energy loss.

The theoretical calculation of $I_{AA}$ is somewhat schematic in this Letter which should be improved with a full calculation including exact pQCD jet cross sections and different energy loss for quarks and gluons. We have also focused on large $p_T^{\text{trig}}$ and $p_T^{\text{assoc}}$, because ideally, the study here is most suitable for large transverse momenta of both triggered and associated hadrons. Large $p_T^{\text{trig}}$ or initial jet energy ensures the validity of the independent fragmentation picture. Otherwise, other non-perturbative and higher twist effects can become important. One of these is the parton recombination process in heavy-ion collisions which has been employed to explain the flavor dependence of the single hadron spectrum suppression and the azimuthal anisotropy [20]. Such recombination process can also modify near-side associated hadron distribution [21], and cause additional trigger bias, especially for small values of $p_T^{\text{trig}}$.

In summary, we have studied the medium modification of dihadron fragmentation functions in both hot and nuclear matter due to multiple parton scattering and induced gluon radiation. The modification is found to follow closely that of single hadron fragmentation functions so that the associated hadron distributions or the ratios of dihadron and single fragmentation functions are only slightly suppressed in DIS off nuclei but enhanced in central heavy-ion collisions due to trigger bias. With no extra parameters, our calculations agree very well with experimental data.

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![FIG. 2: Calculated medium modification of associated hadron distribution from jet fragmentation in $Au + Au$ collisions at $\sqrt{s} = 200 \text{ GeV}$ for different choices of trigger $p_T$ and associated $p_T$ as compared to experimental data [3, 4].](image)

[17] Jet structure in $p + p$ collisions is known to have correlated background due to the multiplicity bias and coherence in hadronization. Such correlated background has non-trivial centrality and azimuthal angle dependence in $p + A$ and $A + A$ collisions. Ref. [3] uses the two-hadron correlation in $p + p$ collisions, including the background, as the baseline while Ref. [10] subtracts out the background both in $p + p$ and $A + A$ collisions. Therefore different methods of background subtraction can affect extracted dihadron correlations and thus have some intrinsic systematic errors.
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