The distribution of red and blue galaxies in groups: an empirical test of the halo model

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ABSTRACT
The popular halo model predicts that the power spectrum of the galaxy fluctuations is simply the sum of the large scale linear halo-halo power spectrum and the weighted power spectrum of the halo profile. Previous studies have derived halo parameters from the observed galaxy correlation function. Here we test the halo model directly for self-consistency with a minimal set of theoretical assumptions by utilising the 2dF Galaxy Redshift Survey (2dFGRS). We derive empirically the halo occupation and galaxy radial distributions in the haloes of the 2dF Percolation-Inferred Galaxy Group (2PIGG) catalogue. We find that the NFW profile provides a good fit to the galaxy radial distributions, with concentration parameters \(c = 4.8, 2.1, 3.4\) for red, blue and all galaxies respectively (with 1-sigma errors of 9-14%). The mean halo occupation number is found to be well-fitted by a power-law, \(\langle N|M\rangle \propto M^{\beta}\) at high masses, with \(\beta = 1.05, 0.88, 0.99\) for red, blue and all galaxies respectively (with 1-sigma errors of 15-19%). Adding the observed linear power spectrum to these results, we compare these empirical predictions of the halo model with the observed correlation functions for these same 2dF galaxy populations. We conclude that subject to some fine tuning it is an acceptable model for the two-point correlations. Our analysis also explains why the correlation function slope of the red galaxies is steeper than that of the blue galaxies. It is mainly due to the number of red and blue galaxies per halo, rather than the radial distribution within the haloes of the two galaxy species.

Key words: large-scale structure of Universe – galaxies: haloes – galaxies: statistics – dark matter

1 INTRODUCTION
Measurements of the galaxy-galaxy two-point correlation function have shown that it is well fitted by a power-law over a wide range of distance scales (e.g. Peebles 1980; Zehavi et al. 2002; Hawkins et al. 2003). Recent studies have further shown that the slope of this power-law is a function of galaxy spectral type or colour (e.g. Zehavi et al. 2002; Norberg et al. 2002; Madgwick et al. 2003): relatively active star-forming galaxies (blue) have a shallower two-point correlation function than passive galaxies (red). The correlation function of the dark matter, on the other hand, has a lower amplitude and is far from being a featureless power-law, thus the galaxy distribution is said to be biased with respect to the dark matter.

Much recent progress towards understanding the nature of galaxy biasing has come through use of the halo model of large scale structure. The model has its origins in the work of Neyman & Scott (1952), and was first applied to continuous density fields by Scherrer & Bertschinger (1991). It has been recently used in the context of the clustering of both dark matter and galaxies (e.g. Peacock & Smith 2000; Seljak 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002; Magliocchetti & Porciani 2003; van den Bosch, Yang & Mo 2003; Cooray & Sheth 2002). Provide a detailed review. In the model, matter in the universe is assumed to reside only in discrete haloes. The distribution of the matter within the haloes gives rise to the non-linear component of the power spectrum, while the large-scale distribution of the haloes in space is responsible for the power on linear scales. The simplicity of the model allows analytic calculations of correlation functions to be made, so that large parameter spaces can be investigated at relatively little cost when compared to numerical simulations of the large scale structure.

Two ingredients are required to extend the model to galaxy clustering: the probability distribution for the number of galaxies hosted by a particular halo, \(P(N|M)\), known as the halo occupation distribution (HOD), and the spatial distribution of galaxies in haloes. The distribution of the galaxies within haloes is commonly assumed, ad hoc,
to be the same as that of the dark matter; differences in the clustering properties are then solely due to the choice of halo occupation distribution. However, it is well known that galaxies of different types are not identically distributed within galaxy groups (manifested in the well-known phenomenon of morphological segregation, e.g. Dressler 1980; Postman & Geller 1984; Goto et al. 2003): red, typically elliptical galaxies, with low star-formation rates are preferentially found towards the centres of large groups, while blue, actively star-forming galaxies dominate in the outskirts of groups and in the field.

Previous studies have assumed the validity of the halo model, and used the observed galaxy correlation functions to constrain the halo occupation distribution, or the radial distribution of galaxies in haloes (e.g., Magliocchetti & Porciani 2003; Scranton 2003; van den Bosch et al. 2003). In contrast, we take an empirical approach as possible. We directly measure the radial and halo occupation distributions of galaxies in the 2dFGRS Percolation-Inferred Galaxy Group (2PIGG) catalogue (Eke et al. 2004), focusing in particular on the distributions of red and blue galaxies. Having measured these distributions, no free parameters remain in the model. Since the correlation functions of these populations have previously been directly obtained (Norberg et al. 2002; Hawkins et al. 2003; Madgwick et al. 2003), the non-trivial success of the halo model in reproducing the observed clustering statistics can be directly tested using fully self-consistent 2dF observations. Requiring the model to account for the differences in the observed correlation functions of the red and blue populations is an especially stringent test.

The structure of this paper is as follows. In Section 2, we describe the halo model in detail. We introduce the 2PIGG catalogue in Section 3, and describe how the properties of interest are inferred from the observational data. In Section 4, we examine the radial distributions of galaxies within the 2PIGGs, and in Section 5 their halo occupation distribution is investigated. Finally, in Section 6 the halo model is applied to our results, and the predicted clustering results compared with observations.

## 2 THE HALO MODEL FORMALISM

In essence, the halo model exemplifies the natural distinction between linear (large-scale) and non-linear (small-scale) clustering. Indeed, the two regimes appear as separate terms in the halo model galaxy power spectrum:

\[ P_{\text{gal}}(k) = P_{\text{gal}}^{(1h)}(k) + P_{\text{gal}}^{(2h)}(k), \]

with \( P_{\text{gal}}^{(1h)}(k) \) the intra-halo, non-linear term, and \( P_{\text{gal}}^{(2h)}(k) \) the inter-halo, linear term. Although we will later require the two-point correlation functions, \( \xi(r) \), explicit calculation of these involves convolutions of the halo profiles; we prefer instead to work in Fourier space, ultimately obtaining the correlation function via the transform

\[ \xi_{ss}(r) = \int \frac{\Delta_{\text{gal}}^{2}(k) \sin(kr) \, dk}{kr}, \]

with \( \Delta_{\text{gal}}^{2}(k) = \frac{A^2}{2\pi^2} P_{\text{gal}}(k) \), the dimensionless form of the power spectrum.

Within the halo model framework, the components of the galaxy power spectrum are (e.g. Cooray & Sheth 2002)

\[ P_{\text{gal}}^{(1h)}(k) = \int dM \, n(M) \frac{\langle N(N-1)|M \rangle}{\delta_{\text{gal}}^2} \bar{u}_{\text{gal}}(k|M)^2, \]

and

\[ P_{\text{gal}}^{(2h)}(k) = P_{\text{dm}}^{(\text{lin})}(k) \left[ \int dM n(M) b(M) \frac{\langle N | M \rangle}{\delta_{\text{gal}}} \bar{u}_{\text{gal}}(k|M) \right]^2. \]

where the integrals are over the halo mass, \( M \). In these expressions, \( n(M) \) is the halo mass function, \( b(M) \) is the halo biasing factor, \( \langle N | M \rangle \) and \( \langle N(N-1) | M \rangle \) are the first and second factorial moments of the halo occupation distribution, \( P(N|M) \), respectively, \( \bar{u}_{\text{gal}}(k|M) \) is the Fourier transform of the normalised radial distribution of galaxies within haloes. We now describe these terms in more detail.

### 2.1 Halo mass function

The number density of haloes of mass \( M \) in space is described by the mass function, \( n(M) \), which we assume to have the form proposed by Sheth & Tormen (1999) (an extension of the Press & Schechter 1974 form):

\[ n(M) dM = \frac{\bar{\rho}}{M} f(\nu) d\nu, \]

\[ \nu f(\nu) = A_\nu \left( 1 + (\nu q)^{-p} \right) \left( \frac{\nu^q}{2\pi} \right)^{1/2} e^{-\nu^q/2}, \]

where \( \bar{\rho} \) is the background density of the universe, \( q = 0.707 \), \( p = 0.3 \), and normalisation implies \( A_\nu \approx 0.322 \). The mass variable is defined as \( \nu \equiv (\delta_{\text{sc}}(z)/\sigma(M))^2 \), where \( \delta_{\text{sc}}(z) \) is the linear-theory prediction for the present day overdensity of a region undergoing spherical collapse at redshift \( z \), and \( \sigma(M) \) is the r.m.s. variance of the present day linear power spectrum in a spherical top-hat which contains an average mass \( M \). Note that the mass function depends on the redshift only through \( \delta_{\text{sc}}(z) = \delta_{\text{sc}}(0)/D(z) \), where \( \delta_{\text{sc}}(0) \approx 1.68 \), and \( D(z) \) is the linear growth factor, normalized so that \( D(0) = 1 \). The value of \( \delta_{\text{sc}}(0) \) is only weakly sensitive to the cosmological model (Eke, Cole & Frenk 1998).

### 2.2 Halo biasing

Haloes are biased tracers of the overall dark matter distribution. The degree of bias is a function of the halo mass. Following Mo & Whitl (1996), we can write the power spectrum of dark matter haloes of given masses, \( M_1 \) and \( M_2 \) as

\[ P_{\text{dm}}(k; M_1, M_2) = b(M_1) b(M_2) P_{\text{dm}}(k). \]

We further assume \( P_{\text{dm}}(k) = P_{\text{dm}}^{(\text{lin})}(k) \), the linear dark matter power spectrum, since inter-halo correlations are only important on large, quasi-linear scales. Sheth & Tormen (1999) derive the required halo bias factors,

\[ b(M) = 1 + \frac{q}{\delta_{\text{sc}}(z)} - \frac{(q^2 - 1)}{2} + \frac{2p}{q} \frac{\delta_{\text{sc}}(z)}{1 + (qv)^p}, \]

with \( p \) and \( q \) taking the values given in Section 2.1.

### 2.3 Galaxy distribution

The halo occupation distribution, \( P(N|M) \), appears in the power spectrum (equations 3 and 4) through its first and
second factorial moments, \( \langle N|M \rangle \) and \( \langle N(N - 1)|M \rangle \) respectively. The galaxy number density is given by

\[
\tilde{n}_{\text{gal}} = \int \langle N|M \rangle \, n(M) \, dM.
\]

(9)

The spatial distribution of galaxies within haloes is assumed to be spherically symmetric about the halo centre, so that the density profile, \( \rho_{\text{gal}}(r|M) \), is a function of \( r \) only for a halo of a given mass \( M \). The profile is normalized so that \( \int r_{\text{vir}} \rho_{\text{gal}}(r|M) \, 4\pi r^2 \, dr = 1 \) \( (r_{\text{vir}} \) will be defined in equation 4), and the Fourier transform of the normalized profile is denoted by \( \tilde{u}_{\text{gal}}(k|M) \).

We attempt to directly measure the radial density and halo occupation distributions in subsequent sections.

2.4 Galaxy bias

On distance scales for which the inter-halo term is important, \( \tilde{n}_{\text{gal}}(k|M) \approx 1 \) is a good approximation. The integral on the right-hand side of equation 9 is then independent of scale and, with reference to equation 10, we can re-write the relations as:

\[
P_{\text{gal}}^{(2h)}(k) = b_{\text{gal}}^{2} P_{\text{dm}}^{(\text{lin})}(k),
\]

(10)

where we have defined the galaxy bias parameter as:

\[
b_{\text{gal}} \equiv \int dM \, n(M) \, b(M) \frac{\langle N|M \rangle}{\tilde{n}_{\text{gal}}}. \]

(11)

2.5 An empirical approach to the halo model

As we have stressed, our aim is to rely on observations wherever possible, avoiding model-dependent assumptions. Our underlying assumption is that the observed galaxy groups represent the haloes. Even if this assumption is not perfect it is very likely that there is a simple ranking relation between the observed groups and the haloes. Although we could take for \( n(M) \) the histogram of the observed groups versus their estimated mass, we prefer to use the more robust mass function \( n(M) \) given by equation 4. Future group samples with accurate masses will allow us to use them directly for \( n(M) \).

Equation 3 requires knowledge of the occupation quantities \( \langle N|M \rangle \), \( \langle N(N - 1)|M \rangle \) and the radial profile \( \rho_{\text{gal}}(r) \) which we shall determine directly from the 2PIGG sample per galaxy type. The mean number of galaxies (equation 9) then follows from the above \( n(M) \) and \( \langle N|M \rangle \).

The halo-halo correlation power spectrum \( P_{\text{gal}}^{(2h)}(k) \) could in principle be taken directly from the group-group power spectrum. The group-group correlation functions have actually been derived by Padilla et al. (2004) and Yang et al. (2004). However, biases in mapping the groups to haloes may make this approach somewhat inaccurate with the present data. While this should be possible with future group samples here we follow the approximation given by equations 10 and 11. In fact, as shown later in Section 6.1, we find that \( b_{\text{gal}} \) is close to unity, interestingly in accord with the biasing derived from the 2dF linear galaxy-galaxy power spectrum (Percival et al. 2001) combined with pre-WMAP CMB measurements (Lahav et al. 2002). In practice we use equations 10 and 11 with the linear power spectrum of the dark matter, \( P_{\text{dm}}^{(\text{lin})}(k) \), which has been well constrained by observations of the Cosmic Microwave Background (CMB). The galaxy power spectrum on linear scales has also been measured observationally, for example by Percival et al. (2001a) from the 2dFGRS, and Heymans et al. (2002) from the Sloan Digital Sky Survey. The shapes of these power spectra have been shown to be consistent with a flat universe Λ-CDM matter power spectrum, with present epoch \( \Omega_m = 0.3 \) [Percival et al. 2001; Efstathiou et al. 2002]. We therefore assume this form for \( P_{\text{dm}}^{(\text{lin})}(k) \), adopting the WMAP normalisation \( \sigma_8 = 0.9 \) [Spergel et al. 2003] and the biasing parameter derived in Section 5.1. We emphasize again that although we have to compromise here by making several theoretical assumptions, it would be possible in the future to use galaxy and group catalogues to test the halo model almost without any theoretical prior.

3 GALAXY GROUP CATALOGUE

We assume that galaxy groups are representative of the underlying dark matter haloes. The 2dFGRS Percolation-Inferred Galaxy Group catalogue (2PIGG; Eke et al. 2004a) is currently the largest homogeneous sample of galaxy groups publicly available. It comprises ~29,000 groups containing at least two galaxy members, which host a total of ~105,000 galaxies. The groups were identified from the 2dFGRS by means of a friends-of-friends percolation algorithm. Eke et al. (2004a) tested the algorithm on mock samples generated from cosmological dark matter simulations in order to optimize the mapping between the recovered galaxy groups and the actual dark matter haloes; this strengthens the case for our prior assumption that galaxy groups may be identified with the true bound structures in the dark matter distribution.

Eke et al. (2004b) tuned their group-finding algorithm so as to maximize the completeness of the recovered groups. Consequently, very few true group members are erroneously excluded, but this is at the expense of increased contamination by interlopers (i.e. field galaxies assigned to groups). The severity of the contamination increases with redshift; following Eke et al. (2004b), we discard groups at redshifts greater than \( z > 0.12 \). At this redshift the total number of field galaxies included in groups rises to ~50 per cent of the total number of true group members, and the number of interlopers increases very rapidly with redshift beyond this point.

3.1 Spectral classification

Madgwick et al. (2002) used a principal component analysis of the 2dFGRS dataset to define, \( \eta \), a continuous parametrization of spectral type. This parameter is most strongly correlated with the current star formation rate in each galaxy, but is also a good indicator for morphological type and colour. Following Madgwick et al. (2003), we broadly classify galaxies in our sample using a cut at \( \eta = -1.4 \). We label galaxies with \( \eta > -1.4 \) (relatively active) as blue galaxies, and those with \( \eta < -1.4 \) (relatively passive) as red. We exclude from our analysis any group which does not have a measurement of \( \eta \) for all its member galaxies. The remaining sample comprises 3,147 groups.
hosting a total of 25,118 galaxies (of which 12,851 are blue and 12,267 are red).

3.2 Mass estimation

The majority of the 2PIGGs have a measurement for the one-dimensional velocity dispersion, \( \sigma_v \). This can be used to estimate the group mass as

\[
M = A \frac{\sigma_v^2 R_{\text{rms}}}{G},
\]

where \( R_{\text{rms}} \) is the r.m.s. projected separation from the central galaxy of the remaining galaxies assigned to the group. Through use of simulated mock surveys, Eke et al. (2004a) derive the value \( A = 5.0 \) by requiring that the estimated mass be unbiased with respect to that of the underlying dark matter haloes. The simulated dark matter haloes are identified using a friends-of-friends algorithm, and the recovered haloes have mean spherical overdensities of \( \sim 200 \) times the background density (Eke, private communication).

Sheth & Tormen (1999) define the mass of a halo to be that enclosed within such an overdensity, thus it is valid to identify the mass estimate of equation (12) with the halo mass enclosed within such an overdensity, thus it is valid to identify the mass estimate of equation (12) with the halo mass used throughout Section 2.

The measurement of the velocity dispersion is very unreliable for groups with a small number of observed galaxies and we therefore discard those with fewer than four members. Even for groups with large memberships we must be wary of the significant scatter in the relation between group mass and the velocity dispersion (see fig. 3 of Eke et al. 2004a). Yang et al. (2004a) have independently constructed a galaxy group catalogue for the 2dFGRS. They investigate the reliability of dynamical mass estimates, and propose an alternative halo mass assignment based on total group luminosity. This gives them a particular advantage at low group masses where the dynamical masses are especially unreliable.

In order to maintain internal consistency regarding the definition of the group mass, we define the group virial radius,

\[
r_{\text{vir}} = \left( \frac{3M}{4\pi\Delta \rho} \right)^{\frac{1}{3}},
\]

to be that enclosing a spherical overdensity \( \Delta = 200 \) times the background density of the universe (at the group redshift).

3.3 Group membership

In order to estimate the true galaxy membership of the 2PIGGs it is necessary to correct for (i) the flux limit, and (ii) the incompleteness of the 2dF survey due to e.g. constraints on fibre positioning. Eke et al. (2004a) compute for each galaxy a weight, \( w_j \), to account for the local incompleteness of the survey. These weights only account for missed galaxies which are brighter than the local magnitude limit, \( b_{j,\text{lim}} \); the flux limit is not yet accounted for.

In order for the group membership to be well-defined observationally, a limiting absolute magnitude, \( M_{b,j,\text{com}} \), must be specified. We explain in Section 4 how our choices for this limit are dictated by the samples used in determining the observed correlation functions; the values we use are \( M_{b,j,\text{com}} - 5 \log_{10} h = -19.25, -19.04, -19.50 \) for red, blue and all galaxies respectively.

The faintest detectable absolute magnitude at distance \( x \) is given by \( M_b^{i,0}(x) = b_{j,\lim} - 25 - 5 \log(x) - K^{i,0}(x) \) for galaxies of spectral type \( i \), where \( K^{i,0}(x) \) is the \( K \)-correction (measured for each spectral type by Madgwick et al. 2002). Of all the galaxies more luminous than the threshold \( M_{b,j,\text{com}} \), the fraction which are detectable at distance \( x \) is given by the selection function:

\[
\phi^{i}(x) = \frac{\int_{M_{b,j}^{i,0}(x)}^{\infty} \phi^{i}(M) \, dM}{\int_{-\infty}^{\infty} \phi^{i}(M) \, dM},
\]

where \( M_{\text{max}} = \max(M_{b,j}^{i,0}(x), M_{b,j,\text{com}}) \). A further complication is that galaxies with \( b_j < 14 \) were removed from the redshift catalogue; the lower integration limit of the numerator in equation (14) is adjusted to account for this.

Since we are dealing with galaxies in groups we choose to use luminosity functions (LFs) specific to the populations of these over-dense regions, rather than those of the whole 2dF sample. Croton et al. (2004) have derived 2dF LFs as a function of density environment and per spectral type. We use their ‘cluster’ LFs for red and blue galaxies to compute the selection function for the 2PIGGs.

We can now assign to each galaxy a weight \( w'_j \), which accounts for both the local incompleteness and the luminosity and flux limits. This weight is defined as

\[
w'_j = \begin{cases} \frac{w_j}{\phi^{i}(x)} & \text{(if } b_j < M_{b,j,\text{com}}), \\ 0 & \text{(if } b_j > M_{b,j,\text{com}}) \end{cases}
\]

for a galaxy of spectral type \( i \) hosted by a group at distance \( x \), where \( M_{b,j} \) is the absolute magnitude of the galaxy in question. The galaxy membership of a group, complete to the absolute magnitude limit, may finally be obtained by summing these weights over the galaxies assigned to the group.

4 HALO OCCUPATION DISTRIBUTION

Having estimated the group membership as described in Section 3, we calculate the mean halo occupation numbers as a function of group mass (Figs 1 and 2). The error bars on these points are purely statistical: the uncertainty in the mass estimates is not directly accounted for.

(i) The exclusion of groups with fewer than four observed members is responsible for \( \langle N \mid M \rangle \) being systematically underestimated at low masses (\( M \lesssim 10^{14} h^{-1} M_{\odot} \)).

(ii) At higher masses, the measured amplitude is likely to be biased relative to the true value. This is due to the decrease in group abundance with increasing mass: of the groups scattered into a particular mass range, the majority

\[1\] Note that, after the imposition of the absolute magnitude limits, \( M_{b,j,\text{com}} \), our sample does in fact include groups with fewer than four members.
This high mass limit the power-law provides a good fit. We and the fits are overlaid on the results in Figs 1 and 2. For power-laws with semi-analytic models of Kauffmann et al. (1999) and find masses are in good agreement with previous work. For sub-populations are considered separately. Unfortunately, as Berlind & Weinberg (2002) predict, we are unable to reliably probe the halo occupation distribution for masses less than $10^{14} h^{-1} M_\odot$. In the interests of keeping our results as empirical as possible, we therefore adopt the simplest possible model for the mean occupation number: a single power-law with a cut-off at low mass. Thus,

$$\langle N|M \rangle = \begin{cases} (M/M_0)^\beta & (M \geq M_{\text{cut}}) \\ 0 & (M < M_{\text{cut}}) \end{cases} \quad (16)$$

where $M_0$ and $\beta$ are free parameters. Berlind & Weinberg (2002) suggest that $\beta$ may be reliably recovered, despite the dispersion in the mass, for $\beta \gtrsim 0.7$.

Using only the data for groups with $M > 10^{14} h^{-1} M_\odot$, we have fitted power-laws to the measured mean occupation numbers; the best-fitting parameters are given in Table 1 and the fits are overlaid on the results in Figs 1 and 2. In this high mass limit the power-law provides a good fit. We find $\beta > 0.7$ for all three samples, so these estimates of the high-mass slope ought not to be seriously affected by the scatter in the mass.

Our results for the power-law slope at high group masses are in good agreement with previous work. For example, Sheth & Diaferio (2001) make simple fits to the semi-analytic models of Kauffmann et al. (1999) and find power-laws with $\beta_{\text{red}} = 0.9$ and $\beta_{\text{blue}} = 0.8$ (but note that their definitions of ‘red’ and ‘blue’ differ from ours). Magliocchetti & Porciani (2003) use the observed 2dF correlation functions to constrain the HOD. They use the same cut on $\eta$ (see Section 4.4) to define red and blue populations, and find for their best-fitting models $\beta_{\text{red}} = 1.1^{+0.1}_{-0.2}$ and $\beta_{\text{blue}} = 0.7^{+0.2}_{-0.1}$ in the high-mass limit.
Table 2. Galaxy number densities (for galaxies brighter than the absolute magnitude limits derived in Section 5) computed from the luminosity functions of Madgwick et al. (2002), and the cut-off masses required to recover the observed number densities from equation 9. The quoted 1-sigma errors are estimated from the uncertainties in the parameters of the luminosity functions.

<table>
<thead>
<tr>
<th></th>
<th>( \bar{n}_g , (10^{-3} h^3 \text{Mpc}^{-3}) )</th>
<th>( M_{\text{cut}} , (h^{-1} \text{M}_\odot) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All galaxies</td>
<td>5.06 ± 0.49</td>
<td>0</td>
</tr>
<tr>
<td>Red</td>
<td>3.90 ± 0.35</td>
<td>0</td>
</tr>
<tr>
<td>Blue</td>
<td>4.80 ± 0.27</td>
<td>3.2 \times 10^4</td>
</tr>
</tbody>
</table>

4.2 Extrapolation to low mass

As we mention above, observational difficulties mean we are unable to recover the halo occupation distribution for haloes less massive than around \( 10^{14} h^{-1} \text{M}_\odot \). In particular, we cannot directly determine the minimum halo mass, \( M_{\text{cut}} \), which may host a galaxy. However, under the assumption that the HOD is well described by equation 16, the one remaining free parameter, \( M_{\text{cut}} \), can be constrained by requiring that we recover the correct average space density of galaxies.

The actual (observed) galaxy number density may be obtained directly from the luminosity function (integrated over the range of luminosity required); for this purpose we use the 2dF luminosity functions per type (regardless of environment) of Madgwick et al. (2002). \( M_{\text{cut}} \) can then be constrained by requiring that equation 9 agrees with this observed value. The number densities and implied values for \( M_{\text{cut}} \) for the different galaxy samples are given in Table 2.

The values we obtain for \( M_{\text{cut}} \) through this method are very low (in fact, for the combined and red samples it is not possible to match the observed number density even by allowing \( M_{\text{cut}} = 0 \)). This might be attributed in part to the underestimation of the amplitude of \( \langle N|M \rangle \) predicted by Berlind & Weinberg (2002), but is most likely due to the overly simplistic HOD we have adopted (equation 16). It is more usual to have \( \langle N|M \rangle \) tend asymptotically to unity at low mass (e.g. Kravtsov et al. 2003), significantly increasing the number of galaxies hosted by haloes in this mass range relative to the pure power-law we have adopted. We consider the impact of these issues on the halo model correlation functions in Section 6.2.

5 RADIAL DISTRIBUTION OF GALAXIES IN GROUPS.

We wish to obtain the group galaxy density profile given the group mass: \( \rho_{\text{gal}}(r|M) \). Unfortunately the virial motions of galaxies in groups give rise to redshift distortions (e.g. the so-called fingers-of-god), so that redshift information alone is not suitable for determining the three-dimensional galaxy distribution in groups. We can therefore only measure the surface density projected along the line-of-sight, \( \Sigma_{\text{gal}}(R) \) (where the upper-case \( R \) denotes projected distance from the group centre). Only if spherical symmetry is assumed can we unambiguously infer \( \rho_{\text{gal}}(r|M) \) from \( \Sigma_{\text{gal}}(R) \).
Eke et al. (2004a) systematically label one member of each 2PIGG as the central galaxy. For each group we calculate the projected angular separation from the central galaxy of the remaining group members. These separations are scaled by the virial radius of the hosting group to allow like-for-like stacking of groups over a range of masses. The total surface density of galaxies is then calculated by summing the weights $w'_j$ in radial bins. Summing the weights ensures that the profile is correctly normalized; although the actual value of the amplitude is of no consequence in the context of the halo model, it is interesting to compare the relative number densities of red and blue galaxies as a function of radius. We find that the shape of the profile is not altered if we use a simple number count instead of summing the weights, and further, that the shape of the profile is insensitive to the limiting absolute magnitude used. We therefore choose to use a considerably fainter limit, $M_{bJ, \text{cut}} - 5 \log_{10} h = -17.5$, than used to determine the group membership, so as to increase the number of galaxies admitted and hence improve the statistics.

The design of the 2dF spectrograph means that fibres cannot be positioned within $\sim 30''$ of one another, hence there is effectively an exclusion zone of this angular radius around the central galaxy in each group. Furthermore, such close-pair incompleteness also exists in the 2dF’s parent APM catalogue; van den Bosch et al. (2004) show that these have an equally important impact on the measured surface density at small separations. The severity of the problem is a decreasing function of the apparent angular size of a group. We have therefore imposed an additional selection on the sample used in this section, discarding all groups having a virial angular diameter less than 600''. Since fibre collisions occur on angular scales $\lesssim 30''$, this confines the affected region to projected separations from the group centre of $R \lesssim 0.05 r_{\text{vir}}$. This choice of cut-off optimizes the balance between the increased statistical noise resulting from using fewer groups, and the improved constraints on the profile shape achieved by probing nearer to the centre. The remaining sample comprises 1,619 groups containing a total of 16,749 galaxies.

The observed surface density profiles are shown in Fig. 4 for all galaxy types, and in Fig. 5 for blue and red galaxies separately. The decline in surface density due to close-pair incompleteness in the region $R \lesssim 0.05 r_{\text{vir}}$ is very apparent. A second feature of the observed profiles is the break at $R \approx r_{\text{vir}}$, beyond which $\Sigma_{\text{gal}}$ falls very rapidly with increasing $R$. This is expected, since the group-finding algorithm only admits galaxies which are deemed to be bound to a group (i.e. they lie within that group’s virial radius). Two effects cause the break to be smoothed somewhat: (i) the scatter in the mass estimates, which translates into a scatter in $r_{\text{vir}}$, and (ii) the fact that real groups are not perfectly spherical. Consequently, this fall in surface density has its onset slightly within $r_{\text{vir}}$.

5.1 Profile fitting

Obtaining an analytic fit to the observed profiles will greatly facilitate their use in the halo model. Simulations suggest that dark matter haloes obey a universal density profile of the form $\Sigma(\frac{r}{r_{\text{vir}}}) = \frac{M_{\text{tot}}}{4\pi r_{\text{vir}}^2} \times \left(1 + \frac{r}{r_{\text{vir}}}ight)^{-3}$ (Navarro, Frenk & White 1997).
where $r_0$ is a characteristic scale radius, and $\rho_s$ sets the amplitude of the profile. This NFW profile is often alternatively expressed in terms of the concentration parameter, $c = r_{\text{vir}} / r_s$. While there is no physical prerequisite that galaxies should follow the same profile as the dark matter, similar previous studies (e.g. Lin, Mohr & Stanford 2004) have found the NFW profile to provide a good fit to the radial distribution of galaxies.

The plane-projected surface density of the NFW profile is given by (e.g. Bartelmann 1996)

$$\Sigma(x) = \frac{2\rho_s r_s}{x^2 - 1} f(x)$$

with

$$f(x) = \begin{cases} 
1 - \frac{2}{\sqrt{x^2 - 1}} \arctan \sqrt{\frac{x - 1}{x + 1}} & (x > 1) \\
1 - \frac{2}{\sqrt{x^2 - 1}} \arctan \sqrt{\frac{1 - x}{1 + x}} & (x < 1) \\
0 & (x = 1)
\end{cases}$$

and $x = R/r_s = cR/r_{\text{vir}}$.

Nagai & Kravtsov (2004) discuss the problem of potential degeneracy between the concentration parameter and amplitude of the NFW profile, and propose a slightly alternative formulation:

$$\Sigma_{\text{gal}}(x) = \frac{c^2 N_{\text{vir}}}{2\pi g(c)(x^2 - 1)} f(x)$$

where $N_{\text{vir}} = \int_{r_{\text{vir}}}^{R_{\text{vir}}} g_{\text{gal}}(r) 4\pi r^2 dr$ is the number of galaxies within the virial radius of the group (in three dimensions) and $g(c) = \ln(1 + c) - c/(1 + c)$. Nagai & Kravtsov (2004) were interested in fitting to simulated data for which $N_{\text{vir}}$ could be trivially obtained, reducing the problem to a one-parameter fit. The peculiar velocity distortions in the 2dF redshifts make it impossible to determine $N_{\text{vir}}$ reliably for the 2PIGGs, hence we are forced to allow the amplitude of the profile to be a second free parameter. We prefer to use this form none the less, since $c$ is far less degenerate with $N_{\text{vir}}$ than it is with $\rho_s$, and we find $N_{\text{vir}}$ a more intuitive parametrization of the amplitude.

As discussed above, the reliability of the galaxy surface density measurements deteriorates towards the centre of the groups and at the fringes. We therefore restrict the profile fitting to points in the range $0.05 < R/r_{\text{vir}} < 0.75$. We find projected NFW profiles are capable of describing the observed profiles very well over the radial range fitted, both for the separate blue and red galaxy profiles and the combined galaxy profile. The best-fitting NFW profiles are overlaid on the data in Figs 3 and 4 and the concentration parameters are given in Table 3. Fig. 4 shows the joint probability distribution for the NFW concentration parameter and amplitude fitted to the data for all galaxies; we find there to be no strong degeneracy between the two parameters.

The best-fitting concentration parameters we give in Table 3 are broadly consistent with the analyses of galaxy clusters in the $K$-band of the 2 Micron All-Sky Survey by Lin et al. (2004) who obtain $c = 2.96^{+0.21}_{-0.24}$, and of 14 CNOC clusters by Carlborg et al. (1997) and van der Marel et al. (2000) who find $c = 3.7$ and 4.2 respectively. Hansen et al. (2004) find $c = 1–3$ for photometrically identified clusters in the SDSS. We caution that the close-pair incompleteness effects described above mean that we cannot reliably constrain the radial distribution for $R < 0.05r_{\text{vir}}$, so it is impossible, for example, to determine whether the profile is cuspy or flat-cored at its centre. If we are still affected by close-pair incompleteness despite the angular diameter selection we have used, we would expect the concentration parameters we have obtained to be underestimated (see also van den Bosch et al. 2004). Furthermore, there is likely to be some discrepancy between the position of the ‘central’ galaxy identified by Eke et al. (2004a) and the true group centre. This will cause a flattening of the profile at small scales, and hence an underestimate of the NFW concentration parameter.

### Table 3. Best-fitting NFW concentration parameters. Errors are 1-sigma, marginalized over the probability distribution of the other fitted parameter. $N_{\text{vir}}$ represents the average number of galaxies within $r_{\text{vir}}$ per group (above the minimum absolute magnitude $M_{b, \text{com}} - 5\log h = -17.5$).

<table>
<thead>
<tr>
<th>$N_{\text{vir}}$</th>
<th>$c = r_{\text{vir}}/r_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All galaxies</td>
<td>34.7 ± 0.7</td>
</tr>
<tr>
<td>Red</td>
<td>19.5 ± 0.6</td>
</tr>
<tr>
<td>Blue</td>
<td>14.9 ± 0.5</td>
</tr>
</tbody>
</table>

![Figure 6. Contours of $\chi^2$ for the concentration parameter and amplitude of the NFW profile fitted to the projected galaxy density profile for all galaxies. Marginalized percentage confidence levels are indicated.](image)

5.2 Mass dependence

The mean concentration parameter of dark matter halos in simulations is found to be a decreasing function of mass, typically modelled as a power law, $c = c_0 (M/M_*)^{-\lambda}$ with $\lambda \sim 0.1$ and $c_0 \sim 10$ (e.g. Bullock et al. 2001).

Fig. 4 shows the measured concentration parameter
Figure 7. Variation of group concentration parameter with mass, for red (squares), blue (circles), and all galaxies (triangles). For comparison, the dashed line shows the trend in the mean concentration parameter of dark matter haloes in simulations (due to Bullock et al. 2001).

against group mass for red and blue galaxies separately, and for the combined sample. These values were obtained by stacking groups within mass bins, and fitting NFW profiles as above. The results suggest a detection of a slight decrease in $c$ with mass, but the trend is barely significant. When calculating halo model-based correlation functions in the following section, we therefore choose to incorporate no mass-dependence in the radial galaxy distribution; we confirm the safety of this assumption below in Section 6.2.

The comparison with the results of Bullock et al. 2001 reveals that, even for the red galaxy distribution, we find the galaxy radial distributions to be significantly less concentrated than those of average dark matter haloes in simulations. Again, we caution that the uncertainty in the position of the group centre and any residual impact of close-pair incompleteness would be expected to depress the profile on small scales, leading to an underestimate of the NFW concentration parameter. The suggestion of a negative correlation of concentration with group mass is in qualitative agreement with the results of Hansen et al. (2004), who find $c$ to be a decreasing function of galaxy occupation number.

6 PREDICTING CORRELATION FUNCTIONS

Armed with the observations of the previous sections, we are now equipped to calculate the halo model predictions for the correlation function. Hawkins et al. (2003) and Madgwick et al. (2003) have measured the two-point correlation functions for the full 2dFGRS sample and for red and blue subsamples. Care must be taken to ensure a fair, like-for-like comparison, since the correlation function is known to depend on redshift and luminosity (e.g. Zheng et al. 2002; Norberg et al. 2002). For this reason, the observed correlation functions are labelled by their effective luminosity and effective redshift. These effective quantities are pair-weighted measures, since the correlation function is based on counting pairs of galaxies; hence, for example, the effective redshifts $z_s = 0.15$ for the all-galaxy sample, and $z_s = 0.11$ for both the red and blue galaxy samples – are somewhat higher than the median redshifts of the samples. We use these redshifts when calculating the halo model correlation functions for the respective populations via the formalism of Section 4.2.

In the case of the luminosity, the situation is complicated by the fact that the 2dF samples are flux-limited, whereas the halo model formalism effectively assumes volume-limited. For a volume-limited sample the effective luminosity is identical to the mean luminosity. The minimum luminosity (required by equation 14) is therefore determined by solving

$$L_s = \int_{L_{\text{min}}}^{\infty} L \phi(L) \, dL$$

for $L_{\text{min}}$. The effective luminosities of the samples used by Hawkins et al. (2003) and Madgwick et al. (2003) are, using $M^* - 5 \log_{10} h = -19.66$: all galaxies, $L_s = 1.4L_*$; red, $L_s = 1.26L_*$; blue, $L_s = 0.95L_*$. The resulting limiting absolute magnitudes are $M_{bcom} - 5 \log_{10} h = -19.25$, $-19.04$, $-19.50$ for red, blue and all galaxies respectively. These limits have been used in determining the halo occupation distribution of the 2PGG groups (Section 4.1) and the number densities of the galaxy populations (Table 2).

In order to negate the effect of redshift distortions in the observed correlation functions we use the radial projection, 

$$\Xi(\sigma) = \frac{2}{\sigma} \int_0^\infty \xi(r) \frac{r \, dr}{(r^2 - \sigma^2)^{1/2}}$$

In Figs 8 and 10 we compare the halo model predictions with the observed projected correlation functions.

In producing these halo model correlation functions we have simply used the best-fitting halo profiles and occupation distributions, deliberately making no attempt to fit to the observed correlation function data. We find that the model successfully recovers the approximate shape and amplitude of the observed correlation functions across the range of distance scales considered. In particular, the model correlation functions trace the obvious divergence from the power-law form at large scales. The different observed HODs and halo profiles of red and blue galaxies derived in Sections 4.1 and 4.2 are seen to give rise to the expected relative strengths of correlations on small scales, although the halo model predictions do show significant deviations from the power-law form on these scales.

6.1 Galaxy bias on large scales

On large scales the differences in the correlation functions of the different galaxy classes are encapsulated in the galaxy bias parameter (equation 10). To allow comparison with other results, we correct the galaxy bias parameter for redshift and luminosity dependence via

$$b_{gal}(L, 0) = b_{gal}(L, z_s)D(z_s)$$

(24)
Table 4. Galaxy bias parameters predicted by the halo model. Note that the effective luminosities and redshifts differ between the three galaxy samples.

<table>
<thead>
<tr>
<th></th>
<th>( b_{\text{gal}}(L_\ast, z_\ast) )</th>
<th>( b_{\text{gal}}(L_\ast, 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All galaxies</td>
<td>1.05</td>
<td>0.92</td>
</tr>
<tr>
<td>Red</td>
<td>1.23</td>
<td>1.12</td>
</tr>
<tr>
<td>Blue</td>
<td>0.85</td>
<td>0.81</td>
</tr>
</tbody>
</table>

and

\[
b_{\text{gal}}(L, 0)/b_{\text{gal}}(L_\ast, 0) = 0.85 + 0.15(L/L_\ast).
\]

The first relation follows from the assumption that galaxy clustering evolves weakly over this redshift range (i.e. \( \sigma_{8,\text{gal}} \) remains approximately constant). The second is found from correlation function analysis by Norberg et al. [2001]. The predicted galaxy bias factors are in Table 4 [Lahav et al. 2002] have measured the galaxy bias factor by combining the 2dFGRS galaxy power spectrum with the post-WMAP CMB data and found

\[
b_{\text{gal}}(L_\ast, 0) \approx 0.96 \pm 0.08,
\]

in good agreement with our predicted value for all galaxies: \( b_{\text{gal}}(L_\ast, 0) = 0.92 \).
6.2 Sensitivity to parameters

The halo bias factor, \( b(M) \), \( (\text{equation } 18) \) is a monotonically increasing function of mass, with the lowest mass haloes in fact being anti-biased. The galaxy bias factor predicted by the halo model \( (\text{equation } 11) \) will therefore be controlled by the relative occupation of high to low mass groups; thus, since the red galaxies have a steeper \( \langle N|M \rangle \) than the blue, they also have a greater large-scale bias factor.

The differences on small scales are, in principle, due to the different radial profiles as well as the relative halo occupation numbers of the galaxy classes. However, it turns out that on the scales we are currently able to access observationally (and given the observational results of the previous sections) the correlation function is much more sensitive to the differences in the halo occupation distribution \( (\text{see also } \text{Scranton } 2003, \text{Berlind } \& \text{ Weingberg } 2002) \). For example, substituting the dark matter concentration parameter relation \( (\text{Bullock et al. } 2001, \text{see Section } 5.2) \) in place of \( \rho(r) \) from galaxy pairs as a function of halo mass: in general lower mass haloes become more centrally concentrated towards smaller scales. By using a more sophisticated extrapolation to low masses it is possible to produce a halo model correlation function which more faithfully reproduces the power-law form of the galaxy correlation functions. Most impressively, the model also successfully recovers the relative occupation of high to low mass groups: thus, the halo model proves successful at reproducing the near power-law form of the halo occupation distribution at low halo masses, directly determined. Despite the uncertainty regarding the form of the halo occupation distribution at low halo masses, the halo model proves successful at reproducing the near power-law form of the galaxy correlation functions. Most impressively, the model also successfully recovers the relative biasing of the red and blue subpopulations. We identify the halo occupation distribution as the dominant factor contributing to the differences in the correlation functions of red and blue galaxies: the HOD for red galaxies gives a greater weighting to high-mass haloes leading to a stronger clustering signal on all scales. The differences in the radial distributions of the galaxies in groups are found to be rela-

![Figure 11](https://example.com/fig11.png)

**Figure 11.** As Fig. 8, but the halo model predictions are based on the more sophisticated HOD described by equations 27 and 28. This form of the HOD better reproduces the power-law on small scales, while still being consistent with the observational constraints of Section 4.

and for the combined sample. The radial distribution of the red galaxies was found to be the most concentrated towards the group centre, but, even for these, the concentration is significantly lower than for dark matter haloes measured in simulations.

We have shown that at high masses the mean occupation number of the 2PIGGs with red, blue, and all galaxies can be described by a power-law in the mass, with the number of red galaxies increasing most steeply with the mass. The difficulty of obtaining reliable dynamical mass estimates for groups with low occupation numbers made it difficult to constrain the halo occupation distribution in this regime.

These observations have allowed us to test the halo model against fully self-consistent observations: the same 2dFGRS galaxy populations used to compile the 2PIGG catalogue have previously had their correlation functions directly determined. Despite the uncertainty regarding the form of the halo occupation distribution at low halo masses, the halo model proves successful at reproducing the near power-law form of the galaxy correlation functions. Most impressively, the model also successfully recovers the relative biasing of the red and blue subpopulations. We identify the halo occupation distribution as the dominant factor contributing to the differences in the correlation functions of red and blue galaxies: the HOD for red galaxies gives a greater weighting to high-mass haloes leading to a stronger clustering signal on all scales. The differences in the radial distributions of the galaxies in groups are found to be rela-

7 DISCUSSION

Motivated by the halo model, we have used the 2dF Percolation-Inferred Galaxy Group catalogue to make direct observational estimates of the distribution functions of galaxies within and amongst haloes. We have found a projected NFW profile provides a good fit to the composite projected surface density of galaxies in red and blue subclasses,
tively unimportant on the distance scales we are presently able to access observationally, but do begin to exert influence at the smallest scales considered.

There are of course any number of forms which could be chosen for the extrapolation of the HOD to low mass (see for example, Kravtsov et al. 2003; Sheth & Diaferio 2001; Scranton 2003; Magliocchetti & Porciani 2003). We have shown that the power-law form of the observed correlation functions does place constraints on this extrapolation. However, in the intra-halo term there is certainly degeneracy between $\alpha(M)$ and $\langle N | M \rangle$, so it is unlikely that the two-point correlation can be used to constrain a unique solution. Direct measurement of the HOD for low-mass haloes requires much improved mass estimates, and represents a difficult observational challenge. Higher order clustering statistics offer the possibility of indirectly constraining the low-mass HOD in a more model-independent fashion than is possible from the two-point function alone (e.g. Ma & Fry 2003; Scoccimarro et al. 2001; Wang et al. 2004).

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