How large could the R-parity violating couplings be?

Pavel Fileviez Pérez
Pontificia Universidad Católica de Chile
Facultad de Física, Casilla 306
Santiago 22, Chile.

We investigate in detail the predictions coming from the $d=4$ operators for proton decay.
We find the most general constraints for the $R$-parity violating couplings coming from proton
decay, taking into account all fermion mixing and in different supersymmetric scenarios.

I. INTRODUCTION

The minimal supersymmetric extension of the standard model (MSSM) is one of the most
popular candidates for the physics beyond the standard model. It is well known that in general it
is possible to write down in the superpotential terms which violate the Baryon($B$) or Lepton($L$)
number, giving us the possibility to describe the neutrino mass through those interactions. For
several phenomenological aspects of $R$ parity violating interactions see reference [1].

Due to the presence of the $B$ violating interactions in the MSSM superpotential it is very
important to understand the constraints coming from the proton decay searches. The decay of the
proton, which was predicted long ago by Pati and Salam [2], is one the most important constraints
for physics beyond the standard model, particularly for grand unified theories [3].

It is well known that in supersymmetric scenarios the $d=4$ and $d=5$ contributions to the
decay of the proton are the most important, while in non-supersymmetric scenarios the gauge $d=6$
operators are the dominant. Assuming that the so-called $R$-parity is an exact symmetry of our
model the dangerous $d=4$ operators are forbidden and there is the possibility to describe the cold
dark matter in the Universe, since the lightest supersymmetric particle (Neutralinos) will be stable.
Those are the main reasons to impose the $R$ parity in the context of the minimal supersymmetric
standard model. See also reference [4] for the possibility to have $R$-parity as an exact symmetry
coming from grand unified theories.

In general, it is very difficult to satisfy the proton decay experimental bounds in the context
of minimal models. For example to know about the status of the minimal supersymmetric $SU(5)$
model see reference [5]. However, in realistic models there is more freedom, and it is still possible
to satisfy all bounds. New experimental bounds [6] are welcome, in order to have better constraints
and if proton decay is found we will able to test the idea of grand unified theories. We understand
all contributions to the decay of the proton, however it is very difficult to know how realize the test of a given model. Recently, it has been pointed out the possibility to make a clear test of grand unified theories with symmetric Yukawa couplings through the proton decay into antineutrinos \[7\]. In the context of the minimal renormalizable supersymmetric flipped SU(5) model it has been pointed out the possibility to use also the ratio between the decays into charged leptons in order to make a clear test \[8\]. At the same time there is a very interesting possibility to rotate proton decay away in the context of flipped SU(5) models \[9\]. A second crucial issue about proton decay is the possibility to find an upper bound on the total proton lifetime, since in this case we will hope that proton decay will be found. Recently, it has been found an very conservative upper bound on the lifetime of the proton \[10\], therefore there is hope to test all those ideas if the decay of the proton is found.

The constraints for the R-parity violating couplings coming from proton decay in low energy supersymmetry have been studied long ago \[11\]. However, up to now it has not been found the most general constraints. In this Letter our main task is study the predictions coming from the $d = 4$ operators in the context of the MSSM. In order to find the most general constraints for the relevant couplings we take into account all flavour mixing, and compute the amplitudes for the different channels using the Chiral Lagrangian Techniques. We compare the constraints for the R-parity violating couplings in the context of the minimal supersymmetric standard model in two scenarios, in low energy SUSY (See for example \[12\]) and in SPLIT supersymmetry \[13\], a new interesting scenario where all scalars, except for one Higgs, are very heavy and the fermions are light. In this scenario it is possible to achieve the unification of gauge couplings and we have a natural candidate for the cold dark matter in the Universe if the R-parity is imposed (See also \[14\] for a possibility to have the Neutralino as cold dark matter candidate even if the R-parity is broken).

II. R-PARITY VIOLATION AND THE DECAY OF THE PROTON

In the minimal supersymmetric standard model we are allowed to write the following terms in the superpotential which violate the so-called R parity:

\[
W_{NR} = \alpha_{ijk} \tilde{Q}_i \tilde{L}_j \tilde{D}^C_k + \beta_{ijk} \tilde{U}_i \tilde{C}^C_j \tilde{D}^C_k + \gamma_{ijk} \tilde{L}_i \tilde{\tilde{L}}_j \tilde{E}^C_k + a_i \tilde{L}_i \tilde{H}
\]  

(1)
In the above equation we use the usual notation for all MSSM superfields (See for example [12]).

The $R$-parity is defined as $R = (-1)^{2S}M$, where $S$ is the spin and $M = (-1)^{3(B-L)}$ is the Matter parity, which is $-1$ for all matter superfields and $+1$ for Higgs and Gauge superfields. The coefficient $\beta_{ijk} = -\beta_{ikj}$ and $\gamma_{ijk} = -\gamma_{jik}$. Notice that in the above equation the second term violates the Baryon number, while the rest of the interactions violate the Leptonic number. It is well known that from the first and second terms we can write at tree level the contributions to proton decay mediated by the $\tilde{d}^C_k$ squarks. These are the most important contributions, and we will use those to understand the constraints. Now let us write the decay rate of the proton using the $R$-parity violating interactions. Using the Chiral Lagrangians and writing all interactions in the physical basis the decay rates of decays of the proton into charged leptons are given by:

$$\Gamma(p \rightarrow \pi^0 e^-_\beta) = \frac{m_p}{64\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 |c(e^+_{\beta}, d^C)|^2$$

$$\Gamma(p \rightarrow K^0 e^-_\beta) = \frac{m_p^2}{32\pi f_\pi^2 m_B} A_L^2 |\alpha|^2 [1 + \frac{m_p}{m_B}(D - F)]^2 |c(e^+_{\beta}, s^C)|^2$$

with:

$$c(e^+_{\beta}, d^C) = \sum_{m=1}^{3} \frac{(\Lambda_3^{\alpha m})*\Lambda_3^{\beta m}}{m_{d_m}^2}$$

where $\alpha, \beta = 1, 2$. $D$ and $F$ are the parameters of the chiral lagrangian, $\alpha$ is the matrix element, and $A_L$ takes into account the renormalization effects from $M_Z$ to 1 GeV. In the case of the decay channels into antineutrinos, the decay rates read as:

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \frac{(m_p^2 - m_K^2)^2}{32\pi f_\pi^2 m_B^2} A_L^2 |\alpha|^2$$

$$\times \sum_{i=1}^{3} \left| \frac{2m_p}{3m_B} D c(\nu_i, d, s^C) + [1 + \frac{m_p}{3m_B}(D + 3F)] c(\nu_i, s, d^C) \right|^2$$

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) = \frac{m_p}{32\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \sum_{i=1}^{3} |c(\nu_i, d, d^C)|^2$$

where:

$$c(\nu_i, d, d^C) = \sum_{m=1}^{3} \frac{(\Lambda_3^{\beta m})*\Lambda_2^{\alpha m}}{m_{d_m}^2}$$

In the above equations the couplings $\Lambda_1$, $\Lambda_2$ and $\Lambda_3$ are given by:

$$\Lambda_1^{\alpha m} = \alpha_{ijk} U_{ij}^{\alpha} E_{j\alpha}^{i} \tilde{D}_{C}^{km}$$

$$\Lambda_2^{\alpha m} = \alpha_{ijk} D_{ij\alpha} N_{jl}^{\alpha} \tilde{D}_{C}^{km}$$

$$\Lambda_3^{\alpha m} = 2\beta_{ijk} U_{ik}^{\alpha} D_{ij}^{\alpha} \tilde{D}_{C}^{km}$$
The mixing matrices in the above expression diagonalize the Yukawa matrices in the following way:

\[
Y^{\text{diag}}_U = U_C^T Y_U U \quad (11)
\]
\[
Y^{\text{diag}}_D = D_C^T Y_D D \quad (12)
\]
\[
Y^{\text{diag}}_E = E_C^T Y_E E \quad (13)
\]
\[
Y^{\text{diag}}_N = N_C^T Y_N N \quad \text{(in the Majorana case)} \quad (14)
\]
\[
Y^{\text{diag}}_N = N_C^T Y_N N \quad \text{(in the Dirac case)} \quad (15)
\]

Now, in order to get the constraints for the \( R \)-parity violating couplings we have to use the decay channels, which could give us the information that we are looking for. From the different equations listed above for the decay rates, it is easy to see that we have to use the decays into charged leptons and mesons, since we cannot get new information from the rest of the channels. In our opinion, it is the best way to get the constraints. Using \( m_p = 938.3 \text{ MeV}, \ D = 0.81, \ F = 0.44, \ MB = 1150 \text{ MeV}, \ f_\pi = 139 \text{ MeV}, \ \alpha = 0.003 GeV^3, \ AL = 1.43 \) and the experimental constraints [16] we get:

\[
|c(e^+, d^C)| < 7.6 \times 10^{-31} \quad (16)
\]
\[
|c(\mu^+, d^C)| < 1.4 \times 10^{-30} \quad (17)
\]
\[
|c(e^+, s^C)| < 4.2 \times 10^{-30} \quad (18)
\]
\[
|c(\mu^+, s^C)| < 4.7 \times 10^{-30} \quad (19)
\]

Now, assuming that all squarks have the same mass \( \tilde{m} \), the quantity \( (\lambda_3^m)^* \lambda_1^{\beta m} \) have to satisfy the following relations:

\[
|\lambda_1^{1m}| \times \lambda_1^{1m} | < 3.8 \times 10^{-31} \tilde{m}^2 \quad (20)
\]
\[
|\lambda_1^{2m}| \times \lambda_1^{2m} | < 7.0 \times 10^{-31} \tilde{m}^2 \quad (21)
\]
\[
|\lambda_1^{1m}| \times \lambda_1^{1m} | < 2.1 \times 10^{-30} \tilde{m}^2 \quad (22)
\]
\[
|\lambda_1^{2m}| \times \lambda_1^{2m} | < 2.3 \times 10^{-30} \tilde{m}^2 \quad (23)
\]

where:

\[
(\lambda_3^m)^* \lambda_1^{\beta m} = \beta_{ijk}^{*} \alpha_{lpk} (U_C^*)^* (D_C^{j\alpha})^* U_1^T E^{p\beta}
\]

As you can appreciate the constraints for \( \alpha_{ijk} \) and \( \beta_{ijk} \) are quite model dependent i.e., depend on the model for fermion mass that we choose. Notice that we can choose for example the basis where the charged leptons and down quarks are diagonal, however still \( U_C \) will remain, and \( U = K_1 V_{CKM} K_2 \)
TABLE I: Upper bounds for the R-parity violating couplings

<table>
<thead>
<tr>
<th>Couplings</th>
<th>Low energy SUSY</th>
<th>SPLIT SUSY ($\tilde{m} = 10^{14}$ GeV)</th>
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(It is well known that the proton decay predictions depend on the fermion mass model). $K_1$ and $K_2$ are diagonal matrices containing three and two CP-violating phases, respectively. Now, let us see the constraints for different values of the scalar mass. In Table I we show the different constraints in two supersymmetric scenarios, in low energy supersymmetry $\tilde{m} = 10^3$ GeV and in split supersymmetry for the case $\tilde{m} = 10^{14}$ GeV, respectively. Notice that the value $\tilde{m} = 10^{14}$ GeV is basically the upper bound for the scalar masses coming from the gluino decay [13].

Therefore as we can appreciate from these results the $R$-parity violating couplings could be large in supersymmetric scenarios with large susy breaking scale (Split Supersymmetry). In the case of low energy SUSY, since those couplings are quite small we usually believe that it is a hint to believe that the $R$-parity has to be a symmetry of our theory. However, in general this question is open. Now, What are the phenomenological implications coming from those constraints?

Those values are particularly interesting in order to study the implications in collider physics and cosmology. For example in the description of neutrino masses, the three body decays of neutralinos and gluinos. As we know one the motivations of the supersymmetric scenarios with large scalar masses is the possibility to describe the cold dark matter in the Universe with neutralinos, since there are light and stable if $R$-parity is impose. Now, here we study the implication of $R$-parity violation for proton decay in those scenarios. It has been shown in reference [14] that if $R$-parity is broken in SPLIT SUSY scenarios still the neutralinos could be a good candidate for dark matter. The reason is the following, if we neglect the last term in equation (1) the neutralinos can decay only into three particles. In these scenarios the three bodies decays are suppressed by the large scalar masses, then they can have a lifetime of the order of the age of the Universe. Notice that in their analysis they use large values for the $R$-parity violating couplings which are in agreement with the constraints coming from proton decay obtained by us.

It is very interesting to know how could be the constraints for those couplings in the context of grand unified theories. In the simplest unified theory, the minimal SUSY $SU(5)$, we can get the $R$-parity violating interactions present in the MSSM from the terms, $\Lambda^{ijk} \ 10_i \ \bar{5}_j \ \bar{5}_k$, $b_i \ \bar{5}_i \ \bar{5}_H$ and $c_i \ \bar{5}_i \ 24_H \ \bar{5}_H$. 


In this case at the GUT scale the couplings satisfy the relations \( \frac{\alpha_{ijk}}{2} = \beta_{ijk} = \gamma_{ijk} = \Lambda_{ijk} = -\Lambda_{ikj} \).

Taking into account these relations we can reduce the number of free parameters, and we could have better constraints. However, in our paper we only try to understand the constraints in the context of the minimal supersymmetric standard model. Notice that in grand unified theories with split supersymmetry the most important contributions for the decay of the proton are the gauge \( d = 6 \) contributions if the R-parity is conserved. Since here we are working in the context of the MSSM, we assumed that the \( d = 6 \) non-renormalizable operators are not present.

As you can appreciate, we have found the most general constraints for the \( R \)-parity violating couplings coming from proton decay, taking into account all important effects. We hope that those results will be useful to study all phenomenological and cosmological implications of those interactions.

### III. SUMMARY

We have investigated the constraints for the \( R \)-parity violating couplings coming from proton decay in different supersymmetric scenarios. Taking into account all fermion mixing, using the Chiral Lagrangian Techniques and imposing the experimental bounds it has been shown how large those couplings could be in the case of split supersymmetry. We hope that these results will useful to discover SPLIT SUSY at future experiments.

### Acknowledgments

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[12] For a review see: H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75.


