Exact Rolling Tachyon
in Noncommutative Field Theory

Yoonbai Kim and O-Kab Kwon

BK21 Physics Research Division and Institute of Basic Science,
Sungkyunkwan University, Suwon 440-746, Korea
yoonbai@skku.edu  okab@skku.edu

Abstract

We study the exact rolling tachyon solutions in DBI type noncommutative field theory with a constant open string metric and noncommutative parameter on an unstable Dp-brane. Functional shapes of the obtained solutions span all possible homogeneous rolling tachyon configurations; that is, they are hyperbolic-cosine, hyperbolic-sine, and exponential under $1/\cosh$ runaway NC tachyon potential. Even if general DBI type NC electric field is turned on, only a constant electric field satisfies the equations of motion, and again, exact rolling tachyon solutions are obtained.
1 Introduction

Rolling tachyon was constructed as classical time dependent solutions in open string theory, describing real-time dynamics of tachyonic degree on unstable D-branes [1, 2, 3]. When NSNS two-form field and the gauge field exist on the D-brane, two descriptions of effective field theory (EFT) can be applied: One is the usual EFT coupled to Dirac-Born-Infeld (DBI) electromagnetism and the other is noncommutative (NC) field theory (NCFT) [4].

When a constant electric component of the NSNS two-form field or an equivalently constant electric field is turned on, various approaches in terms of boundary conformal field theory (BCFT) [5], EFT [5, 6], and NCFT [5] are employed to obtain rolling tachyon solutions which are compared at the level of energy-momentum tensor mostly for large elapsed time. At this new vacuum, tachyon matter [1] does not contribute to pressure but fundamental strings (F1s) in the fluid state do [5, 6]. For any runaway NC tachyon potentials, three species of rolling tachyon solutions can be found, which are identified as two full S-brane solutions and one half S-brane solution [8, 9, 10].

When both electric and magnetic components of the NSNS two-form field are taken into account, the same three types of rolling tachyon solutions are obtained in BCFT [11] and EFT [12] irrespective of various constant electromagnetic field configurations and specific shapes of tachyon potentials. In the EFT of DBI action with 1/cosh type tachyon potential, homogeneous rolling tachyon solutions are given as exact solutions despite of nonlocality and many field strength components for arbitrary Dp-brane [13, 14].

NC electromagnetic field and NC tachyon with its condensation have been an attractive subject since initial investigations on NCFT [15]. Studies on exact tachyon kinks have been performed recently [16, 17], but exact homogeneous rolling tachyon solutions have yet to be reported in the context of NCFT. In this paper we consider an NC DBI action and 1/cosh type NC tachyon potential with constant open string metric and NC parameter, and investigate exact homogeneous NC rolling tachyon solutions. We attempt to find such solutions for cases with or without coupling of the NC electromagnetic field, and they had one-to-one correspondence with those in EFT. For their energy-momentum tensor, two full S-brane solutions are in agreement qualitatively to but half S-brane solution coincides exactly with that of BCFT.

The rest of this paper is organized as follows. In section 2, the most general homogeneous solution of NC rolling tachyon with open string metric and NC parameter is derived from a constant NSNS two-form field. In section 3, DBI NC electromagnetic field is added and we also find homogeneous NC rolling tachyon solution for the case with the
NC electric field. In section 4, results and conclusions are presented.

2 NC Rolling Tachyon

In this section, we study rolling tachyon in NCFT, describing dynamics of an unstable Dp-brane coupled to both constant DBI type electromagnetic field and constant antisymmetric NSNS two-form field. All possible homogeneous NC rolling tachyons are found as exact solutions. In the context of EFT, rolling tachyons with DBI type electromagnetism [5, 6] is described by the action

\[ S = -T_p \int d^p x \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \]  

(2.1)

and, for specific form of tachyon potential [18, 13, 19],

\[ V(T) = \frac{1}{\cosh \left( \frac{T}{R} \right)}. \]  

(2.2)

All possible homogeneous rolling tachyons are obtained as exact solutions [13, 12]. Specifically, for an arbitrary \( p \), every component of the field strength tensor \( F_{\mu\nu} \) satisfying the gauge equation and Bianchi identity should be constant [14], and then the tachyon profiles, \( T(t) \), are obtained by solving a first-order equation given by the definition of the constant Hamiltonian density \( \mathcal{H} \)

\[ \mathcal{H} = \frac{\alpha_{p0} T_p V(T)}{\sqrt{\beta_p - \alpha_{p0} \dot{T}^2}}, \]  

(2.3)

where \( \beta_p = -\det(\eta_{\mu\nu} + F_{\mu\nu}) \) and \( \alpha_{p0} > 0 \) is a 00-component of the cofactor \( C^{\mu\nu} \) of matrix \( \eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T \). For \( \beta_p - \alpha_{p0} \dot{T}^2 = 0 \) and \( \beta_p > 0 \) in order to obtain nontrivial rolling tachyon solutions. This condition leads to critical value of the electromagnetic field. Homogeneous rolling tachyon solutions exist for positive \( \beta_p \) and, by value of the Hamiltonian density \( \mathcal{H} \), solutions of Eq. (2.3) are classified into three forms,

\[ \sinh \left( \frac{T(t)}{R} \right) = \begin{cases} \sqrt{u^2 - 1} \cosh \left( \frac{t}{\zeta} \right) & \text{for } \mathcal{H} < \sqrt{\frac{\alpha_{p0} T_p}{\beta_p}}, \\ \exp \left( \frac{t}{\zeta} \right) & \text{for } \mathcal{H} = \sqrt{\frac{\alpha_{p0} T_p}{\beta_p}}, \\ \sqrt{1 - u^2} \sinh \left( \frac{t}{\zeta} \right) & \text{for } \mathcal{H} > \sqrt{\frac{\alpha_{p0} T_p}{\beta_p}}, \end{cases} \]  

(2.4)

where \( u = \alpha_{p0} T_p / (\beta_p \mathcal{H}) \) and \( \zeta = \sqrt{\alpha_{p0} / \beta_p} R \).

Since the electromagnetic field \( F_{\mu\nu} \) of interest is constant on the D-brane and gauge invariant configuration on this D-brane is given by the sum of the electromagnetic field
and antisymmetric NSNS two-form field, $B_{\mu\nu} + F_{\mu\nu}$, one can regard the constant electro-

magnetic field as a part of constant NSNS two-form field on the D-brane. When we have

an open string theory with flat closed string metric $\eta_{\mu\nu}$ and constant background two-form field $B_{\mu\nu} = F_{\mu\nu}$, a BCFT calculation of the propagator on a disc, which corresponds to a point splitting regularization of string theory, provides an open string metric $G^{\mu\nu}$ and NC parameter $\theta^{\mu\nu}$ in terms of the closed string variables as

$$
G_{\mu\nu} = g_{\mu\nu} - (Bg^{-1}B)_{\mu\nu}, \\
\theta^{\mu\nu} = -\left(\frac{1}{g + B} - \frac{1}{g - B}\right)^{\mu\nu}.
$$

According to the result of Ref. [4], the DBI action (2.1) without the tachyon is equivalent
to NC DBI action $S_\Theta$:

$$
S(g_{\mu\nu}, B_{\mu\nu}; T = F_{\mu\nu} = 0) = \sqrt{-\det(g_{\mu\nu} + B_{\mu\nu})} \cdot S_\Theta(G_{\mu\nu}, \theta^{\mu\nu}; \hat{T} = \hat{F}_{\mu\nu} = 0),
$$

where every product in $S_\Theta$ is replaced by star product (*)

$$
f(x) * g(x) \equiv e^{\frac{i}{2} \theta^{\mu\nu} \delta_{\mu\nu} \delta_{\xi\eta}} f(x + \xi)g(x + \zeta)|_{\xi=\zeta=0}.
$$

Here, $\hat{T}(x)$ is the NC tachyon and $\hat{F}_{\mu\nu}(x)$ is (usually slowly varying) the NC electromagnetic field strength tensor defined by

$$
\hat{F}_{\mu\nu} = \partial_{\mu}\hat{A}_{\nu} - \partial_{\nu}\hat{A}_{\mu} - i\hat{A}_{\mu} \ast \hat{A}_{\nu} + i\hat{A}_{\nu} \ast \hat{A}_{\mu}.
$$

When the NC tachyon with arbitrary spacetime dependence is taken into account, the NC tachyon $\hat{T}$ is related to the tachyon $T$ by a field redefinition, and the equivalence between the DBI type EFT action and the DBI type NC action in Eq. (2.7) is not proved, yet. For the homogeneous rolling tachyon solution depending only on the time coordinate, every star product in $S_\Theta$ is replaced by an ordinary product as

$$
\hat{T}(t) * \hat{T}(t) = e^{\frac{i}{2} \theta^{\mu\nu} \delta_{\mu\nu} \delta_{\xi\eta}} \hat{T}(t + \xi)\hat{T}(t + \zeta)|_{\xi=\zeta=0} = \hat{T}^2(t),
$$

and the field redefinition between $\hat{T}$ and $T$ is identity by the result of Ref. [5]

$$
\hat{T}(t) = T(t).
$$

When $\hat{F}_{\mu\nu} = 0$, a resultant NC version of the action (2.1) is

$$
\hat{S} = -\frac{\hat{T}_0}{2} \int d^{p+1}x \left[ \hat{V}(\hat{T}) * \sqrt{-\det_{\ast}(G_{\mu\nu} + \partial_{\mu}\hat{T} \ast \partial_{\nu}\hat{T}) + (\sqrt{\ast} \hat{V})} \right],
$$

4
where \( \dot{T}_p = T_p/\sqrt{\det(1 + g^{-1}B)} \). In fact, the NC action (2.12) is equivalent to the effective action \( S \) up to leading \( \theta^{\mu\nu} \) order \([10]\). Because \( \tilde{V}(\dot{T}) \Rightarrow V(\dot{T}) \) (see Eq. (2.24)), \( \det \Rightarrow \det \), and \( \partial_\mu \dot{T} \partial_\nu \dot{T} = \partial_\mu \dot{T} \partial_\nu \dot{T} = \delta_{\mu0} \delta_{\nu0} \dot{T}^2 \) from Eqs. (2.10)–(2.11), our action (2.12) with a flat metric \( g_{\mu\nu} \Rightarrow \eta_{\mu\nu} \) is simplified for a homogeneous solution

\[
\dot{S} = \sqrt{-\det(\eta_{\mu\nu} + B_{\mu\nu}) - \det G_{\mu\nu}} S_{\Theta}(G_{\mu\nu}, \theta^{\mu\nu}; \dot{T}, \dot{F}_{\mu\nu} = 0) \tag{2.13}
\]

\[
= -\frac{T_p}{(-G)^{\frac{1}{4}}} \int d^{p+1}x V(\dot{T}) \sqrt{-\det(G_{\mu\nu} + \partial_\mu \dot{T} \partial_\nu \dot{T})} \tag{2.14}
\]

\[
= -\frac{T_p}{(\beta_p)^{\frac{1}{2}}} \int d^{p+1}x V(\dot{T}) \sqrt{\beta_p - \alpha_p \dot{T}^2}, \tag{2.15}
\]

where \( \hat{\beta}_p = -G = -\det(G_{\mu\nu}) \geq 0 \), \( \hat{\alpha}_p > 0 \) is 00-component of the cofactor \( \hat{C}^{\mu\nu} \) of matrix \( (\hat{X})_{\mu\nu} = G_{\mu\nu} + \partial_\mu \dot{T} \partial_\nu \dot{T} \), and \( -\hat{X} \equiv -\det \hat{X}_{\mu\nu} = \hat{\beta}_p - \hat{\alpha}_p \dot{T}^2 \).

NC energy-momentum tensor is read through variation of the open string metric

\[
\hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-G}} \frac{\delta \hat{S}_{\Theta}}{\delta G_{\mu\nu}} = \frac{T_p V}{\sqrt{-G} \sqrt{-X}} (\dot{X} G^{\mu\nu} - G \partial^\mu \dot{T} \partial^\nu \dot{T}), \tag{2.16}
\]

where the tensor indices are lowered and raised by using the open string metric as \( \partial^\mu = G^{\mu\nu} \partial_\nu \). The time-component of the conservation of the energy-momentum tensor

\[
\hat{D}_\mu \hat{T}^{\mu\nu} = 0 \tag{2.17}
\]

forces NC Hamiltonian density to be a constant

\[
\hat{H} = -\sqrt{-G} \hat{T}^0 = \frac{\hat{\beta}_p T_p V(\dot{T})}{\sqrt{\hat{\beta}_p - \hat{\alpha}_p \dot{T}^2}}. \tag{2.18}
\]

This first-order equation is consistent with the second-order NC tachyon equation of motion, and, similar to Eq. (2.3), all possible homogeneous NC rolling tachyons are given by one parameter (\( \hat{H} \)) family of solutions of Eq. (2.18). For any NC tachyon potential \( V(\dot{T}) \) satisfying the runaway property \( V(\dot{T} = 0) = 1 \) and \( V(\dot{T} = \pm\infty) = 0 \), three types of NC rolling tachyons are obtained: (i) When \( \hat{H} < \sqrt{\hat{\beta}_p T_p} \), a convex-up (or down) NC rolling tachyon connecting \( \dot{T}(t = -\infty) = +\infty \) (or \(-\infty\)) and \( \dot{T}(t = \infty) = +\infty \) (or \(-\infty\)); (ii) when \( \hat{H} = \sqrt{\hat{\beta}_p T_p} \), a monotonic increasing (or decreasing) NC rolling tachyon connecting \( \dot{T}(t = -\infty) = 0 \) and \( \dot{T}(t = \infty) = +\infty \) (or \(-\infty\)) and; (iii) \( \hat{H} > \sqrt{\hat{\beta}_p T_p} \), another
monotonic increasing (or decreasing) NC rolling tachyon connecting \( \hat{T}(t = -\infty) = -\infty \) (or \(+\infty\)) and \( \hat{T}(t = \infty) = +\infty \) (or \(-\infty\)). Note that (i) and (iii) are also named as full S-branes and (ii) is a \( \frac{1}{2} \)S-brane \([8, 9]\). If we choose the NC tachyon potential \( V(\hat{T}) \) as \( 1/\cosh \) type (2.2), the exact homogeneous NC rolling tachyon solutions can be obtained as follows,

\[
\frac{\hat{\tau}(t)}{R} \equiv \sinh \left( \frac{\hat{T}(t)}{R} \right) = \begin{cases} 
\sqrt{u^2 - 1} \cosh \left( \frac{t}{\hat{\zeta}} \right) & \text{for } \hat{\mathcal{H}} < \sqrt{\beta_p T_p}, \\
\xi \exp \left( \pm \frac{t}{\hat{\zeta}} \right) & \text{for } \hat{\mathcal{H}} = \sqrt{\beta_p T_p}, \\
\sqrt{1 - \hat{u}^2} \sinh \left( \frac{t}{\hat{\zeta}} \right) & \text{for } \hat{\mathcal{H}} > \sqrt{\beta_p T_p},
\end{cases}
\tag{2.19}
\]

where \( \hat{u} = \sqrt{\beta_p T_p/\hat{\mathcal{H}}}, \hat{\zeta} = \sqrt{\alpha_{p0}/\beta_p R}, \) and \( \xi \) is an arbitrary constant.

Though the functional forms of the exact NC rolling tachyon solutions (2.19) correctly span all possible real solutions of linearized NC tachyon equation in the background of constant NSNS two-form field

\[
-\partial_t^2 \hat{\tau} = -\frac{\hat{\beta}_p}{\alpha_{p0}} \frac{\hat{\tau}}{R^2},
\tag{2.20}
\]

the degree of validity of an effective action can be judged by comparing the obtained classical solutions (2.19) of the EFT with those of the open string theory, described by BCFT \([20]\). Specifically, this comparison is made with respect to the energy density and pressure given by matter responses to small gravitational fluctuations, and current density of F1 given by responses to small NSNS two-form field fluctuations:

\[
\delta \hat{S} = \frac{1}{2} \int d^{p+1}x \sqrt{-\hat{g}} \left( T^{\mu\nu} \delta g_{\mu\nu} + J^{\mu\nu} \delta B_{\mu\nu} \right).
\tag{2.21}
\]

For clarity, let us take into account the single electric component \( E_0 \) of the constant NSNS B-field along the \( x \)-direction on a flat unstable \( D_p \)-brane with closed string metric \( \eta_{\mu\nu} \). When the constant value of the energy density in BCFT is the same one in the EFT

\[
T_{00} = \frac{T_\mu V}{\sqrt{G_0 - \hat{T}^2}} \equiv T_{00}^{\text{BCFT}} = \begin{cases} 
\frac{T_\mu \cos^2(\pi \lambda)}{\sqrt{G_0}} (i) \\
\frac{T_\mu}{\sqrt{G_0}} (ii) \\
\frac{T_\mu \cosh^2(\pi \lambda)}{\sqrt{G_0}} (iii)
\end{cases}
\tag{2.22}
\]

nonvanishing components of the pressure and the string charge density for superstring
theory are \[5\]

If there is no contribution from the F1 represented by nonvanishing electric field where \(\tilde{E}\) is a parameter labelling the energy density (2.22) and we have the following nonvanishing components of the open string metric (2.5) and the NC parameter (2.6)

\[
T_{11}^{\text{BCFT}} = \left\{ \begin{array}{ll}
-\frac{T_p E_0^2 \cos^2(\pi \tilde{\lambda})}{\sqrt{G_0}} & \text{(i)} \\
-\frac{T_p E_0^2 \left[ \cos^4(\pi \tilde{\lambda}) + 4 \sin^2(\pi \tilde{\lambda}) \right]}{\sqrt{G_0}} & \text{(ii)} \\
-\frac{T_p E_0^2 \left[ \cos^4(\pi \tilde{\lambda}) - 4 \sin^2(\pi \tilde{\lambda}) \right]}{\sqrt{G_0}} & \text{(iii)}
\end{array} \right.
\]

\(T_{ab}^{\text{BCFT}} = (T_{11}^{\text{BCFT}} + T_{00}^{\text{BCFT}} E_0^2) \delta_{ab}, \quad (a, b = 2, 3, \cdots, p), \quad (2.24)

\(J_{01}^{\text{BCFT}} = T_{00}^{\text{BCFT}} E_0, \quad (2.25)\)

where \(\tilde{\lambda}\) is a parameter labelling the energy density (2.22) and we have the following nonvanishing components of the open string metric (2.5) and the NC parameter (2.6)

\[
G_0 \equiv 1 - E_0^2 = -G_{00} = G_{11}, \quad G_{ab} = \delta_{ab}, \quad (2.26)
\]

\[
\theta \equiv \frac{E_0}{1 - E_0^2} = \theta^{01} = -\theta^{10}, \quad (2.27)
\]

\[
\hat{\beta}_p = G_0^2, \quad \hat{\alpha}_p = G_0. \quad (2.28)
\]

Here, the cases (i), (ii), and (iii) correspond to the three cases in the NC rolling tachyon solutions (2.19). By comparing Eq. (2.18) with Eq. (2.22), following relation for the pure electric case with \(E_0\) along \(x\)-direction is

\[
\hat{\mathcal{H}} = \left\{ \begin{array}{ll}
T_p G_0 \cos^2(\pi \tilde{\lambda}) & \text{(i)} \\
T_p G_0 & \text{(ii)} \\
T_p G_0 \cos^2(\pi \tilde{\lambda}) & \text{(iii)}
\end{array} \right.
\]

and the nonvanishing pressure components and string charge density are

\[
T_{11} = -T_{00}(1 - \hat{T}^2) = \left\{ \begin{array}{ll}
-\frac{T_p E_0^2 \cos^2(\pi \tilde{\lambda})}{\sqrt{G_0}} & \text{(i)} \\
-\frac{T_p E_0^2 \cos^2(\pi \tilde{\lambda})}{\sqrt{G_0}} & \text{(ii)} \\
-\frac{T_p E_0^2 \cos^2(\pi \tilde{\lambda})}{\sqrt{G_0}} & \text{(iii)}
\end{array} \right.
\]

\[
T_{ab} = -T_{00}(G_0 - \hat{T}^2) = (T_{11} + T_{00} E_0^2) \delta_{ab}, \quad (2.31)
\]

\(J_{01} = T_{00} E_0, \quad (2.32)\)

If there is no contribution from the F1 represented by nonvanishing electric field \(E_0\), the NC tachyon field has \(\lim_{t \to -\infty} \hat{T}^2 \to 1\) and thereby, the homogeneous tachyon matter becomes pressureless, \(\lim_{t \to -\infty} T_{11} = \lim_{t \to -\infty} T_{ab} \to 0\), as long as the production of closed string degrees is neglected.
Due to the equivalence (2.13) between the effective action (2.1) and the NC action (2.12), the pressure (2.30)–(2.31) and the F1 charge density (2.32) coincide exactly with those of EFT as a defining property [13]. In addition, the F1 charge density (2.32) is the same as that in BCFT (2.25). The relative coefficients of $T_{11}$ and $T_{ab}$ computed from EFT or NCFT in Eqs. (2.30) and (2.31) agree qualitatively to but do not coincide exactly with those of BCFT in Eqs. (2.23) and (2.24) for the full S-brane cases (i) and (iii). This mismatch disappears for the half S-brane case (ii); that is, if we choose the origin of time as that satisfying $\xi = \sqrt{2\pi \tilde{\lambda}}$, the stress tensors share the same results with BCFT. This coincidence is also satisfied for the half S-brane in bosonic string theory with identification $\xi = \sqrt{2\pi \tilde{\lambda}}$. The coincidence looks natural since the effective action (2.1) and the tachyon potential (2.2) of our consideration have been derived by open string theory by taking into account the fluctuations around half S-brane configuration with the higher derivatives neglected, i.e., $\partial^2 T = \partial^3 T = \cdots = 0$ [19]. Though the pure electric case is compared, its generalization to arbitrary constant NSNS two-form field is automatic [11]. Comparing the obtained results to those from the string field theory [21] or $c = 1$ matrix model [22] is not yet made at the level of energy-momentum tensor.

3 NC Rolling Tachyon with NC Electric Field

U(1) gauge field lives on a D-brane and, in the context of NCFT, it appears as an NC U(1) gauge field $\hat{F}_{\mu\nu}$ on our single unstable D-brane. Without tachyonic degree the equivalence between the DBI-type action in ordinary field theory and the NCFT (2.7) still holds with inclusion of the NC U(1) gauge field. Various tachyon actions proposed [15, 16, 17], and, for those actions, reduction to the action (2.12) in vanishing $\hat{F}_{\mu\nu}$ limit is a necessary condition. For the following DBI-type action [16, 17], EFT and NCFT including the slowly-varying NC U(1) gauge field and NC tachyon field were proven to be equivalent up to the leading order of NC parameter $\theta^{\mu\nu}$:

$$\hat{S}_F = -\frac{\hat{T}_p}{2} \int d^{p+1}x \left[ \hat{V}(\hat{T}) \sqrt{-\det_*(G_{\mu\nu} + \hat{F}_{\mu\nu} + \hat{D}_\mu \hat{T} \ast \hat{D}_\nu \hat{T}) + (\sqrt{\hat{V}})} \right], \quad (3.1)$$

where

$$\hat{D}_\mu \hat{T} = \partial_\mu \hat{T} - i(\hat{A}_\mu \ast \hat{T} - \hat{T} \ast \hat{A}_\mu). \quad (3.2)$$

Here, let us restrict our interest to pure electric cases. For a flat unstable D-brane of arbitrary $p$ with a constant electric component of NSNS two form field $B_{0i} = E_{0i}$, direction of the electric component can be always chosen as the $x$-direction. Then we have closed
string variables $\eta_{\mu\nu}$ and $E_0 = E_0 x$, and the corresponding open string variables, $G_{\mu\nu}$ and $\theta_{\mu\nu}$, are given in Eqs. (2.26)–(2.27).

In the homogeneous NC rolling tachyon configurations, the NC tachyon and NC U(1) gauge field on the brane depend only on time $\hat{T} = \hat{T}(t)$ and $\hat{F}_{\mu\nu} = \hat{F}_{\mu\nu}(t)$. As we assumed, the field strength tensor has only electric components, $\hat{F}_{0i} = \hat{E}_i(t)$ and $\hat{F}_{ij} = 0$. At a given time $t$, two vectors $E_0$ (or equivalently $\hat{\theta}^1 = \theta x$) and $\hat{E}$ generally form a plane, and we work in the coordinates where the NSNS field is directed in the $x$-direction and the NC electric field lies on the $xy$-plane

$$\hat{F}_{01} = \hat{E}_1(t), \quad \hat{F}_{02} = \hat{E}_2(t).$$

(3.3)

In order to proceed, we choose a gauge

$$\theta^{0\mu} \hat{A}_\mu = 0 \quad (3.4)$$

which leads to $\hat{A}_1 = 0$ since $\theta = \theta^{01}$ is nonvanishing among all the NC parameters. Substituting the gauge (3.4) into the definition of NC field strength tensor (2.9),

$$\hat{E}_1 = \partial_1 \hat{A}_0, \quad \hat{E}_2 = (1 + \hat{E}_1 \theta) \partial_0 \hat{A}_2 - \partial_2 \hat{A}_0.$$  

(3.5)

Note that the NC electric field (3.5) automatically satisfies NC Bianchi identity $\hat{D}_\mu \hat{F}_{\nu\rho} + \hat{D}_\nu \hat{F}_{\rho\mu} + \hat{D}_\rho \hat{F}_{\mu\nu} = 0$ [23, 17].

If we summarize our results under our ansatz (2.11) and (3.3), every star product is replaced by an ordinary product and then $\hat{D}_\mu \hat{T} = \delta_{\mu0} (1 + \hat{E}_1 \theta) \hat{T}^2$. Inserting the results into the action (3.1), we obtain a simplified form of it

$$\hat{S}_F = -\hat{T}_p \int dtdxdydp^d x_\perp \sqrt{-\hat{X}} V(\hat{T}),$$

(3.6)

where $x_\perp$ denote transverse coordinates and

$$-\hat{X} \equiv \beta_F - \alpha_0 \hat{\gamma}^2 = G_0^2 - G_0 \hat{E}_2^2 - \hat{E}_1^2 - G_0 (1 + \hat{E}_1 \theta)^2 \hat{T}^2.$$  

(3.7)

From the action (3.6), the survived components of the equations of motion are the time-component of the conservation of energy-momentum tensor

$$(1 + \hat{E}_1 \theta) \partial_0 \left[ \frac{\hat{T}_p V}{\sqrt{-\hat{X}}} \right] = 0,$$

(3.8)

and $x$- and $y$-components of gauge field equations are

$$(1 + \hat{E}_1 \theta) \partial_0 \left[ \frac{\hat{T}_p V}{\sqrt{-\hat{X}}} \hat{E}_1 \right] = 0, \quad (1 + \hat{E}_1 \theta) \partial_0 \left[ \frac{\hat{T}_p V}{\sqrt{-\hat{X}}} G_0 \hat{E}_2 \right] = 0.$$

(3.9)
Solving the equations (3.8)–(3.9) when \(1 + \hat{E}_1 \theta \neq 0\), we show that the NC electric field \(\hat{E}\) is constant and the only nontrivial equation for the NC tachyon reduces to

\[
\dot{\gamma} = \frac{\hat{T}_0 \hat{V}}{\sqrt{-X}} = \text{constant.} \tag{3.10}
\]

The resultant equation (3.10) is formally the same as Eq. (2.18) under identification, \(\dot{\gamma} \leftrightarrow \dot{H}, \beta_F \leftrightarrow \beta_p, \alpha_{0F} \leftrightarrow \alpha_{p0}\), and \(\hat{T} \leftrightarrow \hat{T}\), so that the spectrum of homogeneous NC rolling tachyons is exactly the same as that in the previous section as given between Eq. (2.18) and Eq. (2.19). Here let us omit displaying the solutions in Eq. (2.19) again.

For the NC rolling tachyon solutions, there are constant nonvanishing electric fluxes

\[
\hat{\Pi}_1 = \gamma \hat{E}_1, \quad \hat{\Pi}_2 = \gamma G_0 \hat{E}_2, \tag{3.11}
\]

which are defined by \(\hat{\Pi}_i = \delta \hat{S}/\delta \hat{E}_i\) and signify the existence of F1 string fluid. The corresponding energy-momentum tensor has constant energy density \(\hat{T}^{00} = \dot{\gamma}\), nonvanishing constant stress along 12-direction \(\hat{T}^{12} = -\frac{\gamma}{\gamma_0} \hat{E}_1 \hat{E}_2\), and zero momentum density \(\hat{T}^{0i} = 0\). On the other hand, all the pressure components are nonvanishing in this reference frame

\[
\begin{align*}
\hat{T}^{11} &= \frac{\gamma}{\gamma_0} \left[-G_0 + \hat{E}_2^2 + (1 + \hat{E}_1 \theta)^2 \hat{T}^2 \right], \tag{3.12} \\
\hat{T}^{22} &= \frac{\gamma}{\gamma_0} \left[-G_0^2 + \hat{E}_1^2 + G_0(1 + \hat{E}_1 \theta)^2 \hat{T}^2 \right], \tag{3.13} \\
\hat{T}^{ij} &= \frac{\gamma}{\gamma_0} \hat{X} \delta^{ij}, \quad (i, j = 3, 4, \cdots, p). \tag{3.14}
\end{align*}
\]

The functional form of the NC rolling tachyon solutions is the same as that of rolling tachyons in EFT, but their equivalence in the action level is guaranteed only up to the leading NC parameter. Therefore, the comparison of the components of energy-momentum tensor given by responses to small gravitational fluctuations \(\delta g_{\mu\nu}\) in NCFT to those in EFT and BCFT can be valid, at most, up to the leading NC parameter. In the analysis of the NC rolling tachyon by the use of different NC tachyon actions [15], all three species of the NC rolling tachyon cannot be obtained as were obtained in the case of NC tachyon kinks [17].

### 4 Summary

We considered a DBI type NCFT action of a real NC tachyon field \(\hat{T}\) as an EFT describing dynamics of a flat unstable Dp-brane. When the background of constant NSNS two-form field is coupled to the brane, this field is expressed in terms of a constant open string metric...
and NC parameter. For the general constant open string metric and NC parameter, we showed the existence of all possible homogeneous NC rolling tachyon solutions irrespective of specific shape of the NC tachyon potential \( \hat{V} \), once \( \hat{V}(\hat{T} = 0) = 1 \) and \( \hat{V}(\hat{T} = \pm\infty) = 0 \) are satisfied. These solutions are classified into two full S-brane solutions, one with \( \hat{T}(t = -\infty) = \pm\infty \) and \( \hat{T}(t = +\infty) = \mp\infty \) and the other with \( \hat{T}(t = -\infty) = \pm\infty \) and \( \hat{T}(t = +\infty) = \pm\infty \), and one half S-brane with \( \hat{T}(t = -\infty) = 0 \) and \( \hat{T}(t = +\infty) = \pm\infty \). Under a specific \( 1/\cosh \) type NC tachyon potential, they are obtained as exact solutions whose functional forms are hyperbolic sine, hyperbolic cosine, and exponential, respectively, which are classical configurations of a linearized tachyon equation in BCFT.

When the energy-momentum tensor and the F1 charge density in NCFT are compared with those in BCFT, pressure and stress components agreed qualitatively but not exactly to those in BCFT for the full S-branes but coincided exactly to those for the half S-brane. This result seems natural since the \( 1/\cosh \) type potential in EFT was derived by considering the fluctuations around half S-brane in open string theory.

The addition of DBI type electromagnetism preserves the structure of NCFT including the covariant derivative of the NC tachyon. However, the allowed configuration of field strengths was constant, and the obtained homogeneous NC rolling tachyons had the same functional form.

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**References**


