Magnetic moments of exotic pentaquark baryons

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Abstract

In this talk, we present our recent investigation on the magnetic moments of the exotic pentaquark states, based on the chiral quark-soliton model, all relevant intrinsic parameters being fixed by using empirical data.
I. INTRODUCTION

Since Diakonov et al. [1] predicted masses and decay widths of the exotic baryon antidecuplet within the framework of the chiral soliton model, the findings of the exotic baryons have been announced by many different experimental collaborations (see, for example, a very recent review [2] and references therein), though some of them are still under debate.

Since the LEPS and CLAS collaborations used photons to produce the $\Theta^+$, it is of great interest to investigate electromagnetic properties of the exotic baryons. In particular, we have to know the magnetic moment of the $\Theta^+$ and its strong coupling constants in order to describe the mechanism of the pentaquark photoproduction. However, information on its static properties is absent to date, so we need to estimate them theoretically. Recently, we calculated the magnetic moments of the exotic pentaquarks within the framework of the chiral quark-soliton model [3, 4] with all relevant dynamic parameters fixed by using empirical data. We would like to present those results briefly in this talk.

II. MAGNETIC MOMENTS IN THE CHIRAL QUARK-SOLITON MODEL

The collective operator for the magnetic moments can be expressed in terms of six constants [5, 6]:

\[
\hat{\mu}^{(0)} = w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot J_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3,
\]

\[
\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{Qq} + w_5 \left( D^{(8)}_{Q3} D^{(8)}_{Q8} + D^{(8)}_{Q8} D^{(8)}_{Q3} \right) + w_6 \left( D^{(8)}_{Q3} D^{(8)}_{Q8} - D^{(8)}_{Q8} D^{(8)}_{Q3} \right).
\]

The parameters $w_{1,2,3}$ are of order $O(m_s^0)$, while $w_{4,5,6}$ are of order $O(m_s)$, $m_s$ being regarded as a small parameter. Though $w_i$ can be calculated numerically within the model, we want to fit them in this work to the experimental data of the octet magnetic moments.

The full expression for the magnetic moments can be decomposed as three different terms:

\[
\mu_B = \hat{\mu}^{(0)} + \hat{\mu}^{(op)} + \hat{\mu}^{(wf)},
\]

where the $\hat{\mu}^{(0)}$ is given by the matrix element of the $\hat{\mu}(0)$ between the purely symmetric collective states $|\mathcal{R}_J, B, J_3\rangle$ [4], and the $\hat{\mu}^{(op)}$ is given as the matrix element of the $\hat{\mu}(1)$ between the symmetry states as well. The wave function correction $\hat{\mu}^{(wf)}$ is given as a sum of the interference matrix elements of the $\hat{\mu}^{(0)}$ between purely symmetric states and those in higher representations.

The measurement of the $\Theta^+$ mass constrains further the parameter space of the model (see Refs. [4, 5] for details). Denoting the set of the model parameters by

\[
\vec{w} = (w_1, \ldots, w_6)
\]

the model formulae for the set of the magnetic moments in representation $\mathcal{R}$ (of dimension $R$)

\[
\vec{\mu}^\mathcal{R} = (\mu_{B_1}, \ldots, \mu_{B_R})
\]
can be conveniently cast into the form of the matrix equations:
\[ \vec{\mu}^R = A^R[\Sigma_{\pi N}] \cdot \vec{w}, \]
where rectangular matrices \( A^8, A^{10}, \) and \( A^{10}_W \) can be found in Refs. [4, 3, 6]. These matrices depend on the pion-nucleon \( \Sigma_{\pi N} \) term.

### III. RESULTS AND DISCUSSION

In order to find the set of parameters \( w_i[\Sigma_{\pi N}] \), we minimize the mean square deviation for the octet magnetic moments: \( \delta \mu^8 = \sqrt{\sum_B (\mu^8_{B,\text{th}}[\Sigma_{\pi N}] - \mu^8_{B,\text{exp}})^2 / 7} \), where the sum extends over all octet magnetic moments, but the \( \Sigma^0 \). The value \( \delta \mu^8 \simeq 0.01 \) is in practice independent of the \( \Sigma_{\pi N} \) in the physically interesting range 45 \( \sim \) 75 MeV. Moreover, the values of the \( \mu^8_{B,\text{th}}[\Sigma_{\pi N}] \) do not depend on \( \Sigma_{\pi N} \).

Similarly, the value of the nucleon strange magnetic moment is independent of \( \Sigma_{\pi N} \) and reads \( \mu^s_N = 0.39 \text{ n.m.} \) in fair agreement with our previous analysis of Ref. [6]. Parameters \( w_i \), however, do depend on \( \Sigma_{\pi N} \), as shown in Table II. Note that parameters \( w_{2,3} \) are formally \( \mathcal{O}(1/N_c) \) with respect to \( w_1 \). For smaller \( \Sigma_{\pi N} \), this \( N_c \) counting is not borne by explicit fits. The \( \mu^0_B \) can be parametrized by the following two parameters \( v \) and \( w \):

\[
\begin{align*}
v &= (2 \mu_p - \mu_\pi + 3 \mu_{\Xi^0} + \mu_{\Xi^-} - 2 \mu_{\Sigma^-} - 3 \mu_{\Sigma^+}) / 60 = -0.268, \\
w &= (3 \mu_p + 4 \mu_n + \mu_{\Xi^0} - \mu_{\Xi^-} - 4 \mu_{\Sigma^-} - \mu_{\Sigma^+}) / 60 = 0.060.
\end{align*}
\]

which are free of linear \( m_s \) corrections [3]. This is a remarkable feature of the present fit, since when the \( m_s \) corrections are included, the \( m_s \)-independent parameters need not be refitted. This property will be used in the following when we restore the linear dependence of the \( \mu^0_B \) on \( m_s \).

Finally, for the baryon antidecuplet we have a strong dependence on \( \Sigma_{\pi N} \), yielding the numbers listed in Table II. They are further depicted in Fig. II. The wave function corrections cancel for the non-exotic baryons and add constructively for the baryon antidecuplet. In particular, for \( \Sigma_{\pi N} = 75 \text{ MeV} \) we have large admixture coefficient of the 27-plet [4]. The magnetic moments of the antidecuplet are rather small in absolute value. For \( \Theta^+ \) and \( p^* \) one obtains even negative values, although the charges are positive. As for \( \Xi^-_{10} \) and \( \Xi^+_{10} \) one gets positive values in spite of their negative signs.

| \( \Sigma_{\pi N} \) [MeV] | \( \Theta^+ \) | \( p^* \) | \( n^* \) | \( \Sigma^+_{10} \) | \( \Sigma^-_{10} \) | \( \Xi^+_{10} \) | \( \Xi^-_{10} \) | \( \Xi^+_{11} \) | \( \Xi^-_{11} \) |
|---|---|---|---|---|---|---|---|---|
| 45 | -1.19 | -0.97 | -0.34 | -0.75 | -0.02 | 0.71 | -0.53 | 0.30 | 1.13 | 1.95 |
| 60 | -0.78 | -0.36 | -0.41 | 0.06 | 0.15 | 0.23 | 0.48 | 0.70 | 0.93 | 1.15 |
| 75 | -0.33 | 0.28 | -0.43 | 0.90 | 0.36 | -0.19 | 1.51 | 1.14 | 0.77 | 0.39 |

**TABLE II:** Magnetic moments of the baryon antidecuplet.
IV. CONCLUSION AND SUMMARY

Our present analysis shows that $\mu_{\Theta^+} < 0$, although the magnitude depends strongly on the model parameters. We anticipate that the measurement of $\mu_{\Theta^+}$ could therefore discriminate between different models. This also may add to reduce the ambiguities in the pion-nucleon sigma term $\Sigma_{\pi N}$.

In the present work, we determined the magnetic moments of the baryon antidecuplet in the model-independent analysis within the framework of the chiral quark-soliton model, i.e. using the rigid-rotor quantization with the linear $m_s$ corrections included. Starting from the collective operators with dynamical parameters fixed by experimental data, we obtained the magnetic moments of the baryon antidecuplet. We found that the magnetic moment $\mu_{\Theta^+}$ is negative and rather strongly dependent on the value of the $\Sigma_{\pi N}$. Indeed, the $\mu_{\Theta^+}$ ranges from $-1.19$ n.m. to $-0.33$ n.m. for $\Sigma_{\pi N} = 45$ and 75 MeV, respectively.

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