We present a technique to calculate the cross sections and spin asymmetries for transversely polarized \( pp \) collisions at NLO in QCD and report on the use of this technique for the processes \( p^\uparrow p^\uparrow \rightarrow \gamma X \), \( p^\uparrow p^\uparrow \rightarrow \pi X \) and \( p^\uparrow p^\uparrow \rightarrow l^+ l^- X \).

1. Introduction

Combined experimental and theoretical efforts in the past few years have led to an improved understanding of the unpolarized parton distributions \( f(x,Q^2) \) and the helicity distributions \( \Delta f(x,Q^2) \) of the nucleon. It is known that the complete understanding of the partonic structure of a spin \( \frac{1}{2} \) object like a nucleon is given in terms of \( f(x,Q^2) \), \( \Delta f(x,Q^2) \) and by the transversity distributions \( \delta f(x,Q^2) \), which give the number densities of partons having the same polarization as the nucleon, when the nucleon is transversely polarized, minus the number with opposite polarization. \( \delta f(x,Q^2) \) remain quantities about which we have the least knowledge and are at present the focus of much experimental activity.

Transversity will be probed in the double transverse spin asymmetries in transversely polarized \( pp \) collisions at the BNL Relativistic Heavy Ion Collider (RHIC). The potential of RHIC in accessing transversity through
double transverse spin asymmetries $A_{TT}$ in the Drell-Yan process was estimated in $^1$. Other relevant processes include high $p_T$ prompt photon and jet production $^2$. Apart from DY, the other calculations were done at leading order (LO). It is known that the next-to-leading order (NLO) QCD calculations are necessary in order to have a firm theoretical prediction.

2. Projection Technique

Apart from the motivations given above, interesting new technical questions arise beyond LO in the calculations of cross sections involving transverse polarization. Unlike the longitudinally polarized case, where the spin vectors are aligned with the momentum, the transverse spin vectors specify extra spatial directions and as a result, the cross section has non-trivial dependence on the azimuthal angle of the observed particle. For $A_{TT}$ this dependence is always of the form $^3$

$$\frac{d^3\delta\sigma}{dp_T d\eta d\Phi} \equiv \cos(2\Phi) \left\langle \frac{d^2\delta\sigma}{dp_T d\eta} \right\rangle,$$  

for a parity conserving theory with vector coupling, here the $z$ axis is defined by the direction of the initial partons in their center-of-mass frame and the spin vectors are taken to point in the $\pm x$ direction. Therefore the integration over the azimuthal angle is not appropriate. This makes it difficult to use the standard techniques developed for NLO calculations of unpolarized and longitudinally polarized processes here because all these techniques usually rely on the integration over the full azimuthal phase space and also on particular reference frames which are related in a complicated way to the center-of-mass frame of the initial protons. In $^4$ a new general technique was introduced which facilitates NLO calculations with transverse polarizations by conveniently projecting on the azimuthal dependence of the cross section in a covariant way. The projector

$$F(p, s_a, s_b) = \frac{s}{\pi tu} \left[ 2 (p \cdot s_a) (p \cdot s_b) + \frac{tu}{s} (s_a \cdot s_b) \right],$$

reduces to $\frac{\cos 2\Phi}{\pi}$ in the center-of-mass frame of the initial protons. Here $p$ is the momentum of the observed particle in the final state. The cross section is multiplied with the projector and integrated over the full azimuthal phase space. Integrations of the terms involving the product of the transverse spin vectors $s_a, s_b$ with the momenta can be performed using a tensor decomposition. After this step, there are no scalar products involving $s_i$ left in the matrix element. For the integration over the phase space, one
can now use the standard techniques from the unpolarized and longitudinally polarized cases. This method is particularly convenient at NLO, where one uses dimensional regularization and the phase space integrations are performed in $n$ dimensions.

3. Applications

As an example, we discuss the use of this technique for high $p_T$ prompt photon production. The LO process is $q\bar{q} \rightarrow \gamma g$. We multiply $\delta|M|^2$ by the projector $F(p, s_a, s_b)$ and integrate over the full $\Delta\phi$ in a covariant way. At NLO, there are two subprocesses contributing, $qq \rightarrow \gamma X$, where $X = qq$ and $q\bar{q} \rightarrow \gamma X$, where $X = q\bar{q} + gg + q'\bar{q'}$. For $2 \rightarrow 3$ processes, one integrates over the phase spaces of the unobserved particles, after multiplying with the projector and eliminating the scalar products with the spin vectors using tensor decomposition. Owing to the presence of the ultraviolet, infrared and collinear singularities, one has to introduce a regulator. We choose dimensional regularization. UV poles in the virtual diagrams are removed by renormalization of the strong coupling constant. Infrared singularities cancel between the real emission and virtual diagrams. After this, only collinear singularities remain, which result from collinear splitting of an initial-state parton into a pair of partons. These correspond to long-distance contribution to the partonic cross section. From the factorization theorem it follows that such contributions need to be factored into the parton distributions. We have imposed an isolation cut to remove the background contribution. All final-state collinear singularities then cancel. The isolation constraint was imposed analytically by assuming a narrow isolation cone.

For our numerical predictions, we model the transversity distribution by saturating Soffer’s inequality at some low input scale $\mu_0 \approx 0.6$ GeV and for higher scales, $\delta f(x, \mu)$ are obtained by solving the QCD evolution equations. Figure 1 shows the results for the prompt photon production in transversely polarized $pp$ collisions. Our numerics apply to the PHENIX detector at RHIC. The lower part of the figure displays the 'K- factor', $K = \frac{d\sigma^{NLO}}{d\sigma^{LO}}$. The scale dependence becomes much weaker at NLO. The corresponding asymmetries are given in 4.

For the Drell-Yan lepton pair production in transversely polarized $pp$ collisions the LO subprocess is $q\bar{q} \rightarrow l^+l^-$. The real emission $2 \rightarrow 3$ subprocess is $q\bar{q} \rightarrow l^+l^-g$. We multiply the squared matrix element by the projector and integrate over the phase space. We obtain the known result for the DY
Figure 1. Predictions for the transversely polarized prompt photon production cross sections at LO and NLO, for $\sqrt{S} = 200$ and 500 GeV. The LO results have been scaled by a factor of 0.01. The shaded bands represent the theoretical uncertainty if $\mu_F (= \mu_R)$ is varied in the range $p_T/2 \leq \mu_F \leq 2p_T$. The lower panel shows the ratios of the NLO and LO results for both c.m.s. energies.

coefficient function in the $\overline{MS}$ scheme at NLO\(^4\).

For inclusive pion production in transversely polarized process the LO channels are $qq \to qX, q\bar{q} \to qX, q\bar{q} \to q'X, q\bar{q} \to gX$. At NLO there are $O(\alpha_s)$ corrections to the above processes and the additional channel $qq \to gX$. We have used the projection technique to calculate the cross section at NLO and the numerical results are in progress.

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References