Chiral dynamics of hadrons in nuclei


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Abstract. In this talk I report on selected topics of hadron modification in the nuclear medium using the chiral unitary approach to describe the dynamics of the problems. I shall mention how antikaons, $\eta$, and $\phi$ are modified in the medium and will report upon different experiments done or planned to measure the $\phi$ width in the medium.

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1. Introduction

Nowadays it is commonly accepted that QCD is the theory of the strong interactions, with the quarks as building blocks for baryons and mesons, and the gluons as the mediators of the interaction. However, at low energies typical of the nuclear phenomena, perturbative calculations with the QCD Lagrangian are not possible and one has to resort to other techniques to use the information of the QCD Lagrangian. One of the most fruitful approaches has been the use of chiral perturbation theory, $\chi PT$ [1]. The theory introduces effective Lagrangians which involve only observable particles, mesons and baryons, respects the basic symmetries of the original QCD Lagrangian, particularly chiral symmetry, and organizes these effective Lagrangians
according to the number of derivatives of the meson and baryon fields.

2. Baryon meson interaction

The interaction of the octet of stable baryons with the octet of pseudoscalar mesons is given to lowest order by the Lagrangian [2, 3]

\[ \mathcal{L}_1 = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{1}{2} D \langle \bar{B} \gamma^\mu \gamma_5 \{ u_\mu, B \} \rangle + \frac{1}{2} F \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle \] 

with the usual definitions for the B and Φ SU(3) matrices, the covariant derivative \( \nabla_\mu \) and the SU(3) matrix \( u_\mu \) [2, 3].

3. Unitarized chiral perturbation theory: N/D or dispersion relation method

One can find a systematic and easily comprehensible derivation of the ideas of the N/D method applied for the first time to the meson baryon system in [4], which we reproduce here below and which follows closely the similar developments used before in the meson meson interaction [5]. One defines the transition T–matrix as \( T_{i,j} \) between the coupled channels which couple to certain quantum numbers. For instance in the case of \( \bar{K}N \) scattering studied in [4] the channels with zero charge are \( K^-p, K^0n, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \pi^0\Lambda, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0 \). Unitarity in coupled channels is written as

\[ \text{Im} T_{i,j} = T_{i,i} \rho_i T_{i,j}^* \]  

where \( \rho_i \equiv 2M_i q_i/(8\pi W) \), with \( q_i \) the modulus of the c.m. three–momentum, and the subscripts \( i \) and \( j \) refer to the physical channels. This equation is most efficiently written in terms of the inverse amplitude as

\[ \text{Im} T^{-1}(W)_{ij} = -\rho(W)_i \delta_{ij} \]  

The unitarity relation in Eq. (3) gives rise to a cut in the T–matrix of partial wave amplitudes, which is usually called the unitarity or right–hand cut. Hence one can write down a dispersion relation for \( T^{-1}(W) \)

\[ T^{-1}(W)_{ij} = -\delta_{ij} \left\{ \tilde{a}_i(s_0) + \frac{s - s_0}{\pi} \int_{s_i}^\infty ds' \frac{\rho(s')_i}{(s' - s)(s' - s_0)} \right\} + T^{-1}(W)_{ij} , \]  

where \( s_i \) is the value of the \( s \) variable at the threshold of channel \( i \) and \( T^{-1}(W)_{ij} \) indicates other contributions coming from local and pole terms, as well as crossed channel dynamics but without right–hand cut. These extra terms are taken directly
from $\chi PT$ after requiring the matching of the general result to the $\chi PT$ expressions. Notice also that

$$g(s)_i = \tilde{a}_i(s_0) + \frac{s - s_0}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho(s')_i}{(s' - s)(s' - s_0)}$$

is the familiar scalar loop integral.

One can further simplify the notation by employing a matrix formalism. Introducing the matrices $g(s) = \text{diag} (g(s)_i)$, $T$ and $T$, the latter defined in terms of the matrix elements $T_{ij}$ and $T_{ij}$, the $T$-matrix can be written as:

$$T(W) = [I - T(W) \cdot g(s)]^{-1} \cdot T(W).$$

which can be recast in a more familiar form as

$$T(W) = T(W) + T(W)g(s)T(W)$$

Now imagine one is taking the lowest order chiral amplitude for the kernel as done in [4]. Then the former equation is nothing but the Bethe Salpeter equation with the kernel taken from the lowest order Lagrangian and factorized on shell, the same approach followed in [6], where different arguments were used to justify the on shell factorization of the kernel.

4. Meson baryon scattering

The low-energy $K^- N$ scattering and transition to coupled channels is one of the cases of successful application of chiral dynamics in the baryon sector. We rewrite Eq. (7) in the more familiar form

$$T = V + V G T$$

with $G$ the diagonal matrix given by the loop function of a meson and a baryon propagators.


5. Strangeness $S = -1$ sector

We take the $K^- p$ state and all those that couple to it within the chiral scheme mentioned above. Hence we have a problem with ten coupled channels. The coupled set of Bethe Salpeter equations were solved in [6] using a cut off momentum of 630 MeV in all channels. Changes in the cut off can be accommodated in terms of changes in $\mu$, the regularization scale in the dimensional regularization formula for $G_l$, or in the subtraction constant $a_l$. In order to obtain the same results as in [6] at low energies, we set $\mu$ equal to the cut off momentum of 630 MeV (in all channels)
and then find the values of the subtraction constants \( a_i \) such as to have \( G_i \) with the same value with the dimensional regularization formula and the cut off formula at the \( KN \) threshold. For the purpose of this talk let us recall that in [6] we obtain the \( \Lambda(1405) \) resonance obtained from the \( \pi\Sigma \) spectrum and the cross sections for \( Kp \) to different channels, some of which are shown in fig. 1. Other resonances are also found but I shall not mention them in this talk.

![Fig. 1. \( K^-p \) scattering cross sections as functions of the \( K^- \) momentum in the lab frame: with the full basis of physical states (solid line), omitting the \( \eta \) channels (long-dashed line) and with the isospin-basis (short-dashed line). Taken from Ref. [6].](image)

6. Strangeness \( S = 0 \) sector

The strangeness \( S = 0 \) channel was also investigated using the Lippmann Schwinger equation and coupled channels in [7]. The \( N^*(1535) \) resonance was also generated dynamically within this approach. Subsequently work was done in this sector in [8], and [9] where the \( N^*(1535) \) resonance was also generated. In [10] the work along these lines was continued and improved by introducing the \( \pi N \rightarrow \pi NN \) channels, which proved essential in reproducing the isospin 3/2 part of the \( \pi N \) amplitude, including the reproduction of the \( N^*(1650) \) resonance.

For total zero charge one has six channels in this case, \( \pi^- p, \pi^0 n, \eta n, K^+ \Sigma^-, K^0 \Sigma^0, \) and \( K^0 \Lambda. \)

Details on the issues discussed in this paper can be seen in the review paper [11].
7. $\bar{K}$ in nuclei

Next we address the properties of the $\bar{K}$ in the nuclear medium which have been studied in [12]. The work is based on the elementary $\bar{K}N$ interaction which has been discussed above, using a coupled channel unitary approach with chiral Lagrangians.

The coupled channel formalism requires to evaluate the transition amplitudes between the different channels that can be built from the meson and baryon octets. For $K^- p$ scattering there are 10 such channels, namely $K^- p$, $\bar{K}^0 n$, $\pi^0 \Lambda$, $\bar{\pi}^0 \Sigma^0$, $\pi^+ \Sigma^-$, $\pi^- \Sigma^+$, $\eta \Lambda$, $\eta \Sigma^0$, $K^+ \Xi^-$ and $K^0 \Xi^0$. In the case of $K^- n$ scattering the coupled channels are: $K^- n$, $\pi^0 \Sigma^-$, $\pi^- \Sigma^0$, $\pi^- \Lambda$, $\eta \Sigma^-$ and $K^0 \Xi^-$. In order to evaluate the $\bar{K}$ selfenergy in the medium, one needs first to include the medium modifications in the $\bar{K}N$ amplitude, $T^\alpha_{\text{eff}}$ ($\alpha = \bar{K}p, \bar{K}n$), and then perform the integral over the nucleons in the Fermi sea:

$$
\Pi^\alpha_{\bar{K}}(q^0, \vec{q}, \rho) = 2 \int \frac{d^3 p}{(2\pi)^3} n(p) \left[ T^K_{\text{eff}}(P^0, \vec{P}, \rho) + T_{\text{eff}}^{\bar{K}N}(P^0, \vec{P}, \rho) \right],
$$

(9)

The values $(q^0, \vec{q})$ stand now for the energy and momentum of the $\bar{K}$ in the lab frame, $P^0 = q^0 + \varepsilon_N(p)$, $\vec{P} = \vec{q} + \vec{p}$ and $\rho$ is the nuclear matter density.

We also include a p-wave contribution to the $\bar{K}$ self-energy coming from the coupling of the $\bar{K}$ meson to hyperon-nucleon hole ($YN^{-1}$) excitations, with $Y = \Lambda, \Sigma, \Sigma^*(1385)$. The vertices $MBB'$ are easily derived from the $D$ and $F$ terms of Eq. (1). The explicit expressions can be seen in [12].

At this point it is interesting to recall three different approaches to the question of the $\bar{K}$ selfenergy in the nuclear medium. The first interesting realization was the one in [13, 14, 15], where the Pauli blocking in the intermediate nucleon states induced a shift of the $\Lambda(1405)$ resonance to higher energies and a subsequent attractive $\bar{K}$ selfenergy. The work of [16] introduced a novel and interesting aspect, the selfconsistency. Pauli blocking required a higher energy to produce the resonance, but having a smaller kaon mass led to an opposite effect, and as a consequence the position of the resonance was brought back to the free position. Yet, a moderate attraction on the kaons still resulted, but weaker than anticipated from the former work. The work of [12] introduces some novelties. It incorporates the selfconsistent treatment of the kaons done in [16] and in addition it also includes the selfenergy of the pions, which are let to excite $ph$ and $\Delta h$ components. It also includes the mean field potentials of the baryons. The obvious consequence of the work of [12] is that the spectral function of the kaons gets much wider than in the two former approaches because one is including new decay channels for the $\bar{K}$ in nuclei.

In the work of [17] the kaon selfenergy discussed above has been used for the case of kaonic atoms, where there are abundant data to test the theoretical predictions. One uses the Klein Gordon equation and obtains two families of states. One family corresponds to the atomic states, some of which are those already measured, and which have energies around or below 1 MeV and widths of about a few hundred KeV or smaller. The other family corresponds to states which are nuclear deeply bound states, with energies of 10 or more MeV and widths around 100 MeV.
With the $K^-$ many body decay channels included in our approach, the resulting widths of the deeply bound $K^-$ states (never bound by more than 50 MeV) are very large (of the order of 100 MeV) and, hence, there is no room for narrow deeply bound $K^-$ states which appear in some oversimplified theoretical approaches. A recent phenomenological work [18] considering the $K^-NN \rightarrow \Lambda\Sigma NN$ nuclear kaon absorption channels, which are also incorporated in [12], also reaches the conclusion that in the unlikely case that there would be deeply bound kaonic atoms they should have necessarily a large width.

8. $\phi$ decay in nuclei

Let us say a few words about the $\phi$ decay in nuclei. The work reported here [19] follows closely the lines of [20, 21], however, it uses the updated $\bar{K}$ selfenergies of [12]. In the present case the $\phi$ decays primarily in $K\bar{K}$, but these kaons can now interact with the medium as discussed previously. For the selfenergy of the $K$, since the $K\Lambda$ interaction is not too strong and there are no resonances, the $t\rho$ approximation is sufficient. In fig. 2 we show the results for the $\phi$ width at $\rho = \rho_0$ as a function of the mass of the $\phi$, separating the contribution from the different channels. What we observe is that the consideration of the s-wave $\bar{K}$-selfenergy is responsible for a sizable increase of the width in the medium, but the p-wave is also relevant, particularly the $\Lambda\Sigma$ excitation and the $\Sigma^*h$ excitation. It is also
interesting to note that the vertex corrections \cite{22} (Yh loops attached to the φ decay vertex) are now present and do not cancel off shell contributions like in the case of the scalar mesons. Their contribution is also shown in the figure and has about the same strength as the other p-wave contributions. The total width of the φ that we obtain is about 22 MeV at ρ = ρ_0, about a factor two smaller than the one obtained in \cite{20,21}. A recent evaluation of the φ self-energy in the medium \cite{23} similar to that of \cite{19}, in which also the real part is evaluated, leads to a width about 20 percent larger than that of \cite{19}. The important message from all these works is, however, the nearly one order of magnitude increase of the width with respect to the free one.

9. η selfenergy and eta bound in nuclei

The method of \cite{12} has also been used recently to determine the η selfenergy in the nuclear medium \cite{24}. One obtains a potential at threshold of the order of (-54 -i29) MeV at normal nuclear matter, but it also has a strong energy dependence due to the proximity of the N*(1535) resonance and its appreciable modification in the nuclear medium.

To compute the η–nucleus bound states, we solve the Klein-Gordon equation (KGE) with the η–selfenergy, Π_η(k^0, r) = Π_η(k^0, θ(r), ρ(r)), obtained using the local density approximation. We have then:

\[
\left[-\nabla^2 + μ^2 + Π_η(\text{Re}[E], r)\right] Ψ = E^2 Ψ
\]

where μ is the η–nucleus reduced mass, the real part of E is the total meson energy, including its mass, and the imaginary part of E, with opposite sign, is the half-width Γ/2 of the state. The binding energy B < 0 is defined as B = Re[E] − m_η.

The results from \cite{25} are shown in Table 1 for the energy dependent potential. On the other hand we see that the half widths of the states are large, larger in fact than the binding energies or the separation energies between neighboring states.

With the results obtained here it looks like the chances to see distinct peaks corresponding to η bound states are not too big.
On the other hand one can look at the results with a more optimistic view if one simply takes into account that experiments searching for these states might not see them as peaks, but they should see some clear strength below threshold in the $\eta$ production experiments. The range by which this strength would go into the bound region would measure the combination of half width and binding energy. Even if this is less information than the values of the energy and width of the states, it is by all means a relevant information to gain some knowledge on the $\eta$ nucleus optical potential.

10. Experiments to determine the $\phi$ width in the medium

Recently there has been an experiment [26] designed to determine the width of the $\phi$ in nuclei. Unlike other experiments proposed which aim at determining the width from the $K^+K^-$ invariant mass distribution and which look extremely difficult [28, 29], the experiment done in Spring8/Osaka uses a different philosophy, since it looks at the A dependence of the $\phi$ photoproduction cross section. The idea is that the $\phi$ gets absorbed in the medium with a probability per unit length equal to

$$-\text{Im}\Pi/q$$

where $\Pi$ is the $\phi$ selfenergy in the medium ($\Gamma = -\text{Im}\Pi/\omega_\phi$). The bigger the nucleus the more $\phi$ get absorbed and there is a net diversion from the A proportionality expected from a photonuclear reaction. The method works and one obtains a $\phi$ width in the medium which is given in [26] in terms of a modified $\phi N$ cross section in the nucleus sizeably larger than the free one. Prior to these experimental results there is a theoretical calculation in [27] adapted to the set up of the experiment of [26], based on the results for the $\phi$ selfenergy reported above. The results agree only qualitatively with the experimental ones, the latter ones indicating that the $\phi$ selfenergy in the medium could be even larger than the calculated one. However, although it has been fairly taken into account in the experimental analysis, there is an inconvenient in the reaction of [26], since there is a certain contamination of coherent $\phi$ production which blurs the interpretation of the data.

In order to use the same idea of the A dependence and get rid of the coherent $\phi$ production, a new reaction has been suggested in [30] which could be implemented in a facility like COSY. The idea is to measure the $\phi$ production cross section in different nuclei through the reaction

$$pA \rightarrow \phi X$$

The calculation is done assuming one step production, and two step production with a nucleon or $\Delta$ in the intermediate states, allowing for the loss of $\phi$ flux as the $\phi$ is absorbed in its way out of the nucleus. Predictions for the cross sections normalized to the one of $^{12}C$ are shown in fig. 3 where it is shown that with a precision of 10 percent in the experimental ratios one could disentangle between the different curves in the figure and easily determine if the width is one time, two
times etc, the width determined in ref. [19, 23] which is about 27 MeV for normal nuclear matter density. An experiment of this type can be easily performed in the COSY facility, where hopefully it will be done in the near future.

![Graph](image)

**Fig. 3.** Ratio of the nuclear cross section normalized to $^{12}\text{C}$ for $T_p = 2.83$ GeV multiplying the $\phi$ width in the medium, $\Gamma$ around 27 MeV, by different factors.

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