Implications of Dynamical Generation of Standard-Model Fermion Masses

Neil D. Christensen and Robert Shrock
C.N. Yang Institute for Theoretical Physics
State University of New York
Stony Brook, NY 11794

We point out that if quark and lepton masses arise dynamically, then in a wide class of theories the corresponding running masses \( m_f(p) \) exhibit the power-law decay \( m_f(p) \propto \Lambda_j^2/p^2 \) for Euclidean momenta \( p \gg \Lambda_j \), where \( f \) is a fermion of generation \( j \), and \( \Lambda_j \) is the maximal scale relevant for the origin of \( m_f \). We estimate resultant changes in precision electroweak quantities and compare with current data. It is found that this data allows the presence of such corrections. We also note that this power-law decay renders primitively divergent fermion mass corrections finite.

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The origin of fermion masses in the standard model (SM) remains mysterious. This model accommodates quark and charged lepton masses via Yukawa couplings to a postulated Higgs boson, but this does not provide insight into these masses, especially since it requires small dimensionless Yukawa couplings for all of the fermions except the top quark, ranging down to \( 10^{-6} - 10^{-5} \) for the first generation, with no explanation. This suggests that one consider models in which quark and lepton masses arise dynamically. The standard model also predicted zero neutrino masses, and has had to be augmented to incorporate the observed nonzero neutrino masses. The standard model also predicted such power-law decays of SM fermion masses are allowed for Euclidean \( p \) large compared to the largest scale \( \Lambda_j \) involved in this mass generation. This renders proper fermion self-energy corrections finite, rather than divergent. An important question for these theories is whether such power-law decays of SM fermion masses are allowed by current precision data. We address this question here and answer it in the affirmative.

We consider a class of theories in which (i) the physics underlying the generation of lepton and (current) quark masses involves the formation of a set of bilinear electroweak symmetry-breaking (EWSB) fermion condensates, denoted generically as \( \langle \psi \psi \rangle \) at a given scale \( \Lambda_{EW} \), (ii) there are interactions connecting the \( \psi \)’s with SM fermions to communicate the EWSB to these fermions, and (iii) the interactions responsible for this mass generation are asymptotically free, so that at sufficiently large momenta all couplings are small, and hence \( \langle \psi \psi \rangle \) and other relevant operators have essentially canonical dimensions with small anomalous dimensions.

We begin by showing the power-law decay of the running mass of an SM fermion in two classes of models satisfying our premises above and then give a general argument. First, consider extended technicolor (ETC) models \( \mathbb{3}, \mathbb{4}, \mathbb{5} \) containing a subsector with a set of massless fermions \( \{F\} \) subject to an asymptotically free, vectorial, confining gauge interaction denoted technicolor (TC) \( \mathbb{3}, \mathbb{5} \). As the momentum decreases from high values to the scale \( \Lambda_{TC} = \Lambda_{EW} \sim 250 \text{ GeV} \), the TC gauge coupling grows sufficiently to produce EWSB technifermion condensates \( \langle FF \rangle \equiv \langle \psi \psi \rangle \). The theory includes a set of vector bosons that transform SM fermions into technifermions, thereby communicating the EWSB to the SM fermions and dynamically generating masses for them. These ETC gauge bosons are denoted \( V_i^j \), where \( j = 1, 2, 3 \) is a generation (family) index and \( t \) is a technicolor index. The ETC group is an asymptotically free, anomaly-free, chiral gauge theory that breaks in stages at the scales \( \Lambda_j \), to the residual exact TC gauge group. Typical values used in recent studies are \( \Lambda_1 \approx 10^3 \text{ TeV} \), \( \Lambda_2 \approx 10^2 \text{ TeV} \), and \( \Lambda_3 \approx 4 \text{ TeV} \) \( \mathbb{3}, \mathbb{4}, \mathbb{5} \). This hierarchy in breaking scales produces a hierarchy in the resultant quark and lepton masses, which have inverse power dependence on these scales. Although fully realistic ETC models have not yet been constructed, and they are subject to strong constraints such as those from precision electroweak data, they serve as useful explicit examples of dynamical fermion mass generation.

To investigate the dependence of the running mass \( m_f(p) \) on momentum, consider the one-loop diagram in which \( f_j(p) \) emits a virtual \( V_i^j \) with momentum \( p-k \) and mass \( M_j \approx \Lambda_j \), transforming to a technifermion \( F^t \) which reabsors the \( V_i^j \). We neglect fermion mixing to start. After Wick rotation, this diagram yields

\[
m_f(p) \sim g_{ETC} N_{TC} \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma_{TC}(k)}{|k^2 + \Sigma_{TC}(k)^2|[(p-k)^2 + M_j^2]^{3/2}}
\]

(1)

where \( \Sigma_{TC}(k) \) is the dynamical technifermion mass \( \mathbb{13} \), with \( \Sigma_{TC}(k) = \Sigma_{TC,0} \approx 2\Lambda_{TC} \) for Euclidean \( k \ll \Lambda_{TC} \). In early TC theories, \( \Sigma_{TC}(k) \) had the power-law decay \( \Sigma_{TC} \approx \Sigma_{TC,0}/k^2 \) for \( k^2 \gg \Lambda_{TC}^2 \), analogous to the momentum dependence of the constituent quark mass in quantum chromodynamics (QCD) \( \mathbb{14} \), with \( \Lambda_{QCD} \) re-
placed by $\Lambda_{TC}$. Current TC theories rely upon a TC gauge coupling that runs slowly (walks) \cite{10} in the interval $\Lambda_{TC} \lesssim k \lesssim \Lambda_{w}$, where typically $\Lambda_{w} \simeq \Lambda_{3}$. In walking TC theories, $\Sigma_{TC}(k)$ falls like $\Sigma_{TC}^{2}/k^{2}$ for $\Lambda_{TC} \lesssim k \lesssim \Lambda_{w}$ and like $\Sigma_{TC}^{2}/k^{2}$ for larger $k$. This yields the pole mass $m_{f} \sim \kappa q \Lambda_{w}^{2}/\Lambda_{3}^{2}$, where $\kappa \sim O(10)$ is a numerical factor (see, e.g., \cite{11} and $\eta$ is a walking factor \cite{12}. The result can be shown from an approximate infrared fixed point in the TC theory; it enhances SM fermion masses, raises pseudo-Nambu-Goldstone boson masses, and can reduce TC contributions to the electroweak $\Sigma$ parameter \cite{13}.

Expanding eq. \textit{4} for small $p$ and using $M_{f}^{2} \gg \Sigma_{TC}^{2}$, we find

$$m_{f}(p) \simeq m_{fj} \left(1 - a_{sp} \frac{p^{2}}{\Lambda_{3}^{2}} \right) \left(2\right)$$

for $p^{2} \ll \Lambda_{3}^{2}$, where $a_{sp}$ is a positive constant $\sim O(1)$ depending on the dynamics responsible for the fermion masses, and we neglect possible $p$-dependent log factors. From eq. \textit{4}, it is evident that softness effects for $m_{f}$ become significant for $p \sim \Lambda_{j}$.

Expanding eq. \textit{4} for $p \gg \Lambda_{j}$, we find

$$m_{f}(p) \simeq m_{fj} \frac{\Lambda_{3}^{2}}{p^{2}} \left(3\right)$$

Here we neglect factors $(\ln(p^{2}/\Lambda_{3}^{2}))^{n/2}$ arising from the anomalous dimensions of the full operator $f_{j}f_{k}\bar{\psi}\psi$, since these are subdominant compared with the power-law decay. Since this $p$-dependence holds above the walking interval $\Lambda_{TC} \lesssim p \lesssim \Lambda_{3}$, it is essentially independent of the presence of walking, the effect of which is contained in $\Sigma_{TC}(k)$ and the pole mass, $m_{fj}$. Hence, one can verify that this type of model yields eq. \textit{4} by inserting an explicit approximate functional form such as $\Sigma_{TC}(k) = \Sigma_{TC}(1 + k^{2}/\Lambda_{3})$ in eq. \textit{4}. Because the TC theory is strongly coupled, higher-loop diagrams are also important; these are subsumed in the low-energy effective Lagrangian containing four-fermion terms of the form $f_{j}f_{k}FF$. Given our premises, these do not significantly modify eq. \textit{4}. As indicated, mixing terms can be included, as in Refs. \cite{11,11,12}. While it is challenging to construct ETC models leading to fully realistic quark mixing, plausible models with small off-diagonal entries in $M_{j}^{(u)}$ and $M_{j}^{(d)}$ maintain the generational dependence of the falloff in eq. \textit{4}.

A second example is provided by topcolor-assisted technicolor (TC2) models \cite{10,15}. For these the set of $\langle \bar{\psi}\psi \rangle$ includes both technifermion condensates and condensates directly involving the $t_{L}$ and $t_{R}$ fields, which contribute importantly to both EWSB and the top quark mass. The relevant scale $\Lambda_{t}$ for the latter condensate(s) is of order a TeV, $\sim \Lambda_{3}$. Since theories of this type can satisfy our premise, it follows that eqs. \textit{4} and \textit{4} hold.

The hierarchy of scales $\Lambda_{3} < \Lambda_{2} < \Lambda_{1}$ means that the power-law decay of $j = 3$ fermion mass sets in at momenta where the mass matrix elements $M_{jk}^{(f)}$ with $j,k \in \{1,2\}$, are, up to logs, still largely momentum-independent (hard). Up to small mixing effects, as $p$ increases above $\Lambda_{3}$, the $t$, $b$, and $\tau$ running masses begin to decay as in \textit{4} with $j = 3$; then as $p$ increases above $\Lambda_{2}$, the $c$, $s$, and $\mu$ masses fall off like $\Lambda_{2}^{2}/p^{2}$, and finally, for $p > \Lambda_{1}$, the $u$, $d$, and $e$ masses fall off like $\Lambda_{1}^{2}/p^{2}$.

We next give a general analysis of the behavior of $m_{fj}(p)$ in a theory with dynamical fermion mass generation satisfying our premises. The relevant low-energy effective Lagrangian contains the terms

$$\mathcal{L}_{eff} = \sum_{f,j,k,\psi} \bar{b}_{jk}^{(f)} f_{jL} f_{kR} \bar{\psi}\psi + h.c., \left(4\right)$$

where $f$ denotes a quark or charged lepton, $j,k \in \{1,2,3\}$ are generation indices, and we suppress other terms involving Fierz rearrangements \cite{16}. The existence of the condensate(s) $\langle \bar{\psi}\psi \rangle$ yields the bilinear mass terms $f_{jL} M_{jk}^{(f)} f_{kR} + h.c.$, where $M_{jk}^{(f)} = b_{jk}^{(f)} \langle \bar{\psi}\psi \rangle$. Diagonalizing this matrix $M_{jk}^{(f)}$, one obtains the (physical, pole) masses $m_{fj}$. To account for the generational hierarchy in SM fermion masses, the dynamics should produce $b_{jk}^{(f)}$ such that $m_{fj} = b_{jk}^{(f)} \langle \bar{\psi}\psi \rangle = c_{jk}^{(f)} \langle \bar{\psi}\psi \rangle / \Lambda_{j}^{2}$ with $\Lambda_{3} < \Lambda_{2} < \Lambda_{1}$ (where a possible TC2 $\Lambda_{1}$ is subsumed as $\sim \Lambda_{3}$). Now, performing the diagonalization at a Euclidean momentum $p$ to obtain the running masses $m_{fj}(p)$, and using the asymptotic freedom property, we guarantee that for large $p$, $\langle \bar{\psi}\psi \rangle$ has operator dimension $d = 3$ with small anomalous dimensions, it follows that for $p \gg \Lambda_{j}$, $m_{fj}(p) \sim c_{jk}^{(f)} \langle \bar{\psi}\psi \rangle / p^{2}$; substituting for $c_{jk}^{(f)}$, one gets eqs. \textit{4} \textit{20} \textit{21}. The generational dependence of the power-law decays of running neutrino masses is more complicated, since it involves both Dirac and Majorana masses of different scales and also generically large mixing effects, as suggested by the observed large leptonic mixing angles.

For the theories considered here, although these power-law decays of SM fermion masses can ultimately be traced to the presence of bilinear fermion condensate(s), they are distinctively different from the well-studied softness of constituent quark masses $\Sigma$ in QCD \cite{17,14} and the softness of dynamical technifermion masses $\Sigma_{TC}$ in either scaled-up QCD-like or modern walking technicolor theories \cite{15,16}. In all three of the latter cases, this softness sets in on the scale of the dynamical mass itself, which is the scale where the respective QCD or TC gauge interaction gets strong and breaks the chiral symmetry. In contrast, the standard-model fermions exhibit softness on scales which (i) are higher than their pole masses, indeed many orders of magnitude higher for the first and second generations; (ii) are associated with the communication of the dynamical EWSB sector to these fermions, and (iii) have a generational hierarchy.

We now use eq. \textit{4} as a new test of theories with dynamical fermion mass generation. We ask the question: Is current precision electroweak data consistent with such power-law decays of SM fermion masses? One of the
clearest tests is provided by SM fermion loop corrections to $W$ and $Z$ self-energies. Standard-model contributions including those due to fermions, are considered to be subtracted in defining the oblique electroweak correction parameters $S$, $T$, and $U$, so that nonzero values of these parameters indicate new physics [6], [22]-[26]. These SM fermion loop contributions are calculated assuming constant fermion masses.

Consider the parameter $\rho = m^2_W / (m^2_Z \epsilon^2)$ [27], where $\epsilon^2 = \cos^2 \theta_{W,\text{MS}} = 1 - s^2$. We focus on the contribution to $\rho$ of the $(t, b)$ quarks, which is the largest among SM fermions. The conventional (one-loop, 1$t$) result for this is [27] $(\Delta \rho)_{\text{th,hard},1t} = \left( 1 - a_T \frac{m^2_{t,b}}{\Lambda^2} \right)$, where $a_T$ is the electromagnetic coupling. In theories with dynamical generation of SM fermion masses, to leading order in $m_t/\Lambda_3$, we find from eq. (5) that

$$(\Delta \rho)_{tb} = (\Delta \rho)_{\text{th,hard}} \left[ 1 - a_T \left( \frac{m^2_{t,b}}{\Lambda^2} \right) \right], \quad (5)$$

where $a_T$ is a positive coefficient $\sim O(1)$ depending on the dynamics responsible for the generation of these fermion masses. The positivity of $a_T$ follows from the fact that the softness of the top quark mass reduces the violation of the custodial SU(2) symmetry and hence the value of $\Delta \rho$. In theories (e.g., TC2) with topcolor, corrections of order $(m_t/\Lambda)^2$ could also be important. Numerically, $(m_t/\Lambda)^2 = 0.03(1 \text{ TeV}/\Lambda)^2$, so that with $\Lambda_3$ on the few TeV scale, the fractional reduction of $\Delta \rho_{tb}$ in eq. (6) is of order $10^{-2}$ - $10^{-3}$.

In passing, we note that the correction [25] is comparable to some of the terms, such as those $\propto x_i^2$, where $x_i = G_F m^2_i / (8\pi^2 \sqrt{2})$, in the two-loop contribution to $\Delta \rho_{\text{th,hard}}$ [25], and to the overall three-loop contribution [28] (including $a_T x_i$ terms $\sim 10^{-3}$) to $\Delta \rho_{tb,\text{hard}}$.

Thus for theories with dynamical generation of SM fermion masses, the conventional subtraction, using the constant-mass expression, to get $T = \Delta \rho^{\text{(new)}} / \hat{\rho}(m_Z^2)$, where $\Delta \rho^{\text{(new)}}$ denotes the change in $\rho$ due to new physics (and $\hat{\rho}(m_Z^2)$ is the electromagnetic coupling), would be a slight oversubtraction. Using a definition of $T$ with the subtraction of the actual, rather than the hard-mass, contribution of $(t, b)$ and correcting for the above oversubtraction thus slightly shifts the allowed region in $T$ upward, by the amount

$$\Delta T_{tb,\text{soft}} \simeq 1.3 a_T \frac{m^2_{t,b}}{\Lambda^3}. \quad (6)$$

The (1-loop) contribution of the hard-mass $(t, b)$ doublet to the full $S$ parameter before subtraction (bf) is

$$S^{(bf)}_{\text{th,hard},1t} \simeq \frac{N_c}{12\pi} \left[ -\frac{1}{9} + \frac{7m^2_Z}{30m^2_t} - \frac{4}{3} \ln \left( \frac{m_t}{m_Z} \right) \right], \quad (7)$$

equal to $-0.075$. Modelling a soft $t$-quark mass approximately by a small decrease in an effective $m_t$ makes this expression slightly less negative. As with $T$, defining $S$ with the subtraction of the actual, rather than the hard-mass, $(t, b)$ contribution and correcting for the difference yields a small shift downward in the allowed region in $S$, by the amount

$$\Delta S_{tb,\text{soft}} \simeq -0.1 a_T \frac{m^2_{t,b}}{\Lambda^3}, \quad (8)$$

where $a_S$ is a positive constant depending on the interactions responsible for the dynamical generation of SM fermion masses and expected to be $\sim O(1)$.

Current global fits yield allowed regions in $(S, T)$ depending on a reference value of the SM Higgs mass, $m_{H,\text{ref}}$. Since theories that dynamically generate SM fermion masses usually also have dynamical electroweak symmetry breaking without any fundamental Higgs, it is appropriate to insert an effective scale of order a TeV for $m_{H,\text{ref}}$. The corresponding allowed region for $m_{H,\text{ref}} = 1$ TeV shown in [6] is roughly elliptical, with semimajor axis having positive slope and central values $(S, T) \simeq (0.21, 0.15)$. The uncertainties listed (for a given $m_{H,\text{ref}}$) are $\pm 0.10$ in $S$ and $\pm 0.12$ in $T$. For plausible values of $\Lambda_3$, the shifts in eqs. (8) and (6) are quite safely smaller than these uncertainties. The corrections to $U$ are also negligibly small.

Thus, we find that this current precision electroweak data is consistent with the power-law decays of running SM fermion masses, eq. (8), that occur in theories with dynamical fermion mass generation. Indeed, we find that these effects are safely small compared with other contributions that can be expected in such theories, such as technifermion loop contributions to $S$ in TC and TC2 theories [17, 18]. An interesting aspect of the corrections [6] and [8] is that even in what are ostensibly standard-model inputs to these precision electroweak quantities there is already a “hidden” effect of this new physics - specifically, the softness of the SM fermion masses.

Soft standard-model fermion masses affect other one-loop processes where top quarks make important SM contributions, such as $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing, and $Z \rightarrow bb$ and $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ decays. Of these, $BR(Z \rightarrow bb)$ and the $\Delta m_{B_d}$ from $B_d - \bar{B}_d$ mixing are the most precisely measured, with fractional accuracies of 0.003 and 0.01, respectively. Soft top quark mass effects could be $\sim \text{few } \times 10^{-3}$ and are consistent with this data. Again, these processes also receive (model-dependent) contributions from the new physics [30]. In ETC models, certain ETC gauge boson exchanges contribute to the above processes, but do not couple to $W$ or $Z$ and hence do not directly affect $S$ or $T$ at one-loop level.

Corrections due to soft SM fermion masses are also present in precisely measured quantities involving first- and second-generation fermions. However, for the anomalous magnetic moments of the electron and muon, $a_e$ and $a_\mu$, these corrections are expected to go like $m_e^2/\Lambda^2$ and $m_\mu^2/\Lambda^2$, respectively. These are much smaller than the respective fractional measurement accuracies of
about $10^{-9}$ and $10^{-6}$ for $a_e$ and $a_{\mu}$. Specific dynamical models yield other corrections, e.g., $[11]$. Finally, we note an important theoretical implication. Consider the one-loop (primitively divergent) corrections to a SM fermion propagator. These yield an inverse fermion propagator $S_f(p)^{-1} = A_f(p)\hat{p} - B_f(p)$. In the SM the divergences in $A_f$ and $B_f$ are cancelled by wavefunction and mass renormalization so that, onshell, $S_f,\text{ren.}(p)^{-1} = \hat{p} - m_f$. However, the power-law decay $[6]$ renders $B_f$ finite. Thus, there is a change in what is divergent versus what is finite, which means a change in the renormalization procedure for the standard model viewed as the low-energy limit of a theory with dynamical fermion mass generation. This is in accord with the greater predictiveness of such theories.

In summary, in asymptotically free theories that dynamically generate SM fermion masses from underlying $\langle \bar{\psi}\psi \rangle$ condensate(s), we have shown that the running masses $m_f(p)\propto m_f/\Lambda_f$ for $p \gg \Lambda_f$. We have explored effects of this and have shown that it is consistent with current precision electroweak data.

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[13] In the TC theory, the inverse technifermion propagator is $S_T(p)^{-1} = A_T(p)\hat{p} - B_T(p) ≅ Z(p)^{-1}(\hat{p} - \Sigma_{TC}(p))$, where $A(p) = Z(p)^{-1} = 1$ in a Landau gauge. The TC gauge invariance of $\Sigma_{TC}(p)$ was shown in T. Appelquist, U. Mamanta, D. Nash, and L.C.R. Wijewardhana, Phys. Rev. D 43, 646 (1991); U. Mamanta, ibid. 45, 1405 (1992).
[19] An early study of four-fermion operators for dynamically generated fermion mass generation is Y. Nambu and G. Jonas-Lasino, Phys. Rev. 122, 345 (1961); here we require them to arise from an asymptotically free theory.
[20] For general $d$ and large $p$, the dynamical $m_f(p)\propto \langle \bar{\psi}\psi \rangle/p^{d-2}$ up to logs, so that the power-law decay is absent for $d = 2$, in agreement with the solution in D. Gross and A. Neveh, Phys. Rev. D 10, 3235 (1974).
[21] Note that eq. [8] would not apply to a hypothetical model in which the SM fermion mass hierarchy arose from a hierarchy in the condensate (mass)$^3$ values themselves, with a (larger) fixed mass $M$ describing the vector bosons that connect SM fermions with the set of $\{\bar{\psi}\}$; such a model could plausibly yield $m_f(p) ≈ m_fM^2/p^2$ for $p \gg M$. The necessity of our asymptotic freedom assumption is illustrated by the fractional power law obtained in the non-asymptotically free QED theory of Ref. [7].
[30] For example, these contributions differ considerably for the ETC models considered in Refs. [9,12] with vectorially or relatively conjugate representations for $Q = -1/3$ (techni)quarks and charged (techni)leptons.
(1999).