Cutoff effects in meson spectral functions

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We study the lattice spacing dependence of meson spectral functions calculated in quenched QCD with domain wall fermions as well as clover Wilson fermions in quenched and partially-quenched QCD. We conclude that for lattice spacing $a \leq 3$ GeV all excited states appearing in the spectral functions are lattice artifacts.

1. Introduction

Temporal meson correlation functions in Euclidean time can be related to meson spectral functions by analytic continuation. Furthermore using the Maximum Entropy Method one can extract the spectral functions from meson correlators calculated on a finite lattice \cite{1}. The method was applied at zero \cite{1,2} as well as non-zero temperature \cite{3,4,5,6}. In such an analysis one usually uses all time slices in the correlation function not just the long distance part. Therefore one is sensitive to short distance physics which may be affected by lattice artifacts. The lattice distortion of the correlators at short distances translates into lattice artifacts in the spectral functions for large energies. So far only Wilson gauge action and Wilson (clover) fermion action have been used to study meson spectral functions. The lattice artifacts seen in numerical simulations for this combination of actions turns out to be quite different from what is expected in the free theory \cite{7}. The purpose of this paper is to study cutoff effects of meson spectral functions using different gauge and fermion actions.

2. Numerical results

We consider the temporal correlators of meson currents

$$G(\tau, T) = \sum_{\vec{x}} \langle J(\vec{x}, \tau) J(\vec{0}, 0) \rangle,$$  \hspace{1cm} (1)

with $J = \bar{q}(\vec{x}, \tau) \Gamma q(\vec{x}, \tau)$ and $\Gamma = \gamma_5, \gamma_\mu$ for pseudo-scalar and vector channels. For temperature $T$ the spectral function is related to the meson correlator by the integral relation

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh \omega (\tau - 1/(2T))}{\sinh \omega/(2T)}.$$ \hspace{1cm} (2)

Although this relation is valid \textit{a priori} only in the continuum, it has been shown recently that in the limit of a non-interacting theory it holds on the lattice \cite{7} as well.

We use quenched domain wall fermion correlators on DBW2 gauge configurations from the RBC collaboration. Furthermore, we calculate

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{The pseudo-scalar (top) and vector (bottom) spectral functions with DWF at $a^{-1} = 2.0$ GeV. The vertical line and band show the mass of the ground and first excited states.}
\end{figure}
tree-level clover fermion correlators on quenched 1-loop tadpole-improved Symanzik lattices and on partially-quenched 2+1 flavor Asqtad fermion lattices from the MILC collaboration.

To extract the spectral function we use the Maximum Entropy Method. We also parametrize the spectral functions as a sum of delta functions

$$\sigma(\omega) = \sum_i F_i \delta(\omega^2 - m_i^2)$$

and determine the parameters $F_i$ and $m_i$ using constrained curve fitting techniques[8] which allows for better systematic control of the results.

We start the discussion of our numerical results with the case of domain wall fermions. Here we performed calculations at three different lattice spacings, $a^{-1} = 1.3, 2.0, 3.0$ GeV and valence quark mass around the strange quark mass. For the coarsest lattice we see a two peak structure in the spectral functions with the second peak located around 2GeV. The positions of the peaks are reasonably reproduced using a two exponential fit. In Fig. 1 we show our results for finer lattices, $a^{-1} = 2.0$ GeV for the pseudo-scalar and vector spectral functions. We see a three peak structure, with the first excited state being close to the position of the physical excited states of the $\eta$ and $\phi$ mesons. The third peak is likely to be a lattice artifact. We also performed a simple two exponential fit of the corresponding meson correlators. The results of the fit are shown as the vertical line and as a band. The ground state mass from the fit (statistical error $\sim 1\%$ not shown) is in reasonable agreement with the MEM analysis while for first excited state there is larger statistical uncertainty indicated by the band. To see whether or not the second and third peak correspond to physical resonances or lattice artifacts, we calculated the spectral functions at $a^{-1} = 3.0$ GeV. The results are shown in Fig. 2 which shows that the ground state peak did not shift (apart from a small shift due to slightly larger quark mass) while the second and the third peaks have moved roughly as $a^{-1}$. This indicates that these peaks are lattice artifacts. Similar results were found for the Wilson action [11,12]. In Ref. [2] they were interpreted as bound states of Wilson doublets. For DWF, however, such an interpretation is more difficult to apply. For this lattice spacing we have also performed a constrained fit with three exponentials. The ground state mass shown as the vertical line in Fig. 2 was obtained from a simple uncostrained fit which was then used as a prior for the constrained fit. The result for the second and third peak from the constrained fit is shown as bands in Fig. 2. Reasonable agreement between the MEM and constrained curve fits is found.

Figure 2. The pseudo-scalar (top) and vector (bottom) spectral functions with DWF at $a^{-1} = 3.0$ GeV. The vertical bands indicate the error on the peak location as determined from a constrained fit.

One may wonder whether the second and third peaks are artifacts of quenching. Absent dynamical quarks, all hadrons are absolutely stable (i.e. have zero width) no matter how large their mass is. If dynamical quarks are present, hadrons may decay, and the higher the mass the larger the width. Therefore higher lying resonances merge into a continuum. To check this we performed calculations in quenched and partially-quenched QCD using the clover action. More precisely we used gauge configurations with Symanzik ac-
tion generated by the MILC collaboration as well the dynamical 2+1 Asqtad configurations. In both cases the lattice spacing was $a \approx 0.09 fm$. We use the tree-level value for the clover coefficient, $c_{sw} = 1$, and $\kappa = 0.14$ which corresponds to $m_{PS}/m_{V} = 0.68$. Results for the spectral functions are shown in Fig. 3. As one can see, the spectral functions for quenched and partially quenched QCD are very similar, indicating that for quark masses around the strange quark mass effects due to quenching are quite small. In addition we have also calculated meson spectral functions wit HYP smeared clover action in 2 + 1 partially-quenched QCD, i.e. we have applied HYP smearing on the dynamical configurations before calculating the meson correlators. We have performed calculations at several quark masses. From extrapolation of the pseudo-scalar mass squared to zero we determine the critical hopping parameter to be $\kappa_c = 0.12718(2)$ (statistical error only). The purpose of calculation with HYP smearing was to see whether the smoothing of the gauge fields removes the artificial peaks, and the corresponding results are shown in Fig. 3. As one can see from the figure, the lattice artifacts are reduced but not removed completely by HYP smearing. However, as a by-product we observe that number of exceptional configurations as well as the number of CG in the fermions matrix inversions are greatly reduced. In fact we did not observe any exceptional configurations in a sample of 59 configurations down to quark masses $\sim m_s/8$ ($m_s$ is the strange quark mass).

Acknowledgment

This work has been authored under the contract DE-AC02-98CH10886 with the U.S. Department of energy. TB was partly supported by the LDRD program of Brookhaven National Laboratory, Project No. 04-041. We thank the RBC collaboration for the DWF correlators. The program for the MEM analysis was provided by Ines Wetzelorke to whom we are very grateful. The calculations were performed on the IBM-SP supercomputer at NERSC using gauge configurations from the MILC collaboration and the MILC code for which we are grateful.

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