Primordial Magnetic Fields from Out of Equilibrium Cosmological Phase Transitions

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Abstract. The universe cools down monotonically following its expansion. This generates a sequence of phase transitions. If a second order phase transition happens during the radiation dominated era with a charged order parameter, spinodal instabilities generate large numbers of charged particles. These particles hence produce magnetic fields. We use out of equilibrium field theory methods to study the dynamics in a mean field or large \( N \) setup. The dynamics after the transition features two distinct stages: a spinodal regime dominated by linear long-wavelength instabilities, and a scaling stage in which the non-linearities and backreaction of the scalar fields are dominant. This second stage describes the growth of horizon sized domains. We implement a formulation based on the non-equilibrium Schwinger-Dyson equations to obtain the spectrum of magnetic fields that includes the dissipative effects of the plasma. We find that large scale magnetogenesis is efficient during the scaling regime. Charged scalar field fluctuations with wavelengths of the order of the Hubble radius induce large scale magnetogenesis via loop effects. The leading processes are: pair production, pair annihilation and low energy bremsstrahlung, these processes while forbidden in equilibrium are allowed strongly out of equilibrium. The ratio between the energy density on scales larger than \( L \) and in the background radiation \( r(L, T) = \rho_B(L, T)/\rho_{cmb}(T) \) is \( r(L, T) \sim 10^{-34} \) at the Electroweak scale and \( r(L, T) \sim 10^{-14} \) at the QCD scale for \( L \sim 1 \) Mpc. The resulting spectrum is insensitive to the magnetic diffusion length and equipartition between electric and magnetic fields does not hold. We conjecture that a similar mechanism could be operative after the QCD chiral phase transition.

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EARLY COSMOLOGY AND FUNDAMENTAL PHYSICS

The history of the universe is determined by its expansion and consequent cooling. During most of its early history the Universe is homogeneous and isotropic to an excellent approximation and is therefore described by the spatially flat Friedmann-Robertson-Walker (FRW) geometry

\[
ds^2 = dt^2 - a^2(t) \, dx^2 \tag{1}
\]

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where the scale factor $a(t)$ grows with $t$. Physical lengths are proportional to $a(t)$ and the temperature decreases as $T(t) \sim \frac{1}{a(t)}$. This monotonous decrease of the temperature generates a sequence of cosmological phase transitions with the ensuing breaking of internal symmetries. The symmetry of the Universe reduces through each phase transition.

The main ingredients to describe the early Universe are:

- **General Relativity: Einstein’s Theory of Gravity**
  - The matter distribution determines the geometry of the spacetime through the Einstein equations. For the geometry eq. (1), the Einstein equations reduce to one scalar equation, the Einstein-Friedman equation
    \[
    \left[ \frac{1}{a(t)} \frac{da}{dt} \right]^2 = \frac{8\pi}{3} G \rho(t),
    \]
  where $G$ stands for Newton’s gravitational constant and $\rho(t)$ for the energy density.

- **Quantum Field Theory and String Theory to describe Matter**
  - Since the energy scale in the early universe is so high (well beyond the rest mass of particles), a quantum field theoretical description for matter is unavoidable. Only such context permits a correct description of particle production and particle decays.

  Electromagnetic, weak and strong interactions are well described by the so-called standard model. That is, quantum chromodynamics (QCD) combined with the electroweak theory (electromagnetic and weak interactions). This a non-abelian gauge theory associated to the symmetry group $SU(3) \otimes SU(2) \otimes U(1)$. The $SU(3)$ corresponding to the color group of QCD while $SU(2) \otimes U(1)$ describes the electroweak sector. To this scheme, one adds presently neutrino masses (through the see-saw mechanism) to explain neutrino oscillations.

  The energy scale in QCD is about $\sim 100\text{MeV} \simeq 10^{12}\text{K}$ corresponding to the chiral symmetry breaking and determined by the pion mass, while the energy scale for the electroweak is the Fermi scale $\sim 100\text{GeV} \simeq 10^{15}\text{K}$, which is determined by the mass of the vector bosons.

  The standard model has been verified experimentally with spectacular precision. However, it is an incomplete quantum field theory and it is the major challenge of our times to understand its extension. It seems obvious that extensions of the Standard model will be symmetric under a group containing $SU(3) \otimes SU(2) \otimes U(1)$ as a subgroup. Proposals for a Grand Unified Theory (GUT) include $SO(10)$, $SU(6)$ and $E_6$ as symmetry group.

  The grand unification idea consists in that at some energy scale all three couplings (electromagnetic, weak and strong) should become of the same strength. The running of the couplings with the energy (or the length) is governed by the renormalization group. For the standard model of electromagnetic, weak and strong interactions, the renormalization group yields that the three couplings get unified approximately at $\sim 10^{16}\text{GeV}$ \[1\]. A better convergence is obtained in supersymmetric extensions of the standard model\[1\].
Grand unified models may possess magnetic monopoles or not according to their symmetry group and to the symmetry of the ground state. Notice that no experimental evidence for magnetic monopoles has been found so far.

Quite generally, the internal symmetry increases with energy. This is true in general, in statistical mechanics, condensed matter as well as in cosmology. For example, a ferromagnet at temperatures higher than the Curie point is in the symmetric phase with zero magnetization. Below the Curie point, the internal symmetry is spontaneously broken by a non-zero spontaneous magnetization.

Current models purport that the universe started with maximal symmetry before inflation and this symmetry reduces gradually while the universe expands and cools. The symmetry breaking transitions includes both the internal symmetry groups (as the GUT’s symmetry group that eventually reduces to the $SU(3) \otimes SU(2) \otimes U(1)$ group) as well as the translational and rotational symmetries which are broken by the density fluctuations amplified by gravitational instabilities leading to structure formation.

It should be noticed, however, that no direct manifestation of supersymmetry is known so far. An indication emerges by studying the energy running of the (electromagnetic, weak and strong) in the standard model and in its minimal supersymmetric extension (MSSM). All three couplings meet at $E \simeq 2 \times 10^{16}$ GeV in the MSSM. The coupling unification becomes quite loose in the Standard Model. This is why the renormalization group running of the couplings in the MSSM supports the idea that supersymmetry would be a necessary ingredient of a GUT.

Neutrino oscillations and neutrino masses are currently explained in the see-saw mechanism as follows:

$$\Delta m_\nu \sim \frac{M_{\text{Fermi}}^2}{M}$$

where $M_{\text{Fermi}} \sim 250$ GeV is the Fermi mass scale, $M \gg M_{\text{Fermi}}$ is a large energy scale and $\Delta m_\nu$ is the difference between the neutrino masses for different flavors. The observed values for $\Delta m_\nu \sim 0.009 - 0.05$ eV naturally call for a mass scale $M \sim 10^{15\text{--}16}$ GeV close to the GUT scale.

The third evidence for an energy scale about $10^{16}$ GeV comes from inflation: data on the cosmic microwave background (CMB) anisotropies indicate that the inflation scale, the grand unification scale and the supersymmetry breaking scale actually coincide.

PRIMORDIAL SEEDS FOR THE MAGNETIC FIELDS IN THE UNIVERSE

A variety of astrophysical observations including Zeeman splitting, synchrotron emission, Faraday rotation measurements (RM) combined with pulsar dispersion measurements (DM) and polarization measurements suggest the presence of large scale magnetic fields. The strength of typical galactic magnetic fields is measured to be $\sim \mu G$ and they are correlated on very large scales up to galactic or even larger reaching to scales of cluster of galaxies $\sim 1$ Mpc. The origin of these large scale magnetic fields is still a subject of much discussion and controversy. It is currently agreed that a variety of dynamo mechanisms are efficient in amplifying seed magnetic fields with typical...
growth rates $\Gamma \sim \text{Gyr}^{-1}$ over time scales $\sim 10 - 12\ \text{Gyr}$ (for a thorough discussion of the mechanisms and models see\cite{5}). The ratio of the energy density of the seed magnetic fields on scales larger than $L$ (today) to that in the cosmic background radiation, $r(L) = \rho_B(L)/\rho_{\text{cmb}}$ must be $r(L \sim 1\text{Mpc}) \geq 10^{-34}$ for a dynamo mechanism to amplify it to the observed value, or $r(L \sim 1\text{Mpc}) \geq 10^{-8}$ for the seed to be amplified solely by the gravitational collapse of a protogalaxy\cite{3}.

There are also different proposals to explain the origin of the initial seed. Astrophysical batteries rely on gradients of the charge density concentration and pressure and their efficiency in producing seeds of the necessary amplitude is still very much discussed\cite{5}. Primordial magnetic fields that could be the seeds for dynamo amplification can be generated at different stages in the history of the early Universe, in particular during inflation, preheating and or phase transitions\cite{3}. Primordial (hyper) magnetic fields may have important consequences in electroweak baryogenesis\cite{7}, Big Bang nucleosynthesis (see\cite{5}), the polarization of the CMB\cite{8} via the same physical processes as Faraday rotation, and structure formation\cite{5,9}, thus sparking an intense program to study the origin and consequences of the generation of magnetic fields in the early Universe\cite{10-16}.

A reliable estimate of the amplitude and correlations of seed magnetic fields must include the dissipative properties of the plasma, in particular the conductivity\cite{11,13}. In ref.\cite{19} we have introduced a formulation that allows to compute the generation of magnetic fields from processes strongly out of equilibrium. This formulation, which is based on the exact set of Schwinger-Dyson equations for the transverse photon propagator is manifestly gauge invariant and is general for any matter fields and any cosmological background (conformally related to Minkowski space-time). In the case in which strongly out of equilibrium effects arise from long-wavelength fluctuations, such as during phase transitions, this formulation allows to separate the contribution of the hard degrees of freedom which are in local thermodynamic equilibrium from that of the soft degrees of freedom that fall out of LTE (local thermal equilibrium) during the phase transition and whose dynamics is strongly out of equilibrium. This separation of degrees of freedom leads to a consistent incorporation of the dissipative effects via the conductivity (for details see\cite{19}). In that reference a study of magnetogenesis in Minkowski space-time during a supercooled phase transitions was presented and the results highlighted the main aspects of the generation of magnetic and electric fields in these situations.

We study the generation of large scale (hyper) magnetic fields by a cosmological phase transition during a radiation dominated era by implementing the formulation introduced in ref.\cite{19}. The setting is a theory of $N$ charged scalar fields coupled to an abelian gauge field (hypercharge). We consider the situation when this theory undergoes a phase transition after the reheating stage and before either the Electroweak or the QCD phase transition, since we expect that these transitions will lead to new physical phenomena. The non-perturbative dynamics out of equilibrium is studied in the limit of a large number $N$ of (hyper) charged fields and to leading order in the gauge coupling. The non-equilibrium dynamics of the charged scalar sector features two distinct stages. The first one describes the early and intermediate time regime and is dominated by the spinodal instabilities which are the hallmark of the process of phase separation and domain formation and growth. This stage describes the dynamics between the time at which the phase transition takes place and that at which non-linearities become
important via the backreaction. The second stage corresponds to a scaling regime which describes the slower non-equilibrium evolution of Goldstone bosons and the process of phase ordering and growth of horizon-sized domains. This scaling regime is akin to the solution found in the classical evolution of scalar field models with broken continuous symmetries after the phase transition that form the basis for models of structure formation based on topological defects.

The solution of the scalar field dynamics is the input in the expression for the spectrum of the magnetic field obtained in to obtain the amplitude of the primordial seed generated during both stages.

We find that scaling stage is the most important for the generation of large scale magnetic fields. Large scale magnetic fields are generated via loop effects from the dynamics of modes that are at the scale of the horizon or smaller. The leading order processes that result in the generation of large scale magnetic fields are: i) pair production, ii) pair annihilation and iii) low energy bremsstrahlung. These processes would be forbidden in equilibrium by energy momentum conservation, but they are allowed strongly out of equilibrium because of the rapid time evolution of the cosmological background and the fast dynamics of the scalar field fluctuations.

The resulting spectrum is rather insensitive to the diffusion length scale which is much smaller than the horizon during the radiation dominated era. The ratio of the magnetic energy density on scales larger than \(L\) (today) to the energy density in the background radiation \(r(L, \eta) = \rho_B(L, \eta) / \rho_{\text{cmb}}(\eta)\) is summarized in a compact formula (eq. 20). For \(L \sim 1\) Mpc (today) we find \(r(L, \eta) \sim 10^{-34}\) at the Electroweak scale and \(r(L, \eta) \sim 10^{-14}\) at the QCD scale, suggesting the possibility that these primordial seeds could be amplified by dynamo mechanisms to the values of the magnetic fields consistent with the observed ones on these scales.

**THE PHYSICAL PICTURE**

The extreme energy scale and energy density during the inflationary and radiation dominated eras call for a quantum field theoretic treatment of the matter and radiation while the geometry is described by the classical metric eq. (1). The fast expansion of the universe can lead to out of thermal equilibrium situations, which require the implementation of out of equilibrium methods. In addition, nonperturbative methods are needed since the energy density is proportional to the inverse of the coupling. We developed nonperturbative field theory methods that successfully treats the inflationary era in various relevant scenarios and allowed to compute the primordial perturbations and the CMB fluctuations as well as to make contact with the customary slow-roll classical treatment.

We consider here a scalar field carrying an abelian charge and coupled to the electromagnetic field in the radiation dominated era. During the second order phase transition the concavity of the potential becomes negative at the origin and strong spinodal fluctuations are generated. These fluctuations in turn generate a magnetic field with a typical wavelength corresponding to the wavelength of the spinodally unstable modes.

This is the main premise of our work: the instabilities which are the hallmark of a non-equilibrium symmetry breaking phase transition lead to strong fluctuations of the
charged scalar fields which in turn, lead to the generation of magnetic fields through the non-equilibrium evolution.

The main ingredients developed in ref. [19] to compute the generation of magnetic fields through this non-equilibrium process were:

- A consistent framework to compute the spectrum of generated magnetic field, namely $(\vec{B}(\vec{k}, t) \cdot \vec{B}(-\vec{k}, t))/V$ with $\vec{B}(\vec{k}, t)$ the spatial Fourier transform of the Heisenberg magnetic field operator and $V$ the (comoving) volume of the system.
- Plasma effects were included to assess the generation and eventual decay of the magnetic fields. For a large conductivity in the medium, the magnetic field diffuses but also its generation is hindered. This point is of particular importance within the cosmological setting.[5, 13, 11].
- A major challenge of any mechanism of large scale magnetogenesis is to generate the seed magnetic fields from microscopic, causal processes. An important aspect of the results presented here is that this generation mechanism is mediated by loop effects and correspond to processes that are forbidden in equilibrium but allowed strongly out of equilibrium.

FIELD THEORETICAL MODEL FOR MAGNETIC FIELDS IN FRIEDMANN-ROBERTSON-WALKER COSMOLOGY

We will not attempt to study a particular gauge theory phenomenologically motivated by some GUT scenario, but will focus our study on a generic scalar field model in which the scalar fields carry an abelian charge. The simplest realization of such model is scalar electrodynamics with $N$ charged scalar fields $\phi_r, r = 1, \ldots, N$ and one neutral scalar field $\psi$ whose expectation value is the order parameter associated with the phase transition. The neutral field is not coupled to the gauge field and its acquiring an expectation value does not break the $U(1)$ gauged symmetry. This guarantees that the abelian gauge symmetry identified with either hypercharger or electromagnetism is not spontaneously broken to describe the correct low energy sector with unbroken $U(1)_{EM}$. We will take the neutral and the $N$ complex (charged) fields to form a scalar multiplet under an $O(2N + 1)$ isospin symmetry. The electromagnetic coupling explicitly breaks the $O(2N + 1)$ symmetry down to $SU(N) \times U(1)$. In the absence of electromagnetic coupling as the neutral field acquires an expectation value the isospin symmetry is spontaneously broken to $O(2N)$. Since by construction only the neutral field acquires a non vanishing expectation value under the isospin symmetry breaking the photon remains massless (it will obtain a Debye screening mass from medium effects).

The action that describes this theory in a general cosmological background and using conformally rescaled fields is given by

$$S = \int d\eta \, d^3x \left[ \frac{1}{2} \partial_\mu \Psi \partial^\mu \Psi + D_\mu \Phi^* D^\mu \Phi - M^2(\eta) \left( \frac{\Psi^2}{2} + \Phi^* \Phi \right) - \frac{\lambda}{4N} \left( \frac{\Psi^2}{2} + \Phi^* \Phi \right)^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right]$$

$$\tag{3}$$
with
\[ M^2(\eta) = -\mu^2C^2(\eta) - \frac{C''(\eta)}{C(\eta)} \quad , \quad D_\mu = \partial_\mu - ieA_\mu \quad ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (4) \]

and the primes refer to derivatives with respect to conformal time. Obviously the conformal rescaling of the metric and fields turned the action into that of a charged scalar field interacting with a gauge field in flat Minkowski space-time, but the scalar field acquires a time dependent mass term. In particular, in the absence of electromagnetic coupling, the equations of motion for the gauge field \( A_\mu \) are those of a free field in flat space time. This is the statement that gauge fields are conformally coupled to gravity and no generation of electromagnetic fields can occur from gravitational expansion alone without coupling to other fields or breaking the conformal invariance of the gauge sector. The generation of electromagnetic fields must arise from a coupling to other fields that are not conformally coupled to gravity, or by adding extra terms in the Lagrangian that would break the conformal invariance of the gauge fields[11].

The dynamics is determined by the Heisenberg equations of motion of the neutral field \( \Psi \) and the charged fields \( \Phi \)[17, 18, 19]. We will consider that at the onset of the radiation dominated era, the system is in the symmetric high temperature phase in local thermal equilibrium with a vanishing expectation value for the scalar fields. In the absence of explicit symmetry breaking perturbations the expectation value of the scalar field will remain zero throughout the evolution, thus \( \varphi \equiv 0 \).

It is convenient to introduce the mode expansion of the charged fields
\[ \Phi_r(\eta, \vec{x}) = \int \frac{d^3k}{\sqrt{2(2\pi)^3}} \left[ a_r(\vec{k}) f_k(\eta) e^{i\vec{k} \cdot \vec{x}} + b^*_r(\vec{k}) f^*_k(\eta) e^{-i\vec{k} \cdot \vec{x}} \right] , \quad r = 1, \ldots, N . \quad (5) \]

In leading order in the large \( N \) limit, the Heisenberg equations of motion for the charged fields translate into the following equations of motion for the mode functions for \( \eta > \eta_R \)[17, 18, 19]
\[ \left[ \frac{d^2}{d\eta^2} + k^2 - M^2(\eta) + \frac{\lambda}{2} \varphi^2(\eta) + \frac{\lambda}{2N} \langle \Phi^\dagger \Phi \rangle \right] f_k(\eta) = 0 . \quad (6) \]

With our choice of the initial state we find the backreaction term to be given by[19]
\[ \lambda \Sigma(\eta) \equiv \frac{\lambda}{2N} \langle \Phi^\dagger \Phi \rangle = \frac{\lambda}{4} \int \frac{d^3k}{(2\pi)^3} |f_k(\eta)|^2 [1 + 2n_k] . \quad (7) \]

This expectation value features quadratic and logarithmic UV divergences which are absorbed in the mass and coupling renormalization[18].

After renormalization and in terms of dimensionless quantities, the non-equilibrium dynamics of the charged scalar fields is determined by[17, 18, 19],
\[ \left[ \frac{d^2}{d\eta^2} + \mathcal{M}^2(\eta) + q^2 + \lambda \Sigma(\eta) \right] f_q(\eta) = 0 \quad ; \quad (8) \]

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2 Here we neglect the effect of the conformal anomaly[15].
with the effective time dependent mass given by

\[ \mathcal{M}^2(\eta) = C^2(\eta) \mu^2 \left[ \frac{T_R^2}{C^2(\eta)} - 1 \right] \]  
\[ T_c^2 = \frac{24 \mu^2}{\lambda + 3 \epsilon^2}. \]  

We see that \( \mathcal{M}^2(\eta) > 0 \) for early times when \( C(\eta) < T/T_c \). For later times \( C(\eta) > T/T_c \) and \( \mathcal{M}^2(\eta) \) becomes negative triggering the phase transition at a time \( \eta_c \) when \( C(\eta_c) = T/T_c \) since we can neglect the nonlinear contribution \( \lambda \Sigma(\eta) \) (recall that \( \lambda \ll 1 \)).

When \( \mathcal{M}^2(\eta) \) is negative, eq. (6) tells us that the modes \( f_q(\eta) \) grow exponentially as

\[ f_q(\eta) \sim a_q e^{\frac{1}{2} (\tilde{\mu} \eta)^2} (\tilde{\mu} \eta) \frac{q^2}{\tilde{\mu}^2 - 1} \left[ 1 + \mathcal{O} \left( \frac{1}{\tilde{\mu}^2 \eta^2} \right) \right] \]  

where \( \tilde{\mu} \equiv \sqrt{\mu H_R} \) and \( H_R \) is the Hubble constant at the reheating time, \( H_R = \eta_R^{-1} \).

This growth continues till the nonlinear term in eq. (8) \( \Sigma(\eta) = \frac{1}{2N} \langle \Phi^\dagger \Phi \rangle \) cannot be neglected anymore and stops the instabilities. An scale invariant stage follows. In this scaling stage the tree level mass term \( -C^2(\eta) \mu^2 \) is compensated by the backreaction of the quantum fluctuations as follows,

\[ \lambda \Sigma(\eta) - \mu^2 C^2(\eta) \eta \sim \frac{15}{4 \eta^2}. \]  

The mode functions can be written during the scaling stage in terms of Bessel functions

\[ f_k(\eta) = A_k \eta^{5/2} \frac{J_2(x)}{x^2} + B_k \frac{x^2 N_2(x)}{\eta^{3/2}}, \]  

where \( x = k \eta \) is the scaling variable. The correlation length of the scalar field is of the order of the Hubble radius in this stage.

**Gauge field dynamics**

The electric conductivity is very large in the high temperature plasma formed after the second order phase transition and dominates the dynamics of the gauge field. The conductivity is obtained from the imaginary part of the photon polarization and it is dominated by charged particles of momenta \( p \sim T \) in the loop with exchange of photons of momenta \( eT < k \ll T \). The effect of Debye (electric) and dynamical (magnetic) screening via Landau damping is crucial leading to the expression,

\[ \sigma(\eta) = \frac{\sigma_R}{C(\eta)}, \quad \sigma_R = \frac{e N T}{\alpha \ln \frac{1}{\alpha N}}. \]  

where \( e = 15.698 \ldots \) Such large conductivity leads to dissipative processes which severely hinders magnetogenesis (see) and also introduces the diffusion length scale which could limit the correlation of the magnetic fields that are generated.
As a consequence, we found for the photon causal correlator for $k \ll \sigma_R$

\[ \mathcal{D}_R(\eta, \eta'; k) = \theta(\eta - \eta') \frac{e^{-\frac{k^2}{\sigma_R^2}(\eta - \eta')}}{\sigma_R}. \]  

(15)

**MAGNETIC FIELD SPECTRUM**

The magnetic energy at wavenumber $k$ is given by the symmetric equal time limit

\[ S_B(\eta, k) = \frac{1}{2} \lim_{\eta' \to \eta} \int d^3x \langle \{ \hat{B}^i(\eta, \vec{x}), \hat{B}^i(\eta', \vec{0}) \} \rangle e^{i\vec{k} \cdot \vec{x}}, \]  

(16)

where $\{ , \}$ denotes the anti-commutator, $B(\eta, \vec{x})$ above is a Heisenberg operator and the expectation value is in the initial density matrix.

The physical magnetic energy density stored on comoving length scales larger than a given $L$ is given by

\[ \Delta \rho_B(L, \eta) = \frac{1}{2\pi^2} \int_0^{2\pi} k^2 S_B(\eta, k) \, dk. \]  

(17)

$\Delta \rho_B(L, \eta)$ stands for the contribution from the non-equilibrium generation (subtracting the local thermodynamic equilibrium contribution), a quantity of cosmological relevance to assess the relative strength of the generated magnetic field is given by the ratio of the power on scales larger than $L$ to the energy density in the radiation background

\[ r(L, \eta) = \frac{\Delta \rho_B(L, \eta)}{\rho_\gamma(\eta)}, \quad \rho_\gamma = \frac{\pi^2 T_\gamma^4}{15}. \]  

(18)

The explicit field theoretic evaluation of $S_B(\eta, k)$ is given in ref.\[19\]. The leading contribution to the power spectrum generated by non-equilibrium fluctuations results expressed in terms of the mode functions $f_k(\eta)$ as follows,

\[ S_B(\eta, k) = (1 + 2n_0)2 \frac{\alpha N}{\pi} k^2 e^{-\frac{k^2}{\sigma_R^2}} \int_0^{\infty} q^4 dq \, d(\cos \theta) \left( 1 - \cos^2 \theta \right) \left| \int_{\eta_R}^{\eta} e^{\frac{k^2}{\sigma_R^2} \eta_1} f_q(\eta_1) f_{\bar{q}+\vec{k}}(\eta_1) \, d\eta_1 \right|^2. \]  

(19)

where $\theta$ is the angle between the vectors $\vec{q}$ and $\vec{k}$ and we have used the result that both the spinodal stage as well as the scaling stage is dominated by the long-wavelength modes that acquire non-perturbatively large amplitudes\[19\].

We then compute $r(L, \eta)$ using the scaling solution eq.\[13\] for the mode functions. The result can be recasted for $L \gg \eta$ as\[19\],

\[ r(\eta, L) = 3.665 \times 10^4 \left[ \frac{\alpha}{\sigma_0} \right]^3 \left[ \frac{1}{L T R} \right]^5 \left[ \frac{\mu}{\sqrt{\lambda} T(\eta)} \right]^4 \left[ \frac{M_*}{T(\eta)} \right]^3. \]  

(20)
where
\[ T(\eta) = \frac{T_R}{C(\eta)} , \quad M_* \simeq \frac{1}{\sqrt{G}} \simeq M_{Pl} , \quad \sigma_0 = \frac{15.698 \ldots}{\ln \alpha} . \]

Several important features of the above result are noteworthy:

- **i:** A typical nonperturbative behaviour $\frac{1}{\sqrt{\lambda}}$ arising from the phase transition.
- **ii:** $r(\eta, L)$ grows with the symmetry breaking scale $\mu$ as $\mu^4$ since the higher is $\mu$ the longer is the scaling stage.
- **iii:** The presence of the huge suppression factor $[L T_R]^{-5}$.

For the symmetry breaking scale $\mu \sim 10^{13}$ Gev and $\lambda \sim \alpha \sim 10^{-2}$, corresponding to a critical temperature of order of a GUT scale $T_c \sim 10^{15}$ GeV and assuming that the scaling regime lasts until the electroweak phase transition scale, i.e. $\eta$ is such that $T(\eta) = T_{EW} \sim 10^2$ GeV. Then the factor
\[
\left( \frac{\mu}{\sqrt{\lambda T_{EW}}} \right)^4 \left( \frac{M_*}{T_{EW}} \right)^3 \sim 10^{100}
\]
compensates for the factor $[L T_R]^{-5}$. Taking $N$ and $g_*$ of the order of 10 we obtain
\[
r[T(\eta), L] \simeq 10^{-34} \left( \frac{L}{1 \text{ Mpc}} \right)^{-5} \left( \frac{T_{EW}}{T(\eta)} \right)^7 . \tag{21}
\]
Therefore,
\[
r[T(\eta), L] \sim \begin{cases} 10^{-34} & \text{at the EW transition} \\ 10^{-14} & \text{at the QCD transition} \end{cases} . \tag{22}
\]
Thus, the amplitude of the large scale magnetic fields turns to be within the range necessary to be amplified by dynamo models.

It must be noticed that eqs. (20)-(22) provide the long wavelength contributions to the magnetic field production which dominate for $L > 0.1 \text{ pc}$.

The results presented here (and in ref. [19]) arise from quantum loop effects taking into account quantum processes that only occur out of equilibrium. These quantum processes are classically forbidden. That is, they are virtual processes that take place off-shell.

In summary, the large scale primordial magnetic fields generated during the scaling stage after a phase transition are a plausible mechanism to generate primordial magnetic fields which are further amplified by the collapse of protogalaxies and by astrophysical dynamos.

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