Mirage cosmology with an unstable probe D3-brane

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We consider the mirage cosmology by an unstable probe brane whose action is represented by BDI action with tachyon. We study how the presence of tachyon affects the evolution of the brane inflation. At the early stage of the brane inflation, the tachyon kinetic term can play an important role in curing the superluminal expansion in mirage cosmology.

PACS numbers: 04.50.+h, 98.80.Cq, 11.10Kk
It is well known that the spectrum of string theory includes unstable non-BPS D-branes besides the stable BPS D-branes. The dynamics of unstable D-branes is described by the rolling of tachyon [1]. The action is represented by Born-Infeld type with decaying potential [2, 3, 4]. One can study the time dependent solutions where tachyon rolls from the top of the potential towards the minimum of the potential. Recently Sen showed that classical decay of unstable D-brane produces pressureless gas with non-zero energy density [5]. The possible effect of rolling tachyon to cosmology was considered by Gibbons [6]. He took into account the gravitational coupling by adding an Einstein-Hilbert term to the effective action and showed that the cosmological evolution of an accelerating universe can be obtained with the rolling of tachyon. This model attracted attention in connection with various issues of cosmology [7], despite the difficulty in the simplest version of this theory [8].

The brane world scenario opened a new frame to think about the problems in cosmology [9]. Many cosmological models based on the brane universe have been studied widely in connection with string theory [10, 11, 12, 13, 14, 15]. The idea of brane universe is that our observed universe is a three-brane (typically D-branes with maximal supersymmetry) embedded in a higher dimensional space. The model of our interest is the mirage cosmology suggested by Kehagias and Kiritsis [13]. The key idea of this model is that the motion of the brane through the bulk, ignoring its back reaction to the ambient geometry, induces cosmological evolution on the brane even when there is no matter field on the brane. The crucial mechanism underlying the construction of this formalism is the coupling of the probe brane to the background fields.

This model was studied extensively in various directions [16, 17, 18]. In the previous work of one of the authors and others [17], the motion of a three-brane in the background with tachyonic fields was studied in the framework of type 0B string theory. It is shown that the presence of tachyon in the background makes the effective matter density on the brane less divergent compared with the case when there is no tachyon in the background. In this letter we will consider the opposite case that there is tachyon field in the probe brane while there is no tachyon field in the background. The presence of tachyonic mode in a probe brane indicates an instability of the probe brane. The point of our interest is to observe how the presence of tachyon affects the evolution of the brane inflation. As a concrete example, we will consider the motion of a probe D3-brane in the bulk background configuration of $\text{AdS}_5 \times S^5$.

We consider a system of supergravity fields coupled to a (DBI+WZ)-type
unstable brane which contains tachyon. To describe the system in general, we need to consider the back reaction of the brane to the bulk geometry. However we assume the probe brane is light enough to neglect its back reaction. We consider the bulk geometry whose action, keeping only the bosonic part, is

\[
S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\phi}(R + 4(\nabla \phi)^2) - \frac{1}{2(n + 1)!} F_{n+1}^2 \right],
\]

(1)

where \( \phi \) is the dilaton field and \( F_{n+1} \) is the field strength tensor of RR potential \( C_n \) made by a stack of many stable D-branes. We will present our analysis with \( n = 4 \) case where the background geometry is \( \text{AdS}_5 \times S^5 \) form.

The metric can be parameterized as

\[
ds^2 = g_{00}(r)dt^2 + g(r)d\vec{x}^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5^2,
\]

(2)

where \((t, \vec{x})\) are the three brane coordinates and \( r \) is the radial coordinate perpendicular to the brane.

Unstable Dp-branes are described by rolling tachyon. Coupling tachyon effective action to gravity has been considered by many authors \([3, 19, 20]\). We start from the action of \([20]\) described by a (DBI+WZ)-type

\[
S_{p-\text{brane}} = -T_p \int dp^{p+1} \xi \left[ e^{-\phi V(T)} \sqrt{-\det(\hat{A}_{\mu \nu})} + f(T) dT \wedge \hat{C}_p \right],
\]

(3)

where

\[
\hat{A}_{\mu \nu} = A_{MN} \frac{\partial x^M}{\partial x^\mu} \frac{\partial x^N}{\partial x^\nu},
\]

\[
A_{MN} = g_{MN} + \partial_M T \partial_N T.
\]

Here \( M, N \) are the 10-dimensional bulk indices and \( \mu, \nu \) are the brane ones. We neglect all fields other than gravity and tachyon for simplicity. The forms of coupling \( V(T) \) and \( f(T) \) are not known precisely. Since the tachyon potential \( V(T) \) measures the varying tension, it should satisfy two boundary values such that \( V(T = 0) = 1 \) and \( V(T = \infty) = 0 \). Specific computation of string theory \([4]\) and a plausible argument based on it \([1]\) predict \( V(T) \sim e^{-T^2} \) for small \( T \) and \( V(T) \sim e^{-T} \) for large \( T \). The potential \( V(T) \) is a smooth function in the intermediate region and connects two asymptotic expressions. Thus the following shape of the potential is widely adopted in tachyonic cosmology for the sake of simplicity

\[
V(T) = \frac{V_0}{\cosh \left( \frac{T}{T_0} \right)},
\]

(4)
where $T_0$ is determined by string theory. We will focus on the early stage of the universe where the precise forms of $V(T)$ and $f(T)$ do not affect the argument significantly.

The WZ term of the brane action contributes only when there is three-from potential in the bulk. Since we consider the case when the probe brane is three-dimensional and there is only four-form potential in the bulk which makes the geometry of AdS$_5 \times S^5$, this term is neglected. Thus the three brane action can be simplified as

$$S_3 = -T_3 \int d^4 \xi e^{-\phi} V(T) \sqrt{-\det(\hat{A}_{\mu\nu})}. \quad (5)$$

The above brane action is a good approximation both for the perturbative vacuum region ($T = 0$) and stable vacuum region ($T = \infty$). The rolling tachyon solution describing the unstable D-branes is time reversal symmetric. So one can take the initial condition $\dot{T}(t = 0) = 0$ without loss of generality. Since $\dot{T} = 0$ initially, there can exist a time interval where $T$ remains fixed near its perturbative vacuum ($T = 0$). We confine our attention on this stage of the cosmological evolution.

To write down the action explicitly, we take the worldvolume coordinates $\xi^\alpha = x^\alpha$ ($\alpha = 0, 1, 2, 3$) in the static gauge. We assume that the tachyon field $T$ does not have spatial dependence $T = T(t)$. It has no $r$ dependence since tachyon exists only on the probe brane. In general, the motion of a probe D3-brane can have a nonzero angular momentum in the transverse directions. Then the Lagrangian of the brane can be expressed as

$$L = -T_3 V(T)e^{-\phi} \sqrt{g^3(|g_{00}(r)| - \dot{T}^2 - g_{rr}r^2 - g_S h_{ij}\dot{\phi}^i\dot{\phi}^j)}, \quad (6)$$

where the dot denotes the derivative with respect to $t$ and $h_{ij}\dot{\phi}^i\dot{\phi}^j$ is the line element on the unit five sphere ($i, j = 5, \ldots, 9$). For convenience of notation, we parameterize the Lagrangian, divided by three brane tension $T_3$, as

$$L = -\sqrt{A(r) - K(r)\dot{T}^2 - B(r)r^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}, \quad (7)$$

where

$$A(r) = V^2(T)g^3(r)|g_{00}(r)|e^{-2\phi},$$
$$K(r) = V^2(T)g^3(r)e^{-2\phi},$$
$$B(r) = V^2(T)g^3(r)g_{rr}(r)e^{-2\phi},$$
$$D(r) = V^2(T)g^3(r)g_S(r)e^{-2\phi}. $$
The momenta of the system are given by

\[ p_r = \frac{B(r)\dot{r}}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}}, \]

\[ p_i = \frac{D(r)h_{ij}\dot{\phi}^j}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}}. \]  

(8)

Calculating the Hamiltonian and demanding the conservation of energy, we have

\[ H = \frac{A(r)}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}} = E, \]

(9)

where \( E \) is the total energy of the brane. Also from the conservation of the total angular momentum \( h_{ij}p_ip_j = \ell^2 \), we have

\[ h_{ij}\dot{\phi}^i\dot{\phi}^j = \frac{\ell^2[A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2]}{D(r)[D(r) + \ell^2]}. \]

(10)

The equation of motion for the tachyon field is

\[
\partial_t \left( \frac{K\dot{T}}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}} \right) - \frac{VV''g^3e^{-2\phi}\dot{T}^2}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}} = 0,
\]

(11)

where \( V' = \frac{dV}{d\ell} \).

We can consider another form of WZ term \(-T_3 \int d^4\xi \hat{C}_4\) which does not contain tachyon. However it is known that the presence of this term results in the shift of the total energy of the brane \( \mathbb{13} \). Adding this four-form potential in the brane action \( \mathbb{3} \)

\[ S_3 = -T_3 \int d^4\xi [e^{-\phi}V(T)\sqrt{-\det(\hat{A}_{\mu\nu}) + \hat{C}_4}], \]

(12)

and defining \( C(r) = C_{0123}(r) \), the Lagrangian \( \mathbb{7} \) is modified to

\[ L = -\sqrt{A(r) - K(r)\dot{T}^2 - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j} + C(r). \]

(13)
From this, the Hamiltonian is calculated as

$$H = \frac{A(r)}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)r^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}} - C. \quad (14)$$

If we require the conservation of energy, the modification is just the shift of energy $E \rightarrow E + C$. It will be straightforward to extend the above result when we turn on the NS/NS field $B_{\mu\nu}$ and gauge field $F_{\mu\nu}$ on the probe brane.

In principle if we know the potential we can solve the coupled system of equations (9), (10) and (11), and use this result to proceed further. However, when the tachyon is rolling from the top of the potential where $dV/dT = 0$, we can neglect the second term as a first approximation. Then the role of tachyon on the probe brane is similar to that of a scalar field. In this case we have

$$\frac{KT}{\sqrt{A(r) - K(r)\dot{T}^2 - B(r)r^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}} \simeq Q, \quad (15)$$

where $Q$ is an integration constant. By solving Eqs (9), (10) and (15) in terms of $\dot{T}^2$, $r^2$ and $h_{ij}\dot{\phi}^i\dot{\phi}^j$, we have

$$\dot{T}^2 = \frac{Q^2 A^2}{K^2 E^2}, \quad (16)$$

$$r^2 = \frac{A}{B} \left(1 - \frac{KD + K\ell^2 + Q^2 D}{KD} \frac{A}{E^2}\right), \quad (17)$$

$$h_{ij}\dot{\phi}^i\dot{\phi}^j = \frac{\ell^2 A^2}{D^2 E^2}. \quad (18)$$

In the point of an observer living on the brane, the induced metric on the brane is the natural metric to see the evolution of its brane. The induced metric on the probe 3-brane universe is

$$ds^2_{4d} = (g_{00} + g_{rr}r^2 + g_{ij}\dot{\phi}^i\dot{\phi}^j)dt^2 + g(d\vec{r})^2. \quad (19)$$

Substituting Eqs. (17) and (18), the induced metric is calculated as

$$ds^2_{4d} = -\frac{g_{00}^2(g^2e^{-2\phi}V_0^2 + Q^2)}{E^2}dt^2 + g(d\vec{r})^2. \quad (20)$$

We define, for the standard form of a flat expanding universe, the cosmic time $\eta$ as

$$d\eta = \frac{|g_{00}|\sqrt{g^2e^{-2\phi}V_0^2 + Q^2}}{E}dt. \quad (21)$$
If we define the scale factor as \( a^2 = g \), we can calculate from the analogue of the four-dimensional Friedman equation the Hubble constant \( H = \dot{a}/a \)

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{E^2 - (g^3 e^{-2\phi} V_0^2 + Q^2 + \ell^2 g_{-1}^1) |g_{00}|}{4 |g_{00}| g_{rr}(g^3 e^{-2\phi} V_0^2 + Q^2)} \left( \frac{g'}{g} \right)^2, \tag{22}
\]

where the dot denotes the derivative with respect to cosmic time \( \eta \) and the prime denotes the derivative with respect to \( r \). The right hand side of Eq. (22) can be interpreted as the effective matter density on the probe brane

\[
\frac{8\pi}{3} \rho_{\text{eff}} = \frac{E^2 - (g^3 e^{-2\phi} V_0^2 + Q^2 + \ell^2 g_{-1}^1) |g_{00}|}{4 |g_{00}| g_{rr}(g^3 e^{-2\phi} V_0^2 + Q^2)} \left( \frac{g'}{g} \right)^2. \tag{23}
\]

The key idea of the above mechanism can be summarized as follows. Though the tachyon does not appear explicitly in the induced metric, it affects the cosmic time through the coupling with \( r \) and \( \phi \) equation.

As an application of the above formalism, we will consider the cosmology of the probe unstable D3-brane under the background configuration given by (1). The near-horizon geometry, corresponding to black hole solution, is AdS\(_5 \times S^5\) form with the metric

\[
d s^2 = \frac{r^2}{L^2} (-f(r) dt^2 + (d\vec{x})^2) + \frac{L^2}{r^2} \frac{dr^2}{f(r)} + L^2 d\Omega_5^2, \tag{24}
\]

where \( f(r) = 1 - (r_0/r)^4 \) and \( r_0 \) is the location of the horizon. The effective density on the probe D3-brane is calculated as

\[
\frac{8\pi}{3} \rho_{\text{eff}} = \frac{1}{a^2 (a^6 e^{-2\phi} V_0^2 + Q^2) L^2} \left[ E^2 - \left( 1 - \frac{r_0^4}{L^4} \frac{1}{a^4} \right) a^2 (a^6 e^{-2\phi} V_0^2 + Q^2 + \frac{\ell^2}{L^2}) \right]. \tag{25}
\]

In the limit \( Q = 0, V_0 = 1 \) and \( \phi = 0 \), which corresponds to mirage cosmology with stable (nontachyonic) probe brane under the nondilatonic background, the above result exactly recovers the known result (see Eq. (5.2) of [13] with \( C = 0 \))

\[
\frac{8\pi}{3} \rho_{\text{eff}} = \frac{1}{L^2} \left[ E^2 - \left( 1 - \frac{r_0^4}{L^4} \frac{1}{a^4} \right) \left( 1 + \frac{\ell^2}{L^2} \frac{1}{a^6} \right) \right]. \tag{26}
\]

For the nontachyonic probe brane, the power \( a^{-8} \) dominates at earlier time. This corresponds to the superluminal equation of state parameter \( \omega = 5/3 \)
\( p = \omega \rho \). Note that this power is smoothed by the tachyonic charge \( Q \) in \( 25 \).

The presence of tachyon on the probe brane makes the expansion less divergent as \( a \to 0 \) where our assumptions are valid. In general, if we add any matter field, the brane universe will inflate faster due to the increased effective density. However, in [17], it is shown that the presence of tachyonic field in the bulk background makes the expansion of the brane less divergent compared with the case without tachyon. Our analysis shows that the presence of tachyonic field in the probe brane can also make the expansion less divergent. So we conclude that the presence of tachyon field which carries the instability of the brane, whether it exists in the bulk background or in the probe brane, makes the mirage inflation less divergent.

In the point of higher dimensional theory, our approximation is valid near the horizon of the higher dimensional black hole made by a stack of heavy stable branes. Conditions in the higher dimensional theory are translated as the constraints in the parameter space. For example, since \( \dot{r}^2 \geq 0 \) for equation (17) this gives the constraint

\[
\frac{A}{B} \left( 1 - \frac{K D + K \ell^2 + Q^2 D A}{K D} \frac{A}{E^2} \right) \geq 0. \tag{27}
\]

Another example is the brane Hamiltonian. For the simple case we considered in this paper, the positivity of energy is automatically satisfied (see Equation(9)). When we add a four-form potential as mentioned before, the allowed values of \( r \) is determined by the energy constraint \( C(r) + E \geq 0 \). Similar argument holds when we consider \( B_{\mu \nu} \) and \( F_{\mu \nu} \).

We would like to point out that our result is valid only when the tachyon is near the top of the potential so that the approximation (15) can be applied. At the early stage of mirage cosmology as we considered here, the effect of tachyon potential is neglected. Then the tachyon kinetic term plays the dominant role for the brane inflation. At late time where one recovers the usual vacuum gravity, it is expected that the cosmology will be driven not only by the brane energy but also by the bulk energy. The behavior of the cosmology during the intermediate time can be studied by solving the tachyon equation of motion (11) together with (9) and (10). In this region both the kinetic term and potential slope should be considered. However, finding the analytic solution seems very difficult for the given potential form. Numerical solution may give some hints.

The intuitive reasoning of the scalar modification to the FRW equation is similar to the modification by a gauge electric field. As far as the tachyon po-
tential is constant, the effect of tachyon is a scalar field affecting the equation of motion of the brane evolution. This case is similar to mirage cosmology with an electric field on the probe brane. When there is an electric field, the square root of BDI action is given by (Equation (4.1) of Ref. [13])

\[- \det(\hat{G}_{\mu\nu} + 2\pi\alpha'\hat{F}_{\mu\nu}) = g^3(|g_{00}(r)| - g_{rr}r^2 - g_{ij}\dot{\varphi}^i\dot{\varphi}^j - \varepsilon^2g^2), \tag{28}\]

where \(\varepsilon^2 = (2\pi\alpha')^2E_iE^i\). Comparing the square root of Equation (6) with the above equation, the only difference is that there is no metric coupling \((g^2\text{ factor})\) in tachyon kinetic term. Thus the mechanism how the tachyonic nature of the scalar enters into the final FRW-like equation is the same as the case with U(1) electric field on the probe brane. The integration constant \(\mu^2 = (2\pi\alpha')^2\mu_i\mu^i\), obtained from the equation of motion for the electric field, enters the final FRW-like equation (Equation(4.7) of Ref.[13]). Similarly the tachyonic nature of the scalar is carried by \(Q^2\) (Equation(25)).

It has been shown that, for large \(T\), the value of the tachyon field at a given space-time point can be identified as the time coordinate when tachyon is coupled only to gravity [21]. Further studies on mirage effect for large value of tachyon field are expected.

Acknowledgements This work was supported by the Korea Research Foundation Grant (KRF-2004-002-C00065). J.Y. Kim is grateful to the Department of Physics at U.C. Davis for hospitality during his visit.

References


