We present calculations of cross sections and spin asymmetries in single-inclusive reactions in $pp$ scattering. We discuss next-to-leading order predictions as well as all-order soft-gluon threshold resummations.

1. Introduction

Single-inclusive reactions in $pp$ scattering, such as $pp \rightarrow \gamma X$, $pp \rightarrow \pi X$, $pp \rightarrow \text{jet} X$, play an important role in QCD. At sufficiently large produced transverse momentum, $p_T$, QCD perturbation theory (pQCD) can be used to derive predictions for these reactions. Since high $p_T$ implies large momentum transfer, the cross section may be factorized at leading power in $p_T$ into convolutions of long-distance pieces representing the structure of the initial hadrons, and parts that are short-distance and describe the hard interactions of the partons. The long-distance contributions are universal, that is, they are the same in any inelastic reaction, whereas the short-distance pieces depend only large scales and, therefore, can be evaluated using QCD perturbation theory. Because of this, single-inclusive cross sections offer unique possibilities to probe the structure of the initial hadrons in ways that are complementary to deeply-inelastic scattering. At the same time, they test the perturbative framework, for example, the relevance of higher orders in the perturbative expansion and of power-suppressed contributions to the cross section.
Of special interest is the case when the initial protons are polarized. At RHIC, one measures spin asymmetries for single-inclusive reactions, in order to investigate the spin structure of the nucleon. A particular focus here is on the gluon polarization in the nucleon, $\Delta g \equiv g^\uparrow - g^\downarrow$.

In the following, we will present some theoretical predictions for cross sections and spin asymmetries for single-inclusive reactions. We will first discuss the double-longitudinal spin asymmetries $A_{LL}$ for pion and jet production at RHIC and their sensitivities to $\Delta g$. In the second part, we will give results for new calculations of the unpolarized cross section for $pp \rightarrow \pi^0X$ in the fixed-target regime, which show a greatly improved description of the available experimental data.

2. Spin asymmetries for $pp \rightarrow (\pi^0, \text{jet})X$ at RHIC

We consider the double-spin asymmetry

$$A_{LL} \equiv \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} \equiv \frac{d\Delta\sigma}{d\sigma},$$

(1)

where the superscripts denote the helicities of the initial protons. According to the factorization theorem the spin-dependent cross section $\Delta\sigma$ can be written in terms of the spin-dependent parton distributions $\Delta f$ as

$$\frac{d\Delta\sigma}{dp_T d\eta} = \sum_{a,b} \Delta f_a(x_a, \mu) \otimes \Delta f_b(x_b, \mu) \otimes \frac{d\Delta\hat{\sigma}_{ab}}{dp_T d\eta}(x_a, x_b, p_T, \eta, \mu),$$

(2)

where the symbols $\otimes$ denote convolutions and where the sum is over all contributing partonic channels. We have written Eq. (2) for the case of jet production; for pion production there is an additional convolution with a pion fragmentation function. As mentioned above, the parton-level cross sections may be evaluated in QCD perturbation theory:

$$d\Delta\hat{\sigma}_{ab} = d\Delta\hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s}{\pi} d\Delta\hat{\sigma}_{ab}^{(1)} + \ldots,$$

(3)

corresponding to “leading order” (LO), “next-to-leading order” (NLO), and so forth. The NLO corrections for the spin-dependent cross sections for inclusive-hadron and jet production were published in [2, 3, 5, 6], respectively. They are crucial for making reliable quantitative predictions and for analyzing the forthcoming RHIC data in terms of spin-dependent parton densities. The corrections can be sizable and they reduce the dependence on the factorization/renormalization scale $\mu$ in Eq. (2). In case of jet production, NLO corrections are also of particular importance since it is only at NLO that the QCD structure of the jet starts to play a role.
Figure 1 shows NLO predictions for the spin asymmetry $A_{LL}$ for high-$p_T$ pion production for collisions at $\sqrt{s} = 200$ GeV at RHIC. We have used various sets of polarized parton densities $g$, which mainly differ in $\Delta g$. As one can see, the spin asymmetry strongly depends on $\Delta g$, so that measurements of $A_{LL}$ at RHIC should give direct and clear information. The “error bars” in the figure are uncertainties expected for measurements with an integrated luminosity of $3/pb$ and beam polarization $P=0.4$. We note that PHENIX has already presented preliminary data $g$ for $A_{LL}$. We also mention that the figure shows that at lower $p_T$ the asymmetry is not sensitive to the sign of $\Delta g$. This is related to the dominance of the $gg$ scattering channel which is approximately quadratic in $\Delta g$. In fact it can be shown that $A_{LL}$ in leading-power QCD can hardly be negative at $p_T$ of a few GeV $g$. One may obtain better sensitivity to the sign of $\Delta g$ by expanding kinematics to the forward rapidity region.

Figure 2 shows predictions for the spin asymmetry $A_{LL}$ for high-$p_T$ jet production. The gross features are rather similar to the pion asymmetry, except that everything is shifted by roughly a factor two in $p_T$. This is due to the fact that a pion takes only a certain fraction of $\sim O(50\%)$ of the outgoing parton’s momentum, so that the hard scattering took place at roughly twice the pion transverse momentum. A jet, however, will carry the full transverse momentum of a produced parton.

We emphasize that PHENIX and STAR have presented measurements $g$ of the unpolarized cross section for $pp \rightarrow \pi^0 X$. These are well described by the corresponding NLO QCD calculations $g$, providing con-
fidence that the NLO pQCD hard-scattering framework is indeed adequate in the RHIC domain. This is in contrast to what was found in comparisons \[^{11}\] between NLO theory and data for inclusive-hadron production taken in the fixed-target regime. We will turn to this issue next.

3. Threshold resummation for inclusive-hadron production

One may further improve the theoretical calculations by an all-order resummation of large logarithmic corrections to the partonic cross sections \[^{4}\]. At partonic threshold, when the initial partons have just enough energy to produce a high-transverse momentum parton (which subsequently fragments into the observed pion) and a massless recoiling jet, the phase space available for gluon bremsstrahlung vanishes, resulting in large logarithmic corrections to the partonic cross section. For the rapidity-integrated cross section, partonic threshold is reached when \(\hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}} = 1\), where \(\sqrt{\hat{s}}\) is the partonic center-of-mass (c.m.) energy, and \(\hat{p}_T\) is the transverse momentum of the produced parton fragmenting into the hadron. The leading large contributions near threshold arise as \(\alpha_s^k \ln^{2k}(1 - \hat{x}_T^2)\) at the \(k\)th order in perturbation theory. Sufficiently close to threshold, the perturbative series will be only useful if such terms are taken into account to all orders in \(\alpha_s\), which is achieved by threshold resummation \[^{12}\]. This resummation has been derived for a number of cases of interest, to next-to-leading logarithmic (NLL) order, in particular also for jet production \[^{13}\] which proceeds through the same partonic channels as inclusive-hadron production.
The larger $\hat{x}_T$, the more dominant the threshold logarithms will be. Since $\hat{s} = x_a x_b S$, where $x_{a,b}$ are the partonic momentum fractions and $\sqrt{S}$ is the hadronic c.m. energy, and since the parton distribution functions fall rapidly with increasing $x_{a,b}$, threshold effects become more and more relevant as the hadronic scaling variable $x_T \equiv 2p_T/\sqrt{S}$ goes to one. This means that the fixed-target regime with $3 \text{ GeV} \lesssim p_T \lesssim 10 \text{ GeV}$ and $\sqrt{S}$ of 20–30 GeV is the place where threshold resummations are expected to be particularly relevant and useful.

The resummation is performed in Mellin-$N$ moment space, where the logarithms $\alpha_s \ln^{2k} (1 - \hat{x}_T^2)$ turn into $\alpha_s \ln^{2k} (N)$, which then exponentiate. For inclusive-hadron production, because of the color-structure of the underlying Born $2 \rightarrow 2$ QCD processes, one actually obtains a sum of exponentials in the resummed expression. Details may be found in [4]. Here, we only give a brief indication of the qualitative effects resulting from resummation. For a given partonic channel $ab \rightarrow cd$, the leading logarithms exponentiate in $N$ space as

$$\hat{\sigma}_{ab \rightarrow cd}^{(res)}(N) \propto \exp \left[ \frac{\alpha_s}{\pi} \left( C_a + C_b + C_c - \frac{1}{2} C_d \right) \ln^2(N) \right], \quad (4)$$

where

$$C_g = C_A = N_c = 3, \quad C_q = C_F = (N_c^2 - 1)/2N_c = 4/3. \quad (5)$$

This exponent is clearly positive for each of the partonic channels, which means that the soft-gluon effects will lead to an enhancement of the cross section. Indeed, as may be seen from Fig. 3, resummation dramatically increases the cross section in the fixed-target regime. The example we give is a comparison of NLO and NLL resummed predictions at $\sqrt{S} = 31.5 \text{ GeV}$ with the data of E706 [14] at that energy. We have used the “KKP” set of pion fragmentation functions [15] and the parton distributions of [16]. We finally note that the results shown in Fig. 3 are also interesting with respect to the size of power corrections to the cross section. Resummation may actually suggest the structure of nonperturbative power corrections. For a recent study of this for single-inclusive cross sections, see [17].

Acknowledgments

I am grateful to D. de Florian, B. Jäger, S. Kretzer, A. Schäfer, G. Sterman, and M. Stratmann for fruitful collaborations on the topics presented here. I thank RIKEN, BNL and the U.S. DoE (contract number DE-AC02-98CH10886) for providing the facilities essential for the completion of this work.
Figure 3. NLO and NLL resummed results for the cross section for $pp \rightarrow \pi^0 X$ for E706 kinematics. Results are given for three different choices of scales, $\mu = \zeta p_T$, where $\zeta = 1/2, 1, 2$. Data are from $^{14}$. 

References
8. Y. Fukao, PHENIX Collab., these proceedings (hep-ex/0501049).