Comment on soft-pion emission in DVCS

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The soft-pion theorem for pion production in deeply virtual Compton scattering, derived by Guichon, Mossé and Vanderhaegen, is shown to be consistent with chiral perturbation theory. Chiral symmetry requires that the nonsinglet operators corresponding to spin-independent and spin-dependent parton distributions have the same anomalous dimensions in cases where those operators are related by chiral transformations. In chiral perturbation theory, their scale-dependences can thus be absorbed in the coefficients of the corresponding effective operators, without affecting their chiral structures.

Deeply virtual Compton scattering (DVCS) promises to provide an important new window on the structure of the nucleon, in the form of generalised parton distributions (GPD’s) [1]. However, as pointed out by Guichon, Mossé and Vanderhaegen [2], the related process \( \gamma^* + p \rightarrow \gamma + N + \pi \), where a low-energy pion is produced, may be hard to disentangle experimentally from DVCS and so could contaminate any determination of GPD’s. Guichon et al. used a soft-pion theorem to relate the amplitude for this process to the same GPD’s that appear in the amplitude for DVCS. This approach has been criticised by Chen and Savage [3], as being inconsistent with chiral perturbation theory (ChPT). More recently, it has been defended by Kivel, Polyakov and Stratman [4]. Here I use the evolution of the spin-dependent parton distributions, as calculated in Refs. [5, 6], to show that the soft-pion theorems and ChPT are consistent, and that the results of Refs. [2, 4] are correct.

The relevant operators for this discussion are the twist-2 ones corresponding to the moments of the nonsinglet quark distributions,

\[
\begin{align*}
\theta^{(n)a}_{V_{\mu_1, \ldots, \mu_n}} &= (i)^{n-1} \bar{q} \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_n} \gamma_a \tau^a q \\
\theta^{(n)a}_{A_{\mu_1, \ldots, \mu_n}} &= (i)^{n-1} \bar{q} \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_n} \gamma_\tau \tau^a q,
\end{align*}
\]

where the Lorentz tensors have been symmetrised and made traceless. (For simplicity I consider here only two flavours of quark and so these operators are isovector.) The structures of the corresponding matrix elements in heavy-baryon chiral perturbation theory (HBChPT) are

\[
\begin{align*}
\theta^{(n)a}_{V_{\mu_1, \ldots, \mu_n}} &\rightarrow M^{n-1} A^{(n)} v_{\mu_1} \cdots v_{\mu_n} \tilde{N} \tau^a \tau^a N + M^{n-1} B^{(n)} v_{\mu_1} \cdots v_{\mu_{n-1}} \tilde{N} S_{\mu_n} \tau^a \tau^a N \\
\theta^{(n)a}_{A_{\mu_1, \ldots, \mu_n}} &\rightarrow M^{n-1} C^{(n)} v_{\mu_1} \cdots v_{\mu_{n-1}} \tilde{N} S_{\mu_n} \tau^a \tau^a N + M^{n-1} D^{(n)} v_{\mu_1} \cdots v_{\mu_n} \tilde{N} \tau^a \tau^a N,
\end{align*}
\]

where

\[
\tau^a = \frac{1}{2} (u \tau^a u \pm u \tau^a u) ,
\]

\( u \) is the usual square root of the matrix \( U = \exp[i \tau \cdot \phi] \) of pion fields in ChPT, and \( S_{\mu_n} \) is the HBChPT spin operator [8]. I have followed here the notation of Ref. [3], omitting the \( \Delta \) term which is irrelevant to the present discussion. The coefficients \( A^{(n)} \) are given by the moments of the nonsinglet, spin-independent quark distributions,

\[
q^-(x) = u(x) - \bar{u}(x) - d(x) + \bar{d}(x).
\]

Similarly the coefficients \( C^{(n)} \) are given by the moments of the spin-dependent distributions,

\[
\Delta q^+(x) = \Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x).
\]

The terms involving \( \tau^a \) involve at least one pion field. Hence they do not appear in the quark distributions of the nucleon, but they do contribute to processes in which a pion is produced.

The structures of the HBChPT operators considered in Ref. [4] are the same as those in Ref. [3], but with the additional constraints that \( D^{(n)} = A^{(n)} \) and \( B^{(n)} = C^{(n)} \). These arise from the fact that Kivel et al. [4] (see also Ref. [7]) construct the twist-2 operators

\[
\begin{align*}
\theta^{(n)a}_{R_{\mu_1, \ldots, \mu_n}} &= \theta^{(n)a}_{V_{\mu_1, \ldots, \mu_n}} + \theta^{(n)a}_{A_{\mu_1, \ldots, \mu_n}} \\
\theta^{(n)a}_{L_{\mu_1, \ldots, \mu_n}} &= \theta^{(n)a}_{V_{\mu_1, \ldots, \mu_n}} - \theta^{(n)a}_{A_{\mu_1, \ldots, \mu_n}},
\end{align*}
\]
corresponding to distributions of right- and left-handed quarks. The corresponding ChPT operators are constructed using \[4\]

\[
\tau^a_+ + \tau^a_- = u\tau^a_+ u^\dagger, \quad \tau^a_+ - \tau^a_- = uu^\dagger \tau^a_+ u^\dagger,
\]

which transform under SU(2)\(_R\times\)SU(2)\(_L\) according to

\[
u^\dagger \tau^a_+ u \rightarrow K(L, R, U)u^\dagger R^\dagger \tau^a_+ RuK(L, R, U)^\dagger, \quad uu^\dagger \tau^a_+ u \rightarrow K(L, R, U)uL^\dagger \tau^a_+ LuK(L, R, U)^\dagger.
\]

Contrary to the claims of Chen and Savage \[3\], this is a consistent implementation of the constraints of chiral symmetry on the operators appearing in the low-energy effective theory. The anomalous dimensions of the operators \(\theta^{(n)a}_{V,\mu_1\ldots\mu_n}\) and \(\theta^{(n)a}_{AS,\mu_1\ldots\mu_n}\) are the same or, equivalently, the parton distributions \(q^-(x)\) and \(\Delta q^+(x)\) evolve according to the same splitting functions. These results have been proved to second order in perturbative QCD \[5, 6\], but they are really just consequences of the chiral symmetry of the massless quarks used in calculating the QCD evolution. (Note that, as discussed by Vogelsang \[6\], some care is needed to show this because the representation of \(\gamma_5\) used in dimensional regularisation does not anticommute with all the other \(\gamma\) matrices.) These right- and left-handed nonsinglet quark distributions thus have well-defined QCD evolution properties. In the low-energy effective theory, their dependences on the scale \(Q^2\) can thus be absorbed in the coefficients of the corresponding effective operators, without affecting the chiral structure of those operators.

It is worth adding that there are no similar constraints on the corresponding singlet twist-2 operators, since they are invariant under SU(2)\(_R\times\)SU(2)\(_L\). Indeed the spin-independent and spin-dependent singlet operators evolve quite differently in perturbative QCD, due to their different mixings with gluonic operators \[5, 6\].

Finally, I would emphasise the point made by Kivel et al. \[4\] that, at least for tree-level amplitudes, ChPT should automatically incorporate the old soft-pion theorems (such as Goldberger-Treiman, Weinberg-Tomozawa, Kroll-Ruderman etc.). Where the older soft-pion methods break down are cases for which pion-loop contributions are needed, most famously for \(\pi^0\) photoproduction at threshold \[9\]. A minor technical complication is that soft-pion results are normally derived using the divergence of the axial current as an interpolating pion field. This differs at order \(m_\pi^2\) from the pion fields in the commonly-used representations of the ChPT Lagrangians. Hence, for example, the soft-pion limit of pion-nucleon scattering in ChPT does not reproduce the pion-nucleon sigma commutator (which is also of order \(m_\pi^2\)). However, in the present context of tree-level amplitudes at order \(m_\pi^2\), neither of these issues arises and so the results derived using soft-pion methods \[2, 4\] should be embodied in the effective Lagrangian of ChPT.

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