I reconsider Hawking's analysis of the effects of gravitational collapse on quantum fields, taking into account interactions between the fields. The ultra-high energy vacuum fluctuations, which had been considered to be an awkward peripheral feature of the analysis, are shown to play a key role. By interactions, they can scatter particles to, or create pairs of particles at, ultra-high energies. The energies rapidly become so great that quantum gravity must play a dominant role. Thus the vicinities of black holes are essentially quantum-gravitational regimes.

Keywords: black holes, quantum gravity, Hawking radiation

1. Introduction

Since black holes are extreme manifestations of general relativity, one might expect that exotic quantum effects would be amplified in their vicinities, perhaps providing clues to quantum gravity. This hope has not been fulfilled, however, by the commonly accepted analysis of the effects of gravitational collapse on quantum theory.\textsuperscript{1,2} That analysis predicts that macroscopic black holes, at least, are essentially classical objects, subject to only minor quantum corrections: small emissions of low-temperature thermal radiation. This prediction of thermal radiation is very beautiful, and attractive in its potential to clarify the connection between thermodynamics and black-hole theory.

But the analysis leading to this prediction is not without its problems.\textsuperscript{3} (See Ref. 4 for a recent review). The most striking is the \textit{trans-Planckian problem}: the model’s reliance on treating vacuum fluctuations at exponentially increasing energy scales by conventional physics. Also the model assumes that interactions between the quantum fields may be neglected, even (or especially) as those energies exponentially increase. While it is acknowledged that there are as yet no very good...
responses to these concerns, I think it is fair to say that the sentiment in the field has largely been that they are peripheral difficulties which will somehow be overcome, leaving essentially intact the picture of a softly thermally radiating black hole.

My aim here is to argue that the correct picture of a black hole is very different. The black hole, far from being an essentially classical object with only minor quantum corrections, has a strong quantum character which profoundly affects physics in a region of space–time, a region extending beyond the hole a distance comparable to the size of the hole itself. In this region, not merely “virtual” vacuum fluctuations, but real Planck-scale physics must be prevalent.

What I shall show is that taking interactions between quantum fields into account completely alters the picture drawn by Hawking. These interactions can promote vacuum fluctuations from “virtual” to real effects. We shall find that real ultra-energetic effects, such as scattering and pair-production of particles, take place in the vicinity of the hole. The energies are of the same order as those of the vacuum fluctuations. This means that they rapidly pass the Planck scale, at which quantum field theory must break down and quantum gravity must take over. The vicinity of a black hole must be a window on quantum gravity.

Conventions and terminology. We shall use natural units throughout, so \( c \), \( G \) and \( \hbar \) are unity. We shall need to discuss processes with energies approaching, but not at, the Planck energy; we shall refer to these as ultra-energetic. We shall only consider the case of a spherically symmetric black hole; its mass will be \( M \).

2. Gravitational collapse and the red-shift

While the event horizon of a black hole is best known as a “point of no return,” it is a different property which is most important both in Hawking’s analysis and in this paper. This is that light rays (and other fields) passing close to the event horizon are red-shifted, the shifts increasing exponentially as later and later rays are considered.

A standard conformal diagram of a spherically symmetric gravitationally collapsing space–time is shown in Fig. 1. The collapsing matter is shaded, and the event horizon is the dashed line. A distant observer, looking back inwards towards the hole, would be represented by a point near future null infinity, and coordinatized by a “retarded time” \( u \). Tracing a light ray perceived by this observer back into space–time, through the origin (where, in the diagram, it appears to reflect from the left-hand edge) and out towards the distant past, it arrives at an “advanced time” \( v = v(u) \). Since the intervals \( du, dv \) between crests of successive rays are related by \( dv = v'(u) \, du \), the factor \( v'(u) \) is the red-shift suffered by a radial ray passing from the past, through the collapsing space–time, into the future.

It can be shown very generally that the red-shift factor has asymptotic form

\[
v'(u) \simeq \exp -u/(4M) \quad \text{as} \quad u \to +\infty ,
\]

where \( M \) is the mass of the hole in geometric units.\textsuperscript{5–6} This exponential decay
drives $v'(u)$ to zero very quickly. For example, for a solar-mass object, the $e$-folding time is $\simeq 2 \times 10^{-5}$ s. This exponential asymptotic form plays a key role in Hawking’s analysis.

3. Fluctuations, Hawking radiation and the trans-Planckian problem

Hawking radiation arises because the vacuum fluctuations, which must be present in the space–time before collapse, are distorted as they propagate through the collapsing space–time. In part they are red-shifted by $v'(u)$; but also the distinction between positive and negative frequencies is distorted. Since particle-content is determined by the frequency-decomposition of the operators, mixing of positive- and negative-frequency terms gives rise to particle production.

In the case of the Hawking process, only certain field modes are distorted in precisely the correct way to produce appreciable particle fluxes. In fact, a computation of which particle modes are produced results in the famous prediction of a thermal spectrum with temperature $T_H = 1/(8\pi M)$.

We can give a diagrammatic picture of the Hawking effect, as follows. While vacuum fluctuations are not ordinarily drawn as Feynman diagrams (because, in the absence of a time-dependent external potential, they can be consistently dis-
In a gravitationally collapsing space–time, the distortion of the vacuum fluctuations can be viewed as causing some of the loops to fail to close, Fig. 3. Although not expressed in these terms, Hawking’s computation is in fact equivalent to evaluating diagrams like the one in Fig. 3. While in principle the virtual portion of the photon line there (that is, everything except the end-points) could pass anywhere in space–time, the dominant contributions come from portions like the one shown, where an ultra-high energy vacuum fluctuation in the past propagates inwards towards the origin, and then outwards, being distorted into real particles along the way.

Fig. 2. A zero-point fluctuation’s contribution to the evolution, represented by a closed virtual photon loop. (Wavy lines represent photons.)

Fig. 3. One can think of propagation through the time-dependent gravitationally collapsing space–time as opening some of the vacuum fluctuation loops, resulting in real particle production. Although, for fixed end-points, one could draw vacuum fluctuation arcs occupying any portion of space–time, the dominant contributions to the Hawking process come from ones like that shown here. Ultra-high frequency fluctuations in the past propagate thought the collapsing space–time, where they are both red-shifted and distorted in a way which produces a low flux of real low-energy photons.

Because the outgoing Hawking quanta have characteristic frequencies $\sim \omega_H =$
1/(8\pi M), the vacuum fluctuations which were their precursors must have had frequencies \( \omega_H/\nu'(u) \simeq \omega_H \exp + u/(4M) \). This is the famous trans-Planckian problem: the prediction of mild thermal radiation requires an appeal to vacuum fluctuations at energy scales increasing exponentially. The scales rapidly pass the Planck scale, at which the model’s reliance on ordinary quantum field theory in curved space–time must break down and quantum gravity must take over.

### 4. Effects of interactions

Hawking’s analysis was for free fields. Let us see what the effects of including interactions are. We shall consider for definiteness quantum electrodynamics, but it will be clear that the basic ideas are more general.

An extraordinary first-order effect occurs when we consider the effect of the interaction in coupling a charged particle to the vacuum fluctuations, Fig. 4. In this diagram, the ultra-high energy arc of the vacuum fluctuation interacts with the charged particle, exchanging energy–momentum. The remaining portion of the arc passes through the gravitationally collapsing region, where it red-shifts (and transmits some of its energy to the hole). The result is that the charged particle has been scattered by an ultra-high energy.

![Diagram](Fig. 4. A charged particle (solid line) can interact with an ultra-high frequency vacuum fluctuation photon pair (wavy line), scattering the charged particle to an ultra-high energy and producing a Hawking photon. (This diagram has been drawn to parallel Fig. 3, and some structure which has conceptual but not literal significance has been retained. The bending of the photon line back on itself conveys the idea of the genesis of an electromagnetic vacuum fluctuation, but is not literally meaningful. The crossing of the photon and charged-particle lines is a consequence of this way of drawing the photon line, and likewise not significant.)]

In field-theoretic terms, the interaction would be given by \( \int A \cdot J \, dt \), where \( A \) is the electromagnetic field mode corresponding to the outgoing Hawking quantum,
and $J$ is the current operator. It is the extension of $A$ into the past, where it becomes an ultra-high frequency mode mixing both positive and negative frequencies, which gives rise to the ultra-energetic scattering.

It is not necessary to have charged particles present initially, however, to have ultra-energetic effects. An ultra-energetic vacuum fluctuation can also engender pair creation, Fig. 5.

![Fig. 5](image)

Fig. 5. An ultra-high energy vacuum electromagnetic fluctuation (wavy line) can produce an ultra-energetic pair of charged particles (solid lines). (Again, the bending-back on itself of the photon line, to suggest its origin in vacuum fluctuations, has been retained, but this is not of literal significance.)

(A technical comment. Feynman diagrams become ambiguous if the definitions of positive and negative frequencies differ in the in- and out-regions. A correct formulation can be inferred by simply considering which two-or three-point function is described by the diagram.)

5. Conclusion

We have seen that the presence of interactions will allow “virtual” vacuum fluctuations to produce real physical effects, such as scattering and pair-creation of particles. In a gravitationally collapsing space-time, the result is that ultra-energetic charged particles are produced and are correlated with the emitted Hawking radiation. However, the energies increase so rapidly that, essentially as soon as the hole can be said to have formed, they have passed the Planck scale. At that point quantum gravity must enter essentially.

These Planck-scale effects occur, not just in a thin layer around the event horizon, but in a region of space whose linear size outwards from the black hole is of the order of the Schwarzschild radius. This is because, at any point in space–time, the
ultra-energetic effects are essentially determined by the possibilities of photons being received which have been exponentially red-shifted by the incipient black hole. This is essentially determined by the fraction of the sky occupied by the hole, as seen by an observer at the point in question, and this fraction will be significant if one is within a few Schwarzschild radii of the hole.

We thus predict that the vicinity of a black hole is a region in which essentially quantum-gravitational, Planck-scale, physics must dominate.

References
