On photon splitting in theories with Lorentz invariance violation.

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In a model with Lorentz invariance violation implemented through modified dispersion relations, we estimate the rate for the decay process $\gamma \rightarrow 3\gamma$ and find that it provides a relevant bound on Lorentz invariance violation.

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Lorentz invariance is not necessarily an exact symmetry of nature. Though all the experimental results show that the laws of physics are Lorentz invariant, several approaches to quantum gravity, for example, suggest that at very high energies Lorentz invariance might be broken. Surprisingly, such violations, even if associated with Planck scale physics, may be constrained by present observations.

To discuss Lorentz invariance violation (LIV) we adopt the kinematic framework based on modified dispersion relations. That is, we alter the relation between the energy and the momentum of a given particle but keep intact the dynamics and the energy-momentum conservation law. Such modifications are indeed suggested by certain approaches to Quantum Gravity\textsuperscript{1}.

Modified dispersion relations manifest in different ways\textsuperscript{2}. They may affect the propagation of photons by inducing dispersion and birefringence over long travel times. They could modify the threshold for known processes, as in $\gamma\gamma \rightarrow e^+ e^-$ and the GZK reaction\textsuperscript{3}. They might also induce new processes not allowed in QED such as photon decay ($\gamma \rightarrow e^+ e^-$), photon splitting ($\gamma \rightarrow n\gamma$), and vacuum Cerenkov radiation $e \rightarrow e\gamma$. The non-observation of such anomalous effects, then, constrains the assumed dispersion relations.

We consider, following Ref.\textsuperscript{2}, the modified dispersion relation for the photon

\[ E_\gamma^2 = p_\gamma^2 + \xi \frac{p_\gamma^3}{M} \tag{1} \]

where $E_\gamma$ and $p_\gamma$ are the energy and momentum of the photon, the scale $M = 10^{19}$ GeV is an energy scale close to the Planck mass, and $\xi$ is a Lorentz invariance violating parameter. This dispersion relation preserves rotation invariance but not boost invariance. Thus it can hold in only one reference frame, which we identify with the rest frame of the cosmic microwave background.

Within an effective field theory (EFT) framework, the photon satisfies the polarization dependent dispersion relation $E_\pm^2 = p^2 \pm \xi (p^3/M)$, where the subscripts $\pm$ refer to the right and left circular polarizations of the photon\textsuperscript{4}. It may be, however, that EFT does not describe the leading effects of Lorentz invariance violation. In any case, EFT is a dynamical assumption that goes beyond the kinematic framework we are following. We will, therefore, neglect polarization dependence in the dispersion relation and assume that all photons satisfy Eq.\textsuperscript{1} with $\xi$ positive.

In standard QED the photon splitting process $\gamma \rightarrow 3\gamma$ does not occur because its phase space has vanishing volume. However, such decay is allowed in the presence of the modified dispersion relation in Eq.\textsuperscript{1} for $\xi > 0$. Since in this case the process $\gamma \rightarrow 3\gamma$ has no threshold, its effectiveness in constraining $\xi$ depends solely on the rate. So far, only a crude guess of this rate exist in the literature. In this letter, we provide an estimate of the decay rate $\Gamma(\gamma \rightarrow 3\gamma)$ and show that the observation of high energy photons from astrophysical sources provides an upper bound on $\xi$ stronger than previously believed.

To compute the rate of the decay $\gamma \rightarrow 3\gamma$ we write the modified dispersion relation in terms of an effective photon mass,

\[ m_\gamma^2 = \xi \left( \frac{p_\gamma^3}{M} \right), \tag{2} \]

so that the photon satisfies $E_\gamma^2 = p_\gamma^2 + m_\gamma^2$. Then, we estimate the decay rate of this massive photon in its rest frame and boost it -with the usual Lorentz factor- to the laboratory frame (which practically coincides with the rest frame of the CMB) to obtain the desired rate.

In order to gain confidence in this procedure, we will verify that when applied to $\gamma \rightarrow e^+ e^-$ it reproduces the results known in the literature\textsuperscript{5} for the photon energy threshold and the decay rate. In the center of mass frame, the decay rate of a massive photon into an electron-positron pair is simply $\Gamma_{CM} \simeq (\alpha/2)m_\gamma$. In the laboratory frame the rate gets an additional inverse gamma factor, $\gamma^{-1} = m_\gamma/E_\gamma$.

\[ \Gamma_{lab} = \frac{\alpha}{2} \frac{m_\gamma^2}{E_\gamma^2} = \frac{\alpha}{2} \frac{\xi E_\gamma^2}{M}. \tag{3} \]

This results coincides with that obtained in Ref.\textsuperscript{5}, which is $\Gamma \simeq (\alpha/2)(c^2-1)E$, for a photon with four momentum $(E, E/c)$, if we replace the effective photon mass in Eq.\textsuperscript{3} by its value in the model of Ref.\textsuperscript{5}, namely $m_\gamma^2 = E^2(c^2-1)$. The energy threshold for this decay

\[ E_\gamma \simeq \frac{m_\gamma^2}{\xi} = m_\gamma \left( \frac{M}{\xi} \right). \]
is obviously given by the condition $m_\gamma^2 = 4m_e^2$, which is equivalent to the condition $E^2 = 4m_e^2/(c^2 - 1)$ found in Ref. [6]. Thus, in terms of the effective photon mass the expression for the energy threshold and the decay rate are particularly simple. In fact, in the footnote 7 of Ref. [3], the procedure we follow is mentioned as providing the right result.

In passing we note that the rate for the $e \to e\gamma$ decay of Ref. [3] can also be obtained from kinematic considerations similar to those used above for $\gamma \to e^+e^-$. Assigning an effective mass to the photon has an additional advantage. It shows clearly that the relevant energy scale for photon decay is not the energy of the initial photon, but its effective mass. And due to the large suppression $M^{-1}$ of this effective mass in Eq. (4), even energetic photons may have a small effective mass (say $m_\gamma \sim 10^{-2}\xi^{1/2}\text{MeV}$ for $E \simeq \text{TeV}$).

Notice that as soon as the decay $\gamma \to e^+e^-$ is kinematically allowed, i.e. for $m_\gamma^2 > 4m_e^2$, this decay mode, that happens at tree level, dominates over the one-loop suppressed $\gamma \to 3\gamma$. In the following we need to make sure that $m_\gamma^2 < 4m_e^2$, because then is the decay $\gamma \to 3\gamma$ important.

At low energies, the interactions among photons are described by the Euler-Heisenberg Lagrangian,

$$\mathcal{L}_{E-H} = \frac{2n^2}{45m^2_\gamma} \left( \frac{1}{2} F_{\mu
u} F^{\mu
u} \right)^2 + 7 \left( \frac{1}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)^2.$$

Using this Lagrangian we estimate $\Gamma(\gamma \to 3\gamma)$ in the laboratory frame (i.e. the CMB rest frame) as

$$\Gamma(\gamma \to 3\gamma) = \left( \frac{2n^2}{45} \right)^2 \frac{1}{3!^22^{15}\pi^8 m^9_\gamma m_\gamma^7 E_\gamma} \times f \quad \text{(5)}$$

$$= 1.5 \times 10^{-20} \left( \frac{E_\gamma^3}{10^5\text{GeV}} \right)^2 \times f \quad \text{(6)}$$

where $f$ is the integral over momenta with some large factors taken out, i.e.

$$f = \frac{4\pi^4}{m^2_\gamma} \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{E_1 E_2 E_3} \delta^4(p_\gamma - k_1 - k_2 - k_3) |\mathcal{M}|^2, \quad \text{(7)}$$

and $\mathcal{M}$ is the invariant matrix element obtained from Eq. (1) by omitting the global factor $2n^2/(45m^2_\gamma)$.

Defined in this manner the factor $f$ is of order one. Indeed, a similar integration was performed numerically in Ref. [3] and $f$ turned out to be 0.2. In our formulas we will keep explicitly the dependence on $f$ and we will see that our final bound depends very weakly on $f$ (in fact as $f^{-1/5}$).

Thus the photon lifetime is

$$\tau(\gamma \to 3\gamma) = 0.025_f^{-5} f^{-1} \left( \frac{50\text{TeV}}{E_\gamma} \right)^4 \text{sec.} \quad \text{(8)}$$

For a given $\xi$ and a particular photon time of flight $t$, the condition $\tau \sim t$ defines a critical value for the photon energy, $E_c$. Due to the strong dependence of the lifetime on the energy, photons with energies above $E_c$ would quickly cascade down before reaching Earth from a distance $ct$ and therefore would not be observed. Thus, photons reaching Earth from a distance $ct$ should satisfy $E_\gamma < E_c$ or equivalently $\tau \geq t$. Since photons with $E_\gamma \approx 50\text{TeV}$ coming from the Crab nebula -1073 seconds away have been detected [2], we get the constraint

$$\xi \leq 1.2 \times 10^{-3} f^{-1/5} \left( \frac{50\text{TeV}}{E_\gamma} \right)^{2.8}.$$  \quad \text{(9)}$$

This bound is considerably more restrictive than the bound $\xi \leq 10^9$ previously obtained [2] using the same decay mode, and it is comparable to the strongest constraint on $\xi$, i.e. $|\xi| \lesssim 10^{-4}$ [5], coming from vacuum birefringence (different speeds for different photon polarizations). This latter bound, however, was derived within the EFT framework, in which the Lorentz invariance violating parameters for left and right circular polarized photons have opposite signs.

The condition $m_\gamma^2 < 4m_e^2$, which guarantees the validity of our approach, translates into a less restrictive bound on $\xi$

$$\xi \leq 10^{-1} \left( \frac{50\text{TeV}}{E_\gamma} \right)^3,$$  \quad \text{(10)}$$

thus it is automatically fulfilled. This last condition insures also that the high energy photons coming from the Crab nebula did not decay into $e^+e^-$ before they reached Earth.

In conclusion, we have estimated the decay rate of the process $\gamma \to 3\gamma$ for photons fulfilling the Lorentz invariance violating dispersion relation $E^2 = p^2 + \xi p^3/M$ and found that the observation of high energy photons from the Crab nebula sets an important constraint on the Lorentz invariance violation parameter $\xi$, much stronger than previously claimed.