Non-thermal leptogenesis via direct inflaton decay without SU(2)\textsubscript{L} triplets

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We present a non-thermal leptogenesis scenario following standard supersymmetric hybrid inflation, in the case where light neutrinos acquire mass via the usual seesaw mechanism and inflaton decay to heavy right-handed neutrino superfields is kinematically disallowed, or the right-handed neutrinos which can be decay products of the inflaton are unable to generate sufficient baryon asymmetry via their subsequent decay. The primordial lepton asymmetry is generated through the decay of the inflaton into light particles by the interference of one-loop diagrams with exchange of different right-handed neutrinos. The mechanism requires superpotential couplings explicitly violating a U(1) R-symmetry and R-parity. We take into account the constraints from neutrino masses and mixing and the preservation of the primordial asymmetry. We consider two models, one without and one with SU(2)\textsubscript{R} gauge symmetry. We show that the former is viable, whereas the latter is ruled out. Although the broken R-parity need not have currently observable low-energy signatures, some R-parity-violating slepton decays may be detectable in the future colliders.

I. INTRODUCTION

One of the most promising scenarios for generating the observed baryon asymmetry of the universe (BAU) is certainly the leptogenesis scenario\textsuperscript{[1,2]}. It applies in all the cases where the light neutrinos (ν) acquire their mass by coupling to heavy standard model (SM) singlet fermions ν\textsc{c}, the right-handed neutrinos (RHNs) (this is known as the seesaw mechanism \textsuperscript{[3]}) or SU(2)\textsubscript{L} triplet Higgs scalars \textsuperscript{[4]}. These heavy particles can decay out of thermal equilibrium generating a primordial lepton asymmetry, which is subsequently converted in part into the observed baryon asymmetry by non-perturbative sphaleron effects at the electroweak phase transition.

In the original realization \textsuperscript{[1]} of this scenario, the heavy particles were assumed to be thermally produced in the early universe. However, there is a tension between correct neutrino masses and this thermal leptogenesis scenario in supersymmetric (SUSY) models because of the gravitino problem \textsuperscript{[5,6]}. Assuming that the gravitino mass is of the order of 1 TeV and employing generic sparticle spectra, one finds that the reheat temperature \( T_{reh} \) should not exceed about \( 10^9 \) GeV since otherwise an unacceptably large number density of gravitinos is thermally produced at reheating. These gravitinos later decay presumably into photons and photinos interfering with the successful predictions of standard big bang nucleosynthesis. On the other hand, adequate thermal production of RHNs or SU(2)\textsubscript{L} triplets, whose decay creates the primordial lepton asymmetry, requires that the mass of these particles does not exceed \( T_{reh} \). This leads to unacceptably large neutrino masses. This problem can be alleviated \textsuperscript{[7,8]} if we allow some degree of degeneracy between the relevant RHNs, which enhances the generated lepton asymmetry, and perhaps also if the branching ratio of the gravitino decay into photons and photinos is less than unity, which somewhat relaxes \textsuperscript{[9]} the gravitino constraint on \( T_{reh} \).

A much more natural solution of the tension between the gravitino bound on \( T_{reh} \) and the masses of light neutrinos is provided by non-thermal leptogenesis \textsuperscript{[10]} at reheating. In existing realizations \textsuperscript{[10]} of this scenario, though, where the inflaton decays into RHN or SU(2)\textsubscript{L} triplet superfields, this still puts a restriction on the masses of these particles: the decay products of the inflaton must be lighter than half its mass \( m_{inf} \). The primordial lepton asymmetry is generated in the subsequent decay of the RHN or SU(2)\textsubscript{L} triplet superfields.

In a recent paper \textsuperscript{[11]}, we considered the consequences of allowing all the RHN and SU(2)\textsubscript{L} triplet superfields to be heavier than \( m_{inf}/2 \) (see also Ref. \textsuperscript{[12]}). Primordial leptogenesis could then take place only through the direct decay of the inflaton into light particles (see also Ref. \textsuperscript{[13]}). We took a simple SUSY grand unified theory (GUT) model which is based on the gauge group \( G_{B-L} = G_{SM} \times U(1)_{B-L} \) (\( G_{SM} \) is the SM gauge group, and \( B \) and \( L \) the baryon and lepton number respectively) and naturally incorporates the standard SUSY realization \textsuperscript{[14,15]} of hybrid inflation \textsuperscript{[16]}. The flatness of the inflationary trajectory at tree level was guaranteed by a U(1) R-symmetry, whereas radiative corrections provided \textsuperscript{[17]} a logarithmic slope along this path, needed for driving the inflaton towards the SUSY vacua. The R-symmetry also guaranteed the conservation of baryon number to all orders in perturbation theory. Therefore, baryon number was only violated by the non-perturbative electroweak...
sphaleron effects.

The model incorporated the solution of the \( \mu \) problem of the minimal supersymmetric standard model (MSSM) proposed in Ref. \[17\]. Although the global R-symmetry forbade the appearance of a \( \mu \) term in the superpotential, it did allow the existence of the trilinear term \( S h_1 h_2 \), where \( S \) is the gauge singlet inflaton of standard SUSY hybrid inflation and \( h_1, h_2 \) are the electroweak Higgs superfields. After the GUT gauge symmetry breaking, the soft SUSY-breaking terms, which generally violated the R-symmetry, gave rise to a suppressed linear term in \( S \) and, thus, this field acquired a vacuum expectation value (VEV) of the order of the electroweak scale divided by a small coupling constant. The above trilinear coupling could then yield a \( \mu \) term of the right magnitude.

The same coupling also gave rise to tree-level couplings of the inflaton, which consisted of two complex scalar fields, to the electroweak Higgs bosons and Higgsinos. After the termination of inflation, the inflaton performs damped oscillations about the SUSY vacuum and eventually decays predominantly into electroweak Higgs superfields via these tree-level couplings, thereby reheating the universe. The model contained both heavy RHN and \( SU(2)_L \) triplet superfields which both contributed to light neutrino masses. A primordial lepton asymmetry could be generated at reheating via the subdominant decay of the inflaton into lepton and Higgs superfields, thereby reheating the universe. The model contained both heavy RHN and \( SU(2)_L \) triplet superfields which both contributed to light neutrino masses. A primordial lepton asymmetry could be generated at reheating via the subdominant decay of the inflaton into lepton and Higgs superfields through the interference between one-loop diagrams with RHN and \( SU(2)_L \) triplet exchange respectively. The simultaneous presence of both RHNs and \( SU(2)_L \) triplets was essential for this particular leptogenesis mechanism to work. Note that, in the model of Ref. \[11\], the generation of a non-zero lepton asymmetry did not rely on the existence of more than one fermion family. However, more than one family was required if the lepton asymmetry was to be preserved down to the electroweak phase transition. Finally, it should be emphasized that the generation of a non-zero lepton asymmetry required the inclusion of some couplings in the superpotential that explicitly violate the \( U(1) \) R-symmetry.

In this paper, we investigate the consequences of doing without \( SU(2)_L \) triplet superfields and instead generating the primordial lepton asymmetry through the exchange of different RHN superfields. Since these heavy fields can only appear in intermediate states of the inflaton decay, we must create the asymmetry directly from this decay. Indeed, leptogenesis can again occur in the subdominant decay of the inflaton into lepton and Higgs superfields, but now through the interference between one-loop diagrams with different RHN exchange. The lepton asymmetry is proportional to a novel CP-violating invariant product of coupling constants.

We study this new leptogenesis scenario within the framework of the model of Ref. \[11\] with the \( SU(2)_L \) triplet superfields removed. All the salient properties of the model are retained except that now the light neutrino masses are generated by the standard seesaw mechanism \[8\] which involves only RHN (super)fields and leptogenesis takes place via the new mechanism mentioned above. In particular, the implementation of the standard SUSY hybrid inflationary scenario \[14, 15\] and the solution of the \( \mu \) problem of Ref. \[17\] remains unaffected. Also, baryon number is still conserved to all orders in perturbation theory.

The model again contains an approximate \( U(1) \) R-symmetry explicitly broken by some superpotential operators. These operators, which are necessary to create a non-zero lepton asymmetry, also violate the \( Z_2 \) matter parity subgroup of the R-symmetry which is preserved by soft SUSY-breaking terms. The matter parity violation may have important observable consequences at low energy. Indeed, if the lightest particle (LSP), which is assumed to be the lightest neutralino, contains a Higgsino component, we find that it could decay predominantly into one or two Higgs bosons and a lepton. These dominant decay channels can then be easily blocked kinematically if the LSP is not too heavy and, under certain conditions, this particle can be made long-lived (see Sec. \[VI\]). However, some matter-parity-violating slepton decays which may be detectable in the future colliders are typically present (see Sec. \[IX\]).

We perform a detailed chemical potential analysis of the evolution of the primordial lepton asymmetry until the time at which the baryon-number-violating electroweak sphaleron effects cease to operate and evaluate the baryon asymmetry at this moment, which yields the observed BAU. We find that there are some lepton-number-violating four-scalar processes involving Higgs bosons and sleptons which are in equilibrium in a range of temperatures just above the SUSY threshold. These processes (which result from the same dimension-four operators that cause the above-mentioned slepton decays) could readily erase the primordial lepton asymmetry, which would lead to the absence of any baryon asymmetry in the present universe. However, it is possible to choose parameters so that one of the three lepton family numbers is conserved by all dimension-four processes. One can further show that all the dimension-five processes which violate this number are well out of equilibrium at all temperatures after reheating. Thus, we can obtain a non-zero BAU at present.

We find that the value of the BAU from the Wilkinson microwave anisotropy probe (WMAP) data \[18\] can be achieved given constraints from other observables, notably the reheating temperature and neutrino masses and mixing, and CP-violating phases of order unity. Note that, contrary to Ref. \[11\] where we worked in the two heaviest family approximation, in the present paper we take fully into account all three neutrino species. We find that only for a small range of values of the lightest neutrino mass \( m_1 \) can the scenario be successful, given the requirement that the GUT breaking scale be perturbative relative to an underlying string or quantum gravity scale, which is also restricted to smaller values than the value used in Ref. \[11\]. The reheating temperature is pushed to higher values relative to this reference. Finally,
the values of the neutrino Yukawa couplings are also restricted. These results reflect the fact that removing the SU(2)$_L$ triplets from the model makes it more restrictive. The prediction for the spectral index of density perturbations is typical of SUSY hybrid inflation models (see e.g. Ref. [10]).

Thus, an acceptable value of the baryon asymmetry can be obtained within a consistent model of cosmology and particle physics, without requiring additional fine-tuned coupling constants, while respecting experimental constraints on neutrino masses and mixing. Moreover, although the scenario requires violation of the R-symmetry, it is not necessary to introduce superpotential terms which would lead to currently observable R-symmetry-violating effects.

We also analyzed the case where SU(2)$_R$ gauge symmetry is imposed, forcing the neutrino Yukawa couplings to be identical to those of the charged leptons at the GUT scale. This case is thus considerably more restrictive than the non-SU(2)$_R$-symmetric case. One of the main differences is that it predicts that the lightest RHN mass is of order $10^7$ GeV, whereas the inflaton mass is greater than $10^{11}$ GeV, thus the inflaton can decay directly to the lightest RHN. However, this does not alter our basic picture for leptogenesis since the corresponding branching ratio and non-thermally generated lepton asymmetry are negligible due to the fact that the lightest RHN mass is very small compared to the inflaton mass. Moreover, thermal leptogenesis from the decay of this particle can be ignored for the same reason. The presence of the lightest RHN in the thermal bath after reheating, however, yields extra restrictions on the parameters of the model from the requirement that the initial lepton asymmetry is not erased. In the end, this case is ruled out because it always yields an unacceptably small BAU. The reason is that our leptogenesis scenario requires strong mixing in the RHN sector but, in this case, the RHN mass matrix has too small off-diagonal elements due to the strong hierarchy of Yukawa coupling constants implied by the SU(2)$_R$ symmetry and the extra restrictions from the preservation of the primordial asymmetry.

In Sec. VIII, we introduce the SUSY GUT models and describe some of their salient features. In Sec. IX, we discuss constraints from the light neutrino masses and mixing, while, in Sec. X, we present the CP-violating invariant products of coupling constants which enter into the primordial lepton asymmetry. The calculation of the primordial lepton asymmetry is sketched in Sec. XI and the effects of R-symmetry (and R-parity) violation are discussed in Sec. XII. Section XIII is devoted to the conditions for the primordial lepton asymmetry to be preserved at high temperatures. We revisit the relation between the initial lepton asymmetry and the final baryon asymmetry in Sec. XIV, finding some novel results for the case when some lepton family numbers are violated. Our numerical results appear in Sec. XV, including a discussion of why the SU(2)$_R$-symmetric case is ruled out. Finally, conclusions appear in Sec. XVI.
the charges of $l^c_i$, $q^c_i$ and $h$ coincide with the charges of their components in Table I. Below the scale $M$, the field content is the same as in the previous case. The superpotential, in this case, is
\[
W = \kappa_S(L^c_i L^c_i - M^2) + \lambda_S h^2 + y_{i1} h \epsilon t^c_j + y_{i2} h q^c_j + (M_{\nu_{ij}}/M^2) L^c_i L^c_j + (\lambda_i/M_S) L^c_i h^2 + \cdots,
\]
where $h^2$ denotes the gauge invariant sum $\frac{1}{2} \text{Tr}(\epsilon^T \epsilon) = (h_1 h_2)$ and $L^c_i$, $t^c_j$, $q^c_j$ are taken as column 2-vectors, while $L^c_i$, $t_i$, $q_i$ as row 2-vectors (the superscript $T$ denotes matrix transposition). Again the only $U(1)_R$-violating superpotential couplings are the ones in the last explicitly displayed term in the RHS of Eq. (2) and $B$ conservation to all orders is automatic. Note that, imposing the SU(2)$_L$ symmetry on the superpotential of Eq. (1), we obtain that the Yukawa coupling constants $y_{i1}$, $y_{i2}$ and $y_{i3}$ $(\nu_{ij})$ become equal [20]. Their common value is the coupling constant $y_{i3}$ in Eq. (2).

In both cases, the classically flat direction in field space along which inflation takes place is as described in Ref. [17]: for $\kappa < \lambda$, it is parametrized by $S$, $|S| > S_c = M$, with the values of all the other fields being equal to zero, and has a constant potential energy density $\kappa^3 M^4$ (at tree level). Here, the dimensionless coupling constants $\kappa$, $\lambda$ and the mass parameter $M$ are taken real and positive by appropriately redefining the phases of the superfields. There are radiative corrections [15] which lift the flatness of this classically flat direction leading to slow-roll inflation which, for $\kappa \ll 1$, terminates practically when $|S|$ reaches the instability point at $|S| = M$ as one can easily deduce from the slow-roll $\epsilon$ and $\eta$ criteria. The quadrupole anisotropy of the cosmic microwave background radiation (CMBR) and the number of e-foldings
\[
N_Q \simeq \ln \left[ 1.88 \times 10^{11} \kappa^4 \left( \frac{M}{\text{GeV}} \right)^4 \left( \frac{T_{\text{reh}}}{\text{GeV}} \right)^4 \right]
\]
suffered by our present horizon scale during inflation are given by the Eqs. (2)-(4) of Ref. [21]; in the non-SU(2)$_R$-symmetric case, the two last terms in the RHS of Eq. (3) must be divided by two since the SU(2)$_R$ doublet chiral superfields $L^c_i$, $\tilde{L}^c_i$ are replaced by the SM singlets $\phi$, $\tilde{\phi}$ (see also Ref. [22]).

For $\kappa < \lambda$, a GUT-symmetry-breaking phase transition takes place when the value of $|S|$ falls below the mass parameter $M$. The fields evolve towards the realistic SUSY minimum at $S = 0$, $\langle \phi \rangle = \langle \tilde{\phi} \rangle = M$, $\langle h_1 \rangle = \langle h_2 \rangle = 0$, where the GUT-symmetry-breaking VEVs are taken real and positive by a $B - L$ rotation. There is also an unreal SUSY minimum which is given below. With the addition of soft SUSY-breaking terms, as required in a realistic model, the position of the vacuum shifts [17] to non-zero $[S] \simeq -m_{3/2}/\kappa$, where $m_{3/2}$ is the mass of the gravitino, and an effective $\mu$ term with $\mu \simeq -\lambda m_{3/2}/\kappa \sim m_{3/2}$ is generated from the superpotential coupling $\lambda S(h_1 h_2)$.

After this phase transition, the (complex) inflaton degrees of freedom are $S$ and $\theta \equiv (\delta \phi + \delta \tilde{\phi})/\sqrt{\Sigma}$, where $\delta \phi = \phi - M$ and $\delta \tilde{\phi} = \tilde{\phi} - M$, with mass $m_{\text{inf}} = \sqrt{2}\kappa M$. These fields oscillate about the minimum of the potential and decay to MSSM degrees of freedom reheating the universe. The predominant decay channels of $S^*$ and $\theta$ are to fermionic and bosonic $h_1$, $h_2$ respectively via tree-level couplings derived from the superpotential terms $S(h_1 h_2)$ and $\kappa S \delta \phi + (\kappa S L^c_i L^c_i)$. Note that, if $\kappa > \lambda$, the system would end up in the unrealistic SUSY minimum at $\phi = \tilde{\phi} = 0$, $|h_1| = |h_2| \simeq (\kappa/\lambda)^{1/2} M$, which is degenerate with the realistic one (up to $m_{3/2}^2/\kappa^2$) and is separated from it by a potential barrier of order $m_{3/2}^2/\kappa^2$.

The RHNs acquire masses $M_{\nu_{ij}}$ after the spontaneous breaking of $U(1)_{B-L}$ by $\langle \phi \rangle$, $\langle \tilde{\phi} \rangle$. The coupling constants of the relevant terms in the third line of the RHS of Eq. (1) (or the second line of Eq. (2)) can also be written as $\lambda_{\nu_{ij}}/M_S$, making it clear that the RHN masses are suppressed by a factor $M/M_S$ relative to $M$. It is possible to redefine the superfields $\nu^c_i$ (or $l^c_i$) to obtain effective mass terms $M_{\nu_{ij}} \nu^c_i \nu^c_j$, which are diagonal in the flavor space with $M_\nu$ real and positive. This basis, which we will call “RHN basis”, is most convenient for calculating the BAU (see Sec. V). For definiteness, we take $M_1 \leq M_2 \leq M_3$.

We will also use a basis in which the lepton Yukawa couplings and the SU(2)$_L$ interactions are diagonal in flavor space, denoting couplings and fields in this “lepton family basis” with a hat. We thus have $\tilde{y}_{i3} = \delta_{ij} \hat{y}_{ij}$, $\hat{y}_{i3} = \delta_{ij} \hat{y}_{ij}$ (or $\tilde{y}_{ij} = \delta_{ij} \hat{y}_{ij}$). The diagonal Yukawa coupling constants $\hat{y}_{ij}$, $\hat{y}_i$ (or $\tilde{y}_i$) are taken real and positive by appropriate rephasing of the fields. Note that, in the non-SU(2)$_R$-symmetric case, it is an additional assumption that the neutrino and charged lepton Yukawa couplings can be diagonalized in the same weak interaction basis. We will elaborate on this assumption in Sec. VII. Since the weak interactions are in equilibrium at temperatures above the critical temperature $T_{\text{cr}}$ for the electroweak phase transition, the lepton family numbers $L_i$ can only be defined in this basis.

Relative to the “unhatted” RHN basis, we have
\[
\hat{l} = l U, \quad \hat{\nu}^c = U^c \nu^c. \tag{4}
\]

Here, $U$, $U^c$ are $3 \times 3$ unitary matrices and we write “left-handed” lepton superfields, i.e. SU(2)$_L$ doublet leptons, as row 3-vectors in family space and “right-handed” anti-lepton superfields, i.e. SU(2)$_R$ singlet anti-leptons, as column 3-vectors. In the SU(2)$_R$-symmetric case, we have the same relation except that the SU(2)$_R$ doublet $l^c_i$ ($\tilde{l}^c_i$) replaces $\nu^c_i$ ($l^c_i$). Then by definition we have, for the neutrino Yukawa couplings,
\[
l_{i1} y_{i1} \nu^c_i = \hat{l}_{i1} y_{i1} U^c l^c_i \nu^c, \quad U^c l^c_i \nu^c_i = \hat{y}_i = \text{diag}(\hat{y}_1, \hat{y}_2, \hat{y}_3). \tag{5}
\]
where $y_\nu$ is the $3 \times 3$ matrix with elements $y_{\nu ij}$. In general, the $\hat{y}_i$ can be any (real positive) numbers. In the SU(2)$_R$-symmetric case, however, they are determined by the “asymptotic” values (at the GUT scale) of the charged lepton masses as $\hat{y}_{12,3} = m_{\nu_i} \tan \beta / (h_2)$. The Higgs VEV is $(h_2) \simeq 174 \sin \beta$ GeV, which, in the large $\tan \beta$ limit, yields $(h_2) \simeq 174$ GeV. Exact SU(2)$_R$ Yukawa coupling relations in the quark sector would fix the ratio of the Higgs VEVs, but at least small deviations are required here to be consistent with the data. Assuming that the corrections are small compared to the third generation Yukawa couplings, $\tan \beta$ should be about 55. (For the conditions under which SU(2)$_R$ implies large $\tan \beta$, see Ref. [17].) It is, of course, possible to write down many other R-symmetry-violating operators. However, they are not necessary for a non-zero primordial lepton asymmetry to be created.

The Yukawa coupling constants $y_{\nu ij}$ (in the original RHN mass eigenstate basis) remain in general complex as one can easily deduce from the rephasing invariants $y_{\nu ij} y_{\nu k}^* C_{ij} C^*_{jk}$ for $j \neq k$ (no summation over repeated indices), as well as the standard rephasing invariant $y_{\nu ij} y_{\nu kj} y_{\nu im} y_{\nu km} M_{ij}^* M_{lm}$ for $j \neq m$ (no summation), which has been used in the original leptogenesis scenario of Ref. [1].

The calculation of lepton asymmetry produced in $S$ and $\theta$ decays is quite straightforward but somewhat lengthy, and differs in detail from the usual case where leptogenesis takes place through the decay of RHN or SU(2)$_L$ triplet superfields. Since we consider the interference of two one-loop diagrams, we will need to find the real parts of loop integrals, which require renormalization. Before proceeding to the calculation, we summarize the constraints derived from the current neutrino experiments.

### III. NEUTRINO MASSES AND CONSTRAINTS

The standard seesaw mechanism yields the light neutrino mass matrix:

$$m = -(h_2)^2 y_{\nu} M^{-1}_{\nu} y^T_{\nu}, \quad (6)$$

which holds in any basis of neutrino states with $M_{\nu}$ being the $3 \times 3$ matrix with elements $M_{\nu ij}$. In the RHN mass eigenstate basis, where $M_{\nu} = \text{diag}(M_1, M_2, M_3)$, it can be rewritten as

$$m = -(h_2)^2 \bar{y}_{\nu} U^c M^{-1}_{\nu} U^T y_{\nu} U^T. \quad (7)$$

Thus, transforming the light neutrino fields to the “hatted” (lepton family) weak interaction basis, we have

$$\hat{m} = U^T m U = -(h_2)^2 \bar{y}_{\nu} U^c M^{-1}_{\nu} U^T y_{\nu}. \quad (8)$$

The light neutrino mass eigenstate basis can be denoted by $\hat{\nu}$ and is given by

$$\hat{\nu} = \nu V^*, \quad (9)$$

where $\nu, \hat{\nu}$ are row 3-vectors in family space and $V$ is the unitary Maki-Nakagawa-Sakata (MNS) matrix which satisfies

$$V^T \hat{m} V = \hat{m} \equiv \text{diag}(m_1, m_2, m_3) \quad (10)$$

with $m_i$ real and positive. Again $V$ is determined only up to a diagonal matrix of complex phases as $V \rightarrow PV$, which reflects the freedom to redefine the phases of the superfields in the “hatted” basis. However, the ambiguity is fixed by requiring $V$ to take the standard form

$$V = \begin{pmatrix} c_3 c_2 & s_3 c_2 & s_2 e^{-i\delta} \\ -c_1 s_3 - c_3 s_1 s_2 e^{i\delta} & c_3 c_1 - s_3 s_1 s_2 e^{i\delta} & c_2 s_1 \\ s_3 s_1 - c_3 c_1 s_2 e^{i\delta} & -c_3 s_1 - c_1 s_3 s_2 e^{i\delta} & c_2 c_1 \end{pmatrix} \cdot \mathcal{P} \quad (11)$$

where $c_i \equiv \cos \theta_i$, $s_i \equiv \sin \theta_i$, with $\theta_i$ $(i = 1, 2, 3)$ being the appropriate mixing angles, $\delta$ is the Dirac phase and $\mathcal{P} = \text{diag}(e^{-i\alpha}, e^{-i\beta}, 1)$ contains the Majorana phases.

In both the SU(2)$_R$- and non-SU(2)$_R$-symmetric cases, we will restrict the light neutrino mass-squared differences and mixing angles to be within the 2$\sigma$ allowed region detailed in Ref. [24]. For most numerical results, however, we will set these observables to their best-fit values, implying that $\hat{m}$ is completely determined once the lightest neutrino mass, the phases $\alpha$ and $\beta$ and the choice of normal or inverted hierarchy are fixed. We will take here the light neutrino mass ordering $m_1 \leq m_2 \leq m_3$ and, unless stated otherwise, adopt the normal hierarchical scheme of neutrino masses, where the solar and atmospheric neutrino mass-squared differences are identified with $\delta m^2_{21} \text{ and } \delta m^2_{31}$ respectively (here $\delta m^2_{ij} = m_i^2 - m_j^2$).

For simplicity, we further take $\alpha = \beta = 0$.

For given values of $m_i$ and $V$, one can find the light neutrino mass matrix in the “hatted” basis by inverting Eq. (11). Then, in the SU(2)$_R$-symmetric case, the
seesaw formula in Eq. (8) can be applied with \( \hat{y}_i \) determined by the known values of the charged lepton masses, renormalization group (RG) evolved to the GUT scale, and for given \( \tan \beta \). Solving the resulting equations, we can simultaneously determine the elements of the unitary matrix \( U^c \) and the RHN mass eigenvalues \( M_i \). In the non-SU(2)\( _L \)symmetric case, the Yukawa coupling constants \( \hat{y}_i \) are not related to the charged lepton masses and, thus, are treated as free input parameters.

For our mechanism to be the exclusive source of baryon asymmetry as desired, either all RHN masses must be greater than \( m_{\text{inf}}/2 \) so that the inflaton decay to RHN superfields is kinematically blocked and the thermal production of RHNs after reheating is suppressed, or any RHNs that are lighter must contribute a negligible net lepton number density via the conventional thermal or non-thermal leptogenesis mechanism, by which they decay out of equilibrium. One way to achieve the latter is to make them adequately light. We also have to check that thermal interactions of any lighter RHNs do not wash out the already created lepton asymmetry. Furthermore, assuming that the dimensionless coefficients \( \lambda_{\nu \nu \nu} \) of the last superpotential terms in the third line of the RHS of Eq. (1) (or the second line of the RHS of Eq. (2)) do not exceed unity, we obtain that the heaviest RHN must not have a mass greater than \( M^2/2M_S \). These requirements place non-trivial constraints on the model parameters which will be further discussed in Sec. IX.

### IV. CP-VIOLATING INVARIANTS

The generation of a \( B - L \) asymmetry from the decay of the inflaton requires that the theory contains at least one CP-violating product of coupling constants which remains unaffected by field rephasing and is non-real corresponding to an operator non-invariant under CP conjugation. Since leptogenesis takes place at reheating, we work in the vacuum where \( \langle \phi \rangle = \langle \bar{\phi} \rangle = M \) and ignore the \( \mu \) term. In previous work [11], when calculating the lepton asymmetry from the CP-violating inflaton decay, we considered only the couplings of the inflaton originating from the renormalizable terms in the superpotential of Eq. (1) (or Eq. (2)). In fact, when one does also include the couplings of the inflaton to RH \( \bar{\nu} \) and Higgs superfields from the last two explicitly displayed non-renormalizable terms in the RHS of Eq. (1) (or Eq. (2)), it turns out that their contribution to the asymmetry vanishes exactly. Thus, we will replace these two superpotential terms by the effective mass terms \( M_i \nu^c_i \nu_i^c \) and couplings \( \zeta \nu^c_i (h_1 h_2) \) respectively. Since the only massive particles at reheating are the inflaton and the RHNs, it is convenient to work in the basis where the RHN Majorana masses are diagonal.

In Ref. [11], we considered rephasing invariants which could survive even if there were only one matter generation. This was possible because of the presence of the SU(2)\( _L \) triplet superfields. In the current models, we must rely on the presence of more than one generation. In addition to the standard leptogenesis CP-violating invariant composed of four Yukawa coupling constants and two RHN masses, there is a CP-violating invariant which can be written in the “hatted” basis, where the neutrino Yukawa couplings are diagonal, as

\[
I_{ijk} = \hat{\zeta}_{ik} \hat{M}^{c*}_{ij} \hat{M}^{c*}_{jk} \hat{M}^{c*}_{ki}
\]

for \( i \neq j \) (no summation). However, this does not survive transformation to the RHN basis and does not on its own give rise to diagrams which can create a lepton number asymmetry in light fields since the corresponding operator does not involve any light lepton superfields. Instead, we “dress” it with Yukawa coupling constants to obtain the CP-violating invariant

\[
I_{ijk} = y_{\nu i j} y_{\nu i k} \hat{\zeta} \hat{M}^{c*}_{ij} \hat{M}^{c*}_{jk} M_k
\]

for \( j \neq k \) (no summation), where we now use the mass eigenstate basis of RHNs. When this invariant is used, the generation of a \( B - L \) asymmetry is independent of the sources of CP violation considered in previous scenarios and we require novel decay diagrams. This CP-violating invariant is minimal in the sense that it involves the smallest possible number of trilinear superpotential couplings. We can transform the first (light lepton) index of this invariant to the “hatted” basis and re-express the invariant in terms of the “hatted” coupling constants as

\[
\hat{I}_{ijk} = \sum_{m, n} \hat{y}^2_{ij} U^c_{ij} U^c_{ik} \hat{\zeta}_m U^c_{mj} \hat{M}^{c*}_{jk} \hat{M}^{c*}_{nk} M_k,
\]

where \( \hat{\zeta}_m \) is defined by

\[
\hat{\zeta}_n = \sum_m \hat{\zeta}_m U^c_{mn},
\]

and we have redefined fields such that \( M_i \) and \( \hat{y}_i \) are real and positive (the two last indices of the invariant remain in the RHN basis). This will be convenient because the Yukawa couplings in this basis preserve the individual lepton numbers \( L_i \).

Note that the invariant \( I_{ijk} \) can be split in two factors \( y_{\nu i j} \hat{\zeta}_j M^*_{jk} \) and \( y_{\nu i k} \hat{\zeta}_k M_k \) which correspond to effective operators with opposite non-zero \( B - L \) charges. These operators involve only light fields since the heavy ones can be contracted. One of these two operators includes bosonic or fermionic \( h_1 \), \( h_2 \), while the other contains their conjugates. These properties are essential for leptogenesis since the inflaton field couples at tree level to the electroweak Higgs superfields \( h_1 \), \( h_2 \) and can decay only to light particles. So, we conclude that \( \hat{I}_{ijk} \) is, in principle, suitable for leptogenesis which requires the interference of two \( (B - L) \)-violating diagrams for the inflaton decay. One can further show that, if the RHN spectrum and couplings are such that the standard leptogenesis scenario (without the \( \zeta \) couplings) fails, any invariant which can be useful for leptogenesis must involve \( \hat{I}_{ijk} \), and thus the effective coupling constants \( \hat{\zeta}_i \). So, the explicit violation of \( U(1)_R \) and matter parity is essential for our scheme. This is another novel feature of this scenario.
V. BARYON ASYMMETRY

In Figs. 1 and 2, we present the diagrams for the $L_i$-violating decay of $S$ and $\theta^*$ respectively. Observe that the CP-violating rephasing invariant $I_{ijk}$ corresponds to the product of coupling constants in the interference of any diagram in Fig. 1 (Fig. 2) with any diagram in the same figure where the exchanged RHN is renamed $\nu'_k$. This interference contributes to the $L_i$ asymmetry due to a partial rate difference in the decays $S \rightarrow l_i h_2$ and $S^* \rightarrow \tilde{l}_i^* h_2^* (\theta^* \rightarrow l_i^* \tilde{h}_2^* \text{ and } \theta \rightarrow \tilde{l}_i \tilde{h}_2)$, where bar and tilde represent the anti-fermion and the SUSY partner respectively. Both the decaying inflaton field ($S$ or $\theta^*$), which is taken at rest, and the decay products must be on mass shell. For simplicity, we consider that all the propagating and external MSSM particles in the diagrams are massless. Moreover, we perform the calculation in the limit of exact SUSY.

We also considered diagrams where the inflaton decays via its couplings to RHN and Higgs superfields which arise from the superpotential terms with coefficients $M_{\nu ij}$ and $\lambda_i$. Such diagrams exist only for the decay of $\theta^*$ and have final states $(l_i \tilde{h}_2, l_i^* h_2, \tilde{l}_i h_2, \tilde{l}_i^* h_2)$ which are different from the final state of the diagrams shown in Fig. 2. However, it can be argued that these diagrams sum to zero in the decay amplitudes if the final state is $\tilde{l}_i h_2$ or $\tilde{l}_i^* h_2$. For the other two cases, the diagrams again sum to zero but only in the amplitude-squared after summing over the phase space of final particles and in the limit where these particles are massless.

We will denote by $F_{ijmn}^S$ and $F_{bij}^S$ the “stripped” diagrams in Figs. 1a and 2a respectively with the dimensionless coupling constants and the $M_j$ mass insertions factored out (we keep, though, the $m_{inf}$ factor appearing in the scalar coupling $\theta^* h_1 h_2$). Here $i$ and $j$ are family indices and $n = 1, 2, 3$ the serial number of the diagram. Similarly, the “stripped” diagrams in Figs. 2b and 2a, are $F_{aij}^g$ and $F_{bij}^g$. In each case, the contribution to the $L_i$ asymmetry is proportional to both $\text{Im} I_{ijk}$ and the imaginary part of the interference of the relevant “stripped” diagrams. Thus, the total net $L_i$ asymmetries $\epsilon_{ij S}$ and $\epsilon_{ij \theta}$ generated per $S$ and $\theta^*$ decay respectively are

$$
\epsilon_{ij S} = -\frac{|\lambda|^2}{\Gamma} \sum_{j,k} \text{Im} \hat{I}_{ijk} \sum_{t,t',n,n'} \text{Im} [F_{t ij n}^S F_{t' t' n'}^{S \dagger}],
$$

$$
\epsilon_{ij \theta} = -\frac{|\lambda|^2}{\Gamma} \sum_{j,k} \text{Im} \hat{I}_{ijk} \sum_{i,i'} \text{Im} [F_{ij i}^\theta F_{i' i' k}^{\theta \dagger}],
$$

(16)

where $\Gamma = |\lambda|^2 m_{inf}/8\pi$ is the common rate of the tree-level decays $S \rightarrow h_1 h_2$ and $\theta \rightarrow h_1 h_2$, the indices $t,t' = a, b$, and integration over the phase space of the particles in the final state is implied.

FIG. 1: The six one-loop diagrams for the $L_i$-violating decay $S \rightarrow l_i h_2$. The solid (dashed) lines represent the fermionic (bosonic) component of the indicated superfield. The arrows depict the chirality of the superfields and the crosses are mass insertions in fermion lines.

FIG. 2: The two one-loop diagrams for the $L_i$-violating decay $\theta^* \rightarrow l_i h_2$. The notation is as in Fig. 1.
The diagrams are both of the vertex (Figs. 1a and 2a) and self-energy (Figs. 1b and 2b) type. Each of the three vertex diagrams in Fig. 1a possesses a logarithmic ultraviolet (UV) divergence. However, it can be easily shown that their sum equals \( m_{\text{inf}} \) times the vertex diagram in Fig. 2b, which is UV finite. Similarly, one can show that the sum of the three quadratically divergent self-energy diagrams in Fig. 1b equals \( m_{\text{inf}} \) times the self-energy diagram in Fig. 2b. The latter is though not UV finite. It rather possesses a logarithmic divergence and, thus, needs renormalization.

The above relations between the diagrams for the \( L_i \)-violating decays of \( S \) and \( \theta^* \) imply that \( \epsilon_{i|S} = \epsilon_{i|\theta} = \epsilon_i \). We thus concentrate on the calculation of \( \epsilon_i \). The vertex diagram in Fig. 2a being finite (both its real and imaginary parts) is independent of the renormalization scheme used. However, the diagram in Fig. 2b involving a divergent self-energy loop requires us to apply a renormalization condition. As argued in Ref. [2], the appropriate renormalization scheme, in this case, is the on-shell (OS) scheme. In a general theory with scalars \( S_i \), the OS conditions on the renormalized self-energies \( \hat{\Pi}_{ij}(p^2) \) are as follows:

\[
\text{Re} \hat{\Pi}_{ij}(\mu_i^2) = \text{Re} \hat{\Pi}_{ij}(\mu_j^2) = 0 \quad (17)
\]

for the off-diagonal self-energies \( (i \neq j) \) and

\[
\lim_{p^2 \to \mu_i^2} \frac{1}{p^2 - \mu_i^2 - \mu_j^2} \text{Re} \hat{\Pi}_{ij}(p^2) = 0 \quad (18)
\]

for the diagonal ones (see e.g. Ref. [27]). Here we take a basis where the renormalized mass matrix is diagonal with eigenvalues \( \mu_i \). The imaginary part of the self-energies is finite and, thus, not renormalized. In Fig. 2b, we have an off-diagonal self-energy diagram between the scalars \( \theta \) and \( \hat{\nu}_i^* \). Given that \( \theta \) is on mass shell, the real part of this diagram vanishes in the OS scheme. The imaginary part, however, gives a finite contribution.

The asymmetry generated in the lepton number \( L_i \) per decaying inflaton is then

\[
\epsilon_i = -m_{\text{inf}}^2 \sum_{j,k} \text{Im} \hat{I}_{ijk} \sum_{t,t'} \text{Im}[F^t_j F^{t'}_{k}]. \quad (19)
\]

Here, the indices \( t, t' \) can be thought of as indices which refer to the topology of the diagrams in Fig. 2. Namely \( t = \text{trg} \) corresponds to the vertex diagram in Fig. 2a, while \( t = \text{self} \) to the self-energy diagram in Fig. 2b. We find that

\[
F^\text{trg}_j = \frac{1}{16\pi^2 m_{\text{inf}}^2} \left[ \frac{\pi^2}{6} - \text{Li}_2 \left( 1 + \frac{m_{\text{inf}}^2}{M_j^2} + i\varepsilon \right) \right], \quad (20)
\]

where \( \text{Li}_2 \) is the dilogarithm, and

\[
F^\text{self}_j = \frac{i}{4\pi m_{\text{inf}}^2 - M_j^2}. \quad (21)
\]

This equation for the contribution of the self-energy diagram holds if \( \theta_i \phi' = 0 \). The only exception is when \( M_j \) is not naturally close to \( m_{\text{inf}} \). Also, note that the contribution to the second sum in the RHS of Eq. (19) originating from the interference of two self-energy diagrams (i.e. the term with \( t = t' = \text{self} \)) vanishes. The reason is that \( F^{\text{self}}_j \) is pure imaginary. Finally, the contributions to the RHS of Eq. (19) which are diagonal in family space (i.e. with \( j = k \)) also vanish since \( \hat{I}_{ijk} \) is real in this case.

The equilibrium conditions including non-perturbative electroweak reactions, for temperatures below the mass scale of superpartners and the critical temperature \( T_c \) of the electroweak phase transition, yield a relation which allows us to find the final baryon asymmetry \( n_B / s \) in terms of the \( B - 3L_i \) asymmetries \( n_{B - 3L_i} / s \), where \( n_X \) is the density of the quantum number \( X \) and \( s \) is the entropy density. Note that, in this regime, all three quantum numbers \( B - 3L_i \) are conserved. This is, however, not true for temperatures above the scale of sparticle masses, which is taken to lie above the critical temperature of the electroweak transition. We assume that at least one of these quantum numbers, which we designate as \( B - 3L_3 \), is conserved at all temperatures after reheating (see Sec. VII). Then, in a temperature range just above the SUSY threshold where \( B - 3L_i \) are violated by processes involving sparticles, we can determine \( n_{B - 3L_3} / s \) in terms of \( n_{B - 3L_i} / s \). Assuming continuity of \( n_{B - 3L_i} / s \) as the temperature crosses the SUSY threshold, we can express \( n_B / s \) in terms of \( n_{B - 3L_3} / s \) (see Sec. VIII). The result can be written as

\[
\frac{n_B}{s} = k \frac{n_{B - 3L_3}}{s}. \quad (22)
\]

Then if we imagine the inflaton to decay instantaneously out of equilibrium creating initial \( L_3 \) lepton number density \( n_{L_3,\text{init}} \), we have

\[
\frac{n_B}{s} = -3k \frac{n_{L_3,\text{init}}}{s} = -3k \epsilon_3 \frac{n_{\text{inf}}}{s} \frac{s}{t_{\text{reheat}}} \frac{T_{\text{reheat}}}{4 m_{\text{inf}}} \quad (23)
\]

using the standard relation \( n_{\text{inf}} / s = (n_S + n_\theta) / s = 3T_{\text{reheat}} / 4 m_{\text{inf}} \) for the inflaton number density. The reheat temperature is given by

\[
T_{\text{reheat}} = \left( \frac{45}{2\pi^2 g_*} \right)^{1/4} (\Gamma_{m_P})^{1/4}, \quad (24)
\]

where \( m_P \simeq 2.44 \times 10^{16} \) GeV is the reduced Planck scale, and \( g_* \) counts the effective number of relativistic degrees of freedom taking account of the spin and statistics and is equal to 228.75 for the MSSM spectrum.

VI. EFFECTS OF R-SYMMETRY VIOLATION

We will now explore the possible phenomenological and cosmological consequences of the explicit violation
of $U(1)_R$ in the superpotential. Our models contain a $Z_2$ matter parity symmetry under which all the matter (quark and lepton) superfields change sign. This symmetry is actually the $Z_2$ subgroup of $U(1)_R$ and is left unbroken by the soft SUSY-breaking terms. Matter parity combined with the $Z_2$ fermion parity under which all fermions change sign yields R-parity, which, if unbroken, guarantees the stability of the LSP. In the present case, however, matter parity is violated along with the $U(1)_R$ by the last explicitly displayed term in the RHS of Eq. (1) (or Eq. (2)), which is necessary for generating the observed BAU. Consequently, R-parity is explicitly violated and the LSP can decay rapidly, rendering it unsuitable for dark matter and leading to distinctive collider signatures.

The LSP could decay into a pair of electroweak Higgs bosons and a lepton if it contains a Higgsino component. The dominant diagrams are constructed from the $U(1)_R$- and R-parity-violating Yukawa vertices $\zeta \nu^c_i (h_1 h_2)$ with the fermionic $\nu^c_i$ connected to the fermionic $\nu^c_j$ of the Yukawa couplings $y_{\nu ij} (l, h_2) \nu^c_j$ via a mass insertion. For the values of the parameters considered here (see Sec. IX), we find that the resulting LSP life-time can be as low as about $10^{-3}$ sec. However, it is easy to block kinematically these decay channels of the LSP by taking its mass to be smaller than twice the mass of the lightest Higgs boson, which is very reasonable. Even then, the LSP could decay to a lepton and an electroweak Higgs boson with similar life-time. The relevant (one-loop) diagrams may be obtained by connecting the two external Higgs lines of the previous diagrams to a trilinear Higgs vertex. These two-body decay channels can though also be blocked kinematically if the LSP mass is taken smaller than the lightest Higgs boson mass. In this case, the dominant diagrams for the LSP decay may be constructed from the above one-loop diagrams by coupling the external Higgs line to a pair of fermions (excluding the $t$-quark). So, we obtain one-loop diagrams leading to three-body decay of the LSP. They involve an extra small Yukawa coupling constant which together with the one-loop factor yields a suppressed decay rate. The lifetime can be of order $10^9$ sec or higher and can be further enhanced if the LSP is predominantly a gaugino. Thus, it may be possible to rescue the LSP as dark matter candidate, especially if its Higgsino component is suppressed. Alternatively, if the LSP decays fast, we would have to ensure that its life-time were less than 1 sec to avoid conflict with primordial nucleosynthesis (see e.g. Ref. 23).

Besides the LSP decay, we could also have other low energy processes which violate R-parity. The superpotential terms which break R-parity involve at least one superheavy field. On integrating out the superheavy fields, we generically obtain effective R-parity-violating operators involving only MSSM fields. If these operators have dimension five or higher, they do not lead to detectable processes since they are suppressed by some powers of $M_f$. On the contrary, operators with dimension four such as the effective scalar vertex $h_1 h_2 h^*_2 l^* i$ which originates from the superpotential couplings $\hat{y}_{\nu i j} (l_i h_2) \nu^c_i k^c_l$, which can lead to low-energy R-parity-violating processes which may be detectable in the future colliders (see Sec. IX).

As in any leptogenesis scenario with RHNS, we must ensure [30] that the primordial $L_1$ asymmetry (for at least one value of $i$) is not erased by lepton-number-violating scattering processes such as $l_i l_j \rightarrow h_2^* l^*_2$ or $l_i h_2 \rightarrow h_2^* l^*_j$ at all temperatures between $T_{\text{reh}}$ and about 100 GeV. In our case, due to the presence of the R-parity-violating superpotential terms, there exist some extra processes of this type such as $h_1 l_j \rightarrow h_2 l^*_2$ or $h_1 h_2 \rightarrow h_2^* l^*_j$, which come from diagrams similar to the ones mentioned above for the fast LSP decay. In addition to all these processes which correspond to effective operators of dimension five (or higher), we also have dimension-four processes which violate R-parity (and lepton number) such as the process $h_1 h_2 \rightarrow h_2 l_i$ derived from the effective four-scalar vertex in the previous paragraph. We will return to this issue in the next section.

As already mentioned, the classical flatness of the inflationary trajectory in the limit of global SUSY is ensured, in our case, by a continuous R-symmetry enforcing a linear dependence of the superpotential on $S$. This is retained even after supergravity corrections, given a reasonable assumption about the Kähler potential. The solution to the $\mu$ problem is also reliant on the R-symmetry. These properties are not affected by the explicit R-symmetry breaking considered here. Note that, in our scheme, some R-symmetry-violating couplings are included in the superpotential and some not. Thanks to the non-renormalization property of SUSY, this situation is stable under radiative corrections, but it may be considered unnatural since there is no symmetry to forbid the missing terms.

VII. PRESERVATION OF $B - 3L_3$

In this section, we will discuss the conditions for at least one quantum number $B - 3L_3$ to be preserved from reheating all the way down to the electroweak phase transition. This is important since, in the opposite case, the primordial lepton asymmetry will be erased. Note that the non-perturbative electroweak sphaleron effects preserve all these three quantum numbers. So, violation of some $B - 3L_4$ can only take place if the corresponding $L_4$ is violated perturbatively. ( Recall that $B$ is conserved to all orders in perturbation theory and the non-perturbative QCD instanton interactions conserve $B$ and all the $L_i$‘s.)

Dangerous $L_i$-violating reactions that could wash out any primordial asymmetry in $B - 3L_4$ include dimension-four scalar interactions arising from the F-term $|F_{\nu c_{\mu}}|^2$, which violate $L_i$ either through the R-parity-violating operator $\xi_{\nu c_{\mu}} (h_1 h_2)$ or if they involve two Yukawa couplings $\hat{y}_{\nu i j} (l_i h_2) \nu^c_i k^c_l$ and $\hat{y}_{\nu j k} (l_j h_2) \nu^c_j k^c_l$ with $i \neq j$. Note that the latter possibility exists if $y_{\nu c}$ is not simultane-
ously diagonalizable with $y_e$ (the $3 \times 3$ matrix with elements $y_{eij}$). In the case where the lightest RHN ($\nu_1^c$) has a mass less than $T_{\text{reh}}$, direct thermal production of $\nu_1^c$ also takes place. We then have an extra source of $L_i$ violation from the Yukawa coupling of $\nu_1^c$ to $\tilde{h}_i$. Finally, we must consider $L_\nu$-violating effective dimension-five operators arising from fermionic $\nu^c$ exchange with mass insertion (and also without mass insertion in the case $T_{\text{reh}} > M_1$).

A. Dimension-four operators

As mentioned earlier, individual lepton numbers $L_i$ for the MSSM superfields can only be defined in the “hatted” basis where the charged lepton Yukawa coupling constant matrix $\hat{y}_e$ is diagonal. In the SU(2)$_R$-symmetric case, $y_e$ can be diagonalized simultaneously with $y_e$. In the non-SU(2)$_R$-symmetric case, however, this is in general not possible. It is though possible to strongly restrict the form of $\hat{y}_e$ by using the remaining freedom of performing unitary rotations in the flavor space of the RHN superfields and applying the condition that $L_i$ (for some particular value of $i$) is conserved by dimension-four operators. We start by considering the four-scalar operator $l_1^h h_2^h l_2^h h_2^h$ with coupling constant $\hat{y}_{eik}\hat{y}_{ejk}^*$ which, for $j \neq i$, violates $L_i$. In order to retain the $L_i$ asymmetry, we must impose the condition

\[ \sum_k \hat{y}_{eik}\hat{y}_{ejk}^* = 0 \]  

(25)

for $j \neq i$. For definiteness, let us choose to preserve the $L_3$ asymmetry, thus $i = 3$. An appropriate rotation in the $\nu_3^c$ space can bring the 3-vector $\hat{y}_{e3k}$ on the $\nu_3^c$ axis, $i.e.$ can make $\hat{y}_{e31}$, $\hat{y}_{e32}$ to vanish. Equation (25) then implies that the 3-vectors $\hat{y}_{e1k}$ and $\hat{y}_{e2k}$ lie in the $\nu_1^c - \nu_2^c$ subspace, $i.e.$ $\hat{y}_{e13} = \hat{y}_{e23} = 0$. Using the remaining freedom of rotations in this subspace, we can bring $\hat{y}_{e2k}$ on the $\nu_2^c$ axis, $i.e.$ make $\hat{y}_{e21} = 0$. So the only non-vanishing $\hat{y}_{e1k}$’s are the diagonal ones and $\hat{y}_{e12}$. The diagonal elements $\hat{y}_{ii}$ of $\hat{y}_e$ can be further made real and positive by absorbing their complex phases into the RHN superfields, while $\hat{y}_{e12}$ remains arbitrary and complex. For simplicity, we have chosen $\hat{y}_{e12} = 0$ so that $y_e$ is simultaneously diagonalizable with $y_e$. We still have to consider the four-scalar operator $l_1^h h_2^h l_2^h h_3^h$ with coupling constant $\hat{\zeta}_3\hat{y}_3^c$ which also violates $L_3$. Therefore, in order to retain the $L_3$ asymmetry, we must further impose the condition

\[ \hat{\zeta}_3 = 0. \]  

(26)

Note that it is by no means necessary for this equation, or the other constraints on $L_3$-violating couplings, to be satisfied exactly. The dangerous dimension-four operators will not come into thermal equilibrium if their effective coupling constants are less than about $10^{-7}$. However, we take them to vanish exactly in order to simplify the discussion.

B. Dimension-five operators

In Sec. VIII we will assume that, for a temperature range just above the scale of the superpartner masses, the $L_1$ and $L_2$ quantum numbers are violated due to the four-scalar operators involving $\zeta$ couplings, whereas $L_3$ is perturbatively conserved for all temperatures after reheating. The latter is guaranteed for dimension-four operators by Eq. (26), but needs to be checked for effective dimension-five operators with $\nu^c$ exchange, since the RHN mass terms explicitly violate the lepton numbers $L_i$. We consider these operators first for the case without SU(2)$_R$ symmetry where the RHN masses are all at or above the inflaton mass scale (see Sec. IX), and then for the case of SU(2)$_R$ symmetry. In both cases, we estimate the rate of reaction mediated by a chirality-conserving RHN exchange as

\[ \Gamma \approx T^3 \left| \sum_k \frac{A_k T}{\max(M_k^2, T^2)} \right|^2, \]  

(27)

where $A_k$ is the product of the dimensionless coupling constants which corresponds to the diagram with an exchange of a RHN with mass $M_k$. In the case of a chirality-changing transition with mass insertion on the $\nu^c$ propagator, the relevant formula is

\[ \Gamma \approx T^3 \left| \sum_k \frac{A_k M_k}{\max(M_k^2, T^2)} \right|^2. \]  

(28)

1. Non-SU(2)$_R$-symmetric case

When all RHNs have masses of the order of $m_{\text{inf}}$ or greater, we have $M > T$. Equation (28) then becomes

\[ \Gamma \approx |A|^2 T^3, \]  

(29)

where $A = \sum_k A_k M_k$. For scattering processes of the type $l_1 h_2 \rightarrow l_1 h_2^*$, which are mediated by a RHN exchange with mass insertion, this is

\[ A_{ij} = -\hat{\zeta}_3(M_{\nu_i}^{-1})_{ij} \hat{y}_3^c \approx \frac{2}{v^2} m_{ij}, \]  

(30)

where, as explained above, we take the neutrino Yukawa couplings to be diagonal in the charged lepton flavor basis (“hatted” basis) and also approximate the up-type Higgs VEV to $v \approx 174$ GeV (large $\tan \beta$ limit). The light neutrino mass matrix in the charged lepton basis is found as $m = V^* m V$ for given values of the MNS matrix $V$ and light neutrino masses $m_i$. Hence, we find that $A_{ij}$ has entries of at most $10^{-15}$ GeV$^{-1}$, assuming a hierarchical spectrum of light neutrinos. The ratio of the rate of the $L_3$-violating scattering processes involving two Yukawa couplings and a chirality-changing RHN exchange to the Hubble rate is thus

\[ \frac{\Gamma}{H} \approx \frac{m_p T |A_{3i}|^2}{5} \approx T |A_{3j}|^2 \times 4.9 \times 10^{17} \text{ GeV}. \]  

(31)
Therefore, even for reheat temperatures which are as high as $3 \times 10^{10}$ GeV, these scattering processes are well out of equilibrium after reheating.

We also have to take into account scattering processes of the type $l_3 h_2 \rightarrow \tilde{h}_1 h_2^*$ with one Yukawa and one $\hat{\zeta}_j$ coupling, $j \neq 3$, mediated by a chirality-changing RHN exchange. The amplitude for these processes is

$$\sum_{j=1,2} A_{3j} \hat{\zeta}_j \frac{\hat{y}_j}{y_j}$$  \hspace{1cm} (32)

For given values of the light neutrino parameters, $\hat{\zeta}_j$ and $\hat{y}_j$, the rate may be found. The worst case occurs in the small $m_1$ limit for which the rate of these scattering processes at $T = 3 \times 10^{10}$ GeV is estimated as

$$\frac{\Gamma}{H} \approx 3.4 \left( \frac{\hat{\zeta}}{m_D^2 / \text{GeV}} \right)^2 + 190 \left( \frac{\hat{\zeta}}{m_D^2 / \text{GeV}} \right)^2,$$  \hspace{1cm} (33)

where $m_D$ are the (diagonal) Dirac neutrino masses and we take $\hat{\zeta}_1 = i\hat{\zeta}_2$ with $|\hat{\zeta}_1| = |\hat{\zeta}_2| \equiv |\hat{\zeta}|$ (see Sec. IX). Hence $L_3$ is safely preserved after reheating if we have

$$m_D^2 / \text{GeV} \gtrsim 1.8|\hat{\zeta}|, \quad m_D^2 / \text{GeV} \gtrsim 14|\hat{\zeta}|.$$  \hspace{1cm} (34)

which are easily satisfied within the region of parameter space where the model constraints are fulfilled. Note that dimension-five operators mediated by a chirality-conserving RHN exchange are even more suppressed as one can easily deduce by comparing Eqs. (27) and (28).

So, no $L_3$ violation by dimension-five operators is encountered in the non-SU(2)$_R$-symmetric case after reheating.

2. SU(2)$_R$-symmetric case

In the SU(2)$_R$-symmetric case, the neutrino Yukawa coupling constants in the “hatted” basis are unambiguously determined to be given by the asymptotic relation $\hat{y}_\nu = (1/v \cos \beta) \text{diag}(m_e, m_\mu, m_\tau)$. For the purposes of calculating thermal effects near the reheat temperature, we can use the asymptotic values of Yukawa coupling constants since these constants do not run very much between the GUT scale and $T_{\text{reh}}$. Our best-fit value of $\tan \beta$ is around 55, thus the numerical values of the $\hat{y}_i$’s are given by $\hat{y}_\nu = \text{diag}(0.00014, 0.028, 0.66)$ approximately, taking into account the RG evolution of the charged lepton Yukawa coupling constants to the GUT scale. The $\nu^c$ masses are also determined uniquely by the light neutrino masses and the MNS matrix. As mentioned, the RHN masses must satisfy $M_3 \lesssim M^2_2/M_S$ and $M_2 > m_{\text{rad}}/2$. The lightest RHN mass ($M_1$) turns out to be very small (of order $10^7$ GeV) in this case (see Sec. IX). However, for our chosen parameter values, these bounds may be satisfied with a similar choice of light neutrino parameters to that for the non-SU(2)$_R$-symmetric case. Hence the entries in the matrix $A$ defined in Eq. (30) are of similar size. Thus, for temperatures that are smaller than all the RHN masses, the $L_3$-violating dimension-five operators involving two Yukawa couplings and a chirality-changing RHN exchange are out of equilibrium.

However, we also have to take into account the fact that we have a range of temperatures which are higher than the lightest RHN mass eigenstate $M_1$. In this case, the contribution of $\nu^c_i$ to scattering amplitudes will be different from its contribution in the previous case. Consider the propagator for $\nu^c_i$ exchange with mass insertion, instead of the factor $1/M_1$ assumed in Eq. (30), we obtain $M_1/T^2$. But this will evidently give a smaller rate than the estimate in Eq. (29) (which is incorrect in this case) if there are no cancellations between contributions of different RHNs (cancellations appear very unlikely given the large hierarchy in the $\nu^c_i$ masses, and this can be checked numerically). Thus, the $L_3$-violating scattering rate, in this case, is always equal to or smaller than the estimate in Eq. (31), which is still smaller than $H$. Consequently, the $L_3$-violating scattering processes from dimension-five operators which involve two Yukawa couplings and a chirality-changing RHN exchange are expected to be out of equilibrium at all temperatures after reheating in the SU(2)$_R$-symmetric case too.

We also have scattering through $\nu_i^c$ exchange without mass insertion involving two Yukawa couplings. This does not violate the total lepton number, but could convert $L_3$ to $L_j$ ($j \neq 3$). The $\nu^c_i$ propagator now varies as $1/T$ for $T > M_1$ or $T/M_2$ for $T < M_1$, which is, in both cases, suppressed relative to the dependence of $1/M_1$ in Eq. (30). The situation is similar in the case of $\nu^c_3$ exchange where the discussion at the end of Sec. IX applies. Hence the scattering rate is suppressed relative to the rate in Eq. (31), and is again smaller than the Hubble rate for temperatures below $3 \times 10^{10}$ GeV. One can further show that this conclusion remains valid even if we replace one of the Yukawa couplings by a $\zeta$ coupling in all cases (with and without mass insertion).

Since we are considering temperatures above $M_1$, there will be a thermal density of $\nu^c_3$ particles and $L_3$ may be violated, say, by direct scattering of $l_3$ and $h_2$ into $\nu^c_3$ (see also Ref. 29), where the RHN will subsequently decay mainly to $l_1 h_2^*$ (or $l_1^* h_2$). The rate for this is similar to the rate for any dimension-five process through $\nu^c_3$ exchange without mass insertion (an equivalent process if $\nu^c_3$ is put on mass shell), except that the coupling constant is simply $\hat{g}_3 U_{31}^c$. The relevant bound is now

$$|\hat{g}_3 U_{31}^c|^2 \times 4.9 \times 10^{17} \text{ GeV} \lesssim T,$$  \hspace{1cm} (35)

which, for $T \approx 10^7$ GeV, implies

$$|U_{31}^c| \lesssim 10^{-5}.$$  \hspace{1cm} (36)

This bound may in fact be respected for some parameter choices, but we will see that the SU(2)$_R$-symmetric case is ruled out by other considerations, namely the smallness of off-diagonal elements of $U^c$. 


VIII. CHEMICAL POTENTIALS AND \( n_B/s \)

In order to find the final baryon asymmetry, we consider the evolution of the net number densities of all the relativistic species between the time of reheating and the time at which all \( B \)-violating interactions are out of equilibrium and the baryon number is effectively conserved. At any given temperature, we determine which reactions (including the non-perturbative ones, i.e. the sphaleron or QCD instanton effective vertices) are in equilibrium. This is the case if the rate of reaction normalized to the particle number density is greater than the Hubble rate. Then one can deduce which additive quantum numbers are conserved by these reactions. The equilibrium number density \( n \) of a relativistic species minus the number density \( \bar{n} \) of its antiparticle is given, in terms of its chemical potential \( \mu \), by the standard formula

\[
\frac{n - \bar{n}}{s} = \frac{15g}{4\pi^2g_*} \begin{cases} 
2 \text{ (bosons)} \\
1 \text{ (fermions)} 
\end{cases} \frac{T}{\mu},
\]

where \( g \) is the number of internal degrees of freedom of the species. For each reaction in equilibrium, the sum of chemical potentials of the reaction products is equal to the sum of chemical potentials of the reactants. Using the equilibrium equations, one can express \[30, 33\] the net number density of any species as well as the density of any quantum number in terms of a small set of chemical potentials. Eliminating these chemical potentials, one can determine the densities of all quantum numbers given the densities of the conserved ones. When some reactions go out of equilibrium, one may find the new densities of the quantum numbers which cease to be violated by imposing that the densities remain unchanged. We use the convention \[33\] that the same symbol denotes the species and its chemical potential. For fermions, we count each left-handed species separately, hence a non-zero chemical potential for the gluino \( \tilde{g} \) (say) implies an asymmetry between left- and right-handed gluinos.

We will first perform the chemical potential analysis in the period which follows reheating and for temperatures higher than the mass scale of the superpartners and the critical temperature \( T_c \) for the electroweak phase transition. In this regime, the SM gauge group is unbroken and we will assume that the spectrum coincides with the MSSM one. This is certainly the case in our model provided that all the RHN masses are larger than \( T_{\text{reh}} \). In the SU(2)_{\text{R}}-symmetric case, however, this analysis will only hold for temperatures which are lower than the lightest RHN mass, which is smaller than \( T_{\text{reh}} \). Gauge interactions imply \[31\] that the gluons, the photon and the \( B \) gauge boson have vanishing chemical potential at all times, as do the \( W^\pm \) bosons above the electroweak phase transition. The interactions of squarks with all three types of SM gauge bosons and the corresponding gauginos also ensure \[32\] that all gauginos have the same chemical potential, which we denote by \( \tilde{g} \).

Yukawa interactions originating from the superpotential are found to be in equilibrium above the electroweak scale, implying the relations

\[
\begin{align*}
\tilde{a}^c &= -q - h_2, \\
\tilde{d}^c &= -q - h_1, \\
\tilde{e}_i^c &= -l_i - h_1.
\end{align*}
\]

(38)

Due to the Cabbibo-Kobayashi-Maskawa mixing, the quark chemical potentials are independent of the generation number. In the leptonic sector, the situation is different: due to the very small scale of masses, neutrino oscillations are not \[29\] in equilibrium at the temperatures of interest; and, as has been shown, interactions mediated by massive RHN exchange may not be either. Thus the charged lepton and neutrino chemical potentials may differ between the three generations in the “hatted” basis, where the lepton Yukawa couplings are diagonal. Note that, although the chemical potentials \( l_i \) (or \( \nu_i, \epsilon_i \)) and \( \epsilon_i^c \) are, throughout, in the “hatted” basis, we suppress the hats for simplicity of notation.

SUSY-breaking soft terms mediate transitions which are \[33\] in equilibrium for temperatures below about \( 10^7 \) GeV and above the scale of soft scalar masses (for simplicity, we will assume a common SUSY threshold). In this regime, the gaugino chemical potential vanishes and all members of a supermultiplet have equal chemical potential. At higher temperatures, we have the following relations \[34\] between the components of the chiral superfields:

\[
\begin{align*}
\tilde{q} &= q + \tilde{g}, \\
\tilde{u}^c &= u^c + \tilde{g}, \\
\tilde{d}^c &= d^c + \tilde{g}, \\
\tilde{l}_i &= l_i + \tilde{g}, \\
\tilde{e}_i^c &= e_i^c + \tilde{g}. \\
\tilde{h}_{1,2} &= h_{1,2} - \tilde{g}.
\end{align*}
\]

(39)

Then the baryon and lepton number densities are

\[
\begin{align*}
\frac{n_B}{s} &= \frac{15}{4\pi^2g_*T} 3N_g (4q + h_1 + h_2), \\
\frac{n_L}{s} &= \frac{15}{4\pi^2g_*T} [3(3l_i + h_1) + 2\tilde{g}],
\end{align*}
\]

(40)

where \( N_g \) is the number of generations. The condition that the electric charge density should vanish is

\[
\frac{n_Q}{s} = \frac{15}{4\pi^2g_*T} 3[2N_gq - 2\sum_i l_i + (2N_g + 1)(h_2 - h_1)] = 0.
\]

(41)

We must now also consider the SU(2)_{\text{L}} and SU(3)_{\text{c}} non-perturbative interactions (‘t Hooft vertices), which involve all the left-handed fermions transforming non-trivially under the corresponding group and are unsuppressed at these temperatures. They give the following relations:

\[
\begin{align*}
\text{SU}(2)_{\text{L}} &: 3N_gq + \sum_i l_i + h_1 + h_2 + 2\tilde{g} = 0, \\
\text{SU}(3)_{\text{c}} &: N_g(h_1 + h_2) - 6\tilde{g} = 0.
\end{align*}
\]

(42)
which together with Eq. (41) yield
\[
q = -\frac{1}{3N_g} \sum_i l_i - \frac{2(N_g + 3)}{3N_g^2} \tilde{g},
\]
\[
h_1 = -\frac{4}{3(2N_g + 1)} \sum_i l_i + \frac{16N_g + 3}{3N_g(2N_g + 1)} \tilde{g},
\]
\[
h_2 = \frac{4}{3(2N_g + 1)} \sum_i l_i + \frac{20N_g + 15}{3N_g(2N_g + 1)} \tilde{g}. \tag{43}
\]

So, we are left only with four independent chemical potentials \((l_i, \tilde{g})\), in terms of which the baryon and lepton number densities are given by
\[
n_B/s = \frac{15}{4\pi^2 g_* T} \left( -4 \sum_i l_i + \frac{10N_g - 24}{N_g} \tilde{g} \right),
\]
\[
n_{L_i}/s = \frac{15}{4\pi^2 g_* T} \left( 9l_i - \frac{4}{2N_g + 1} \sum_j l_j + \frac{4N_g^2 + 18N_g + 3}{N_g(2N_g + 1)} \tilde{g} \right), \tag{44}
\]

which hold regardless of the details of the model.

The scalar potential term \(|\partial W/\partial \nu_i|^2\) gives rise to operators mediating the transition \(h_1 h_2 \leftrightarrow h_2 l_i\) for \(i = 1, 2\) (recall that we imposed the condition that \(\zeta_3 = 0\)). These reactions are in equilibrium just above the superpartner threshold and imply that \(l_{1,2} = -\tilde{g} + h_1\). Moreover, as already mentioned, below a temperature \(10^7\) GeV and above the superpartner mass threshold, \(\tilde{g}\) vanishes. So, just above the SUSY threshold, we have only one independent chemical potential (say \(l_3\)) and Eq. (44) yields
\[
n_B/s = \frac{15}{4\pi^2 g_* T} \left( -12(2N_g + 1) \right) l_3,
\]
\[
n_{L_3}/s = \frac{15}{4\pi^2 g_* T} \left( -48 \right) l_3,
\]
\[
n_{L_3}/s = \frac{15}{4\pi^2 g_* T} \left( 3(18N_g + 29) \right) l_3. \tag{45}
\]

Eliminating \(l_3\), we can express all the quantum numbers in terms of one of them. In particular, we find that
\[
n_B/s = \frac{4(2N_g + 1)}{6N_g + 11} \frac{n_{B-3L_3}}{s}, \tag{46}
\]
\[
n_{B-3L_3}/s = \frac{4(2N_g - 11)}{62N_g + 91} \frac{n_{B-3L_3}}{s}, \tag{47}
\]

which also gives
\[
\sum_i \frac{n_{B-3L_3}}{s} = \frac{3(26N_g + 1)}{62N_g + 91} \frac{n_{B-3L_3}}{s}. \tag{48}
\]

Note that \(B-3L_3\) is the only conserved quantum number just above the SUSY threshold. It is actually the only quantum number which remains conserved at all times after reheating, and, thus, the corresponding asymmetry is equal to \(-3\) times \(n_{L_3,\text{init}}/s\). Equation (46) then yields
\[
n_B/s = \frac{12(2N_g + 1)}{62N_g + 91} \frac{n_{L_3,\text{init}}}{s}. \tag{49}
\]

This gives the correct result for the final baryon number density provided that \(i\) the electroweak phase transition is strongly first order so that no \(B\)-violating \(SU(2)_L\) sphaleron interactions are in equilibrium in the broken phase, and \(ii\) the scale of the superpartner masses is smaller than the critical temperature \(T_c\), i.e. the final value of \(n_B/s\) is fixed at a temperature at which reactions involving superpartners are in equilibrium.

However, if superpartners decouple while sphalerons are still active, one must consider the equilibrium equations without the superpartner contributions, and perform a non-SUSY analysis \[30\]. In this case, we have two possibilities: either the electroweak phase transition is strongly first order (i.e. the sphalerons freeze out very quickly after the transition) and the final baryon number is fixed in the unbroken phase, or it is second order (or weakly first order) and the sphalerons continue to be in equilibrium in the broken phase. Recent data indicate that the weakly first-order transition is more likely (see e.g. Ref. \[31\]). We will thus analyze this case in detail.

With superpartners and heavy Higgs bosons decoupled, we are left \[30\] with the following chemical potentials which \(a\ priori\) may be non-zero: \(u, d\) (corresponding, respectively, to the up- and down-type component of the left-handed \(SU(2)_L\) doublet \(g\)), \(u^c, d^c, e_i, e_i^c, \nu_i\), \(h\) (corresponding to the lightest Higgs boson), and \(W^-\). For \(T > T_c\), the chemical potential of the \(W^-\) gauge boson vanishes, while below \(T_c\), the \(W^-\) boson may have a non-zero chemical potential, but \(h = 0\). This immediately fixes the \(SU(2)_L\) singlet chemical potentials as \(u^* = -u\), \(d^* = -d\), \(e_i^* = -e_i\). Gauge interactions imply \(d = u + W^-\), \(e_i = \nu_i + W^-\), leaving us, for \(T < T_c\), with \(N_g + 2\) independent chemical potentials \(u, \nu_i, W^-\). The condition that the electric charge vanish leads to the relation
\[
N_g u - \sum_i \nu_i = -(2N_g + 3)W^-, \tag{50}
\]
which determines \(W^-\) in terms of \(u\) and \(\nu_i\). The baryon and lepton number densities can be easily calculated and, after eliminating \(W^-\) by using Eq. (50), are given by
\[
n_B/s = \frac{15}{4\pi^2 g_* T} \frac{2N_g}{2N_g + 3} \left( (5N_g + 6)u - \sum_i \nu_i \right),
\]
\[
n_{L_3}/s = \frac{15}{4\pi^2 g_* T} \left( 3\nu_i + 2 \frac{N_g u - \sum_j \nu_j}{2N_g + 3} \right). \tag{51}
\]

Now consider the \(SU(2)_L\) sphaleron interaction. Its equilibrium condition implies that \(N_g (u + 2d) + \sum_i \nu_i\) should vanish, which fixes \(u\) in terms of \(\nu_i\):
\[
u_i = - \frac{3 \sum_i \nu_i}{N_g (8N_g + 9)}. \tag{52}
\]
Equation (51) then yields
\[ \frac{n_B}{s} = \frac{15}{4\pi^2 g_* T} \frac{4(2N_g + 3)}{8N_g + 9} \sum \nu_i, \tag{53} \]
\[ \frac{n_{B-3L_i}}{s} = \frac{15}{4\pi^2 g_* T} \left( 9\nu_i + \frac{4(2N_g - 3)}{8N_g + 9} \sum \nu_j \right). \]

The three quantum numbers \( B - 3L_i \) are separately conserved after superpartners decouple, because reactions violating \( L_{1,2} \) are out of equilibrium. We solve for \( \nu_i \) in terms of \( n_{B-3L_i} \) and substitute back into the baryon number density to obtain
\[ \frac{n_B}{s} = \frac{4(2N_g + 3)}{3(32N_g + 15)} \sum \frac{n_{B-3L_i}}{s}. \tag{54} \]

From continuity, the constant values of the asymmetries \( n_{B-3L_i}/s \) for temperatures lower than the SUSY threshold should be identical to the values of these asymmetries just above this threshold where the superpartners are still in equilibrium and \( \dot{q} = 0 \). In this regime, \( B - 3L_{1,2} \) are violated and the values of the corresponding asymmetries are expressed in terms of the conserved \( B - 3L_3 \) asymmetry. Assuming that the SUSY threshold lies higher than \( T_{\text{reh}} \), the \( B - 3L_{1,2} \) asymmetries are given by Eq. (10). Then, applying Eqs. (18) and (53), we find that the final baryon number density is given by
\[ \frac{n_B}{s} = \frac{4(26N_g + 1)(2N_g + 3)}{(62N_g + 91)(32N_g + 15)} \frac{n_{B-3L_3}}{s} \]
\[ = -\frac{12(26N_g + 1)(2N_g + 3)}{(62N_g + 91)(32N_g + 15)} \frac{n_{L_i,\text{init}}}{s} \]
\[ = -\frac{2844}{10249} \frac{n_{L_i,\text{init}}}{s}. \tag{55} \]
for \( N_g = 3 \).

IX. NUMERICAL RESULTS

We make the assumption that the LSPs are long-lived and their thermal production is negligible, which holds in many cases. So, the LSPs which possibly contribute to the cold dark matter (CDM) in the present universe can originate solely from the late decay of the gravitinos which were thermally produced at reheating. Their relic abundance is
\[ \Omega_{\text{LSP}} h^2 = 0.037 \left( \frac{m_{\text{LSP}}}{100 \text{ GeV}} \right) \left( \frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right), \tag{56} \]
where \( \Omega_{\text{LSP}} \) is the relic density of the LSPs in units of the critical density of the universe, \( h \) is the present Hubble parameter in units of \( 100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \), and \( m_{\text{LSP}} \) is the LSP mass. We further assume that the CDM in the universe consists exclusively of LSPs. Taking the best-fit value for the CDM abundance from the WMAP data [18], i.e. \( \Omega_{\text{CDM}} h^2 \approx 0.1126 \), and putting \( m_{\text{LSP}} = 100 \text{ GeV}, \) Eq. (50) yields \( T_{\text{reh}} \approx 3.04 \times 10^{10} \text{ GeV} \). This high value of \( T_{\text{reh}} \), chosen here to maximize the BAU, can well be compatible with the gravitino constraint [8] provided that the branching ratio of the gravitino decay to a photon and a photino is adequately small.

The R-symmetry-violating effective coupling constants \( \hat{\zeta}_1, \hat{\zeta}_2 \) are taken to satisfy the relation \( \hat{\zeta}_1 = i\hat{\zeta}_2 \), i.e. the CP-violating relative phase between them is taken equal to \( \pi/2 \). The magnitude of these coupling constants is maximized, i.e. \( |\hat{\zeta}_1| = |\hat{\zeta}_2| = M/\hat{M}_S \). These choices maximize the generated baryon asymmetry. In order to preserve perturbativity and to have a good effective field theory, we require \( M \lesssim 0.1M_S \).

As already mentioned, the light neutrino mass-squared differences and mixing angles are generally taken to lie in the 2\( \sigma \) confidence intervals in Ref. [24]. In the non-SU(2)\( _R \)-symmetric case, in particular, we take them to coincide with their best-fit values in Ref. [24]: \( \delta m^2_{21} = 8.1 \times 10^{-5} \text{ eV}^2 \), \( \delta m^2_{31} = 2.2 \times 10^{-3} \text{ eV}^2 \), \( \sin^2 \theta_1 = 0.5 \), \( \sin^2 \theta_2 = 0 \), and \( \sin^2 \theta_3 = 0.3 \). For simplicity, in the MNS matrix \( V \), we set the Dirac phase \( \delta \) and the Majorana phases \( \alpha \) and \( \beta \) to zero. This highlights the fact that our mechanism works in the absence of any CP-violating phases in the standard (R-parity-conserving) couplings.

A. Non-SU(2)\( _R \)-symmetric case

We will first examine the non-SU(2)\( _R \)-symmetric case. We fix the parameter \( \kappa \) to the value \( 10^{-4} \). The cosmic microwave background explorer (COBE) value of the quadrupole anisotropy of the CMBR (\( \langle \delta T/T \rangle_Q \approx 6.6 \times 10^{-5} \) [38] is then reproduced for \( \lambda \approx 3.48 \times 10^{-4} \) \( \lambda > \kappa \) as it should) and \( M \approx 5.63 \times 10^{15} \text{ GeV} \). Thus, \( m_{\text{inf}} \approx 7.96 \times 10^{11} \text{ GeV} \). The spectral index of density perturbations comes out practically equal to unity.

For definiteness, we set \( \tan \beta = 50 \) although in this case it scarcely affects the calculation of baryon asymmetry. We are still free to vary \( M_S \), \( m_1 \) and the three neutrino Dirac masses \( m_D^i \). As previously mentioned, we require that all three RHN masses be greater than \( m_{\text{inf}}/2 \) and less than \( M_S^2/M_S \), i.e. \( M_1 > m_{\text{inf}}/2 \) and \( M_2 M_3 < M^2 \). By placing one RHN mass \( M_j \) near the inflaton pole \( M_j = m_{\text{inf}} \), the contribution of the corresponding self-energy diagram to the baryon asymmetry is resonantly enhanced (see Eq. (21)). However, we regard this as an unnatural fine-tuning since it apparently cannot be ensured by any symmetry, and reject solutions where any \( M_j \) is less than 10% from the pole, i.e. \( |M_j - m_{\text{inf}}|/m_{\text{inf}} < 0.1 \). We also impose the requirement that \( L_3 \) is not violated by any dimension-five processes and that the observed baryon asymmetry \( n_B/s \approx 8.66 \times 10^{-11} \), derived from the recent WMAP data [18], is reproduced. Under all these restrictions and for fixed values of \( m_D^i \), we find the value of \( m_1 \) which maximizes \( M_S \) (and thus minimizes \( |\hat{\zeta}_1| \) and \( |\hat{\zeta}_2| \)). We have chosen to maximize \( M_S \) in order to obtain values as
FIG. 3: The contours with fixed maximal $M_S$ in the $m_{D1} - m_{D2}$ plane for $m_{D3} = 30 \text{ GeV}$. The cyan, green, magenta, red and blue contours correspond, respectively, to $M_S = 6, 7, 8, 9$ and 10 in units of $10^{16}$ GeV. In black and white, contours with higher $M_S$ appear to be darker. The red/dark shaded area is excluded by both the requirements that, for the maximal $M_S$, $M_1 > m_{\text{inf}}/2$ and $L_3$ is conserved by dimension-five processes, while the blue/lightly shaded one only by the latter.

close as possible to the standard scale ($\approx 5 \times 10^{17}$ GeV) of the weakly-coupled heterotic string. We keep only values of maximal $M_S$ which exceed $10M \approx 5.63 \times 10^{16}$ GeV so that the perturbativity requirement is satisfied.

In Figs. 3 and 4, we present the contours with fixed value of this maximal $M_S$ and the corresponding $m_1$ respectively in the $m_{D1} - m_{D2}$ plane for $m_{D3} = 30 \text{ GeV}$. In Fig. 3 we include only contours with fixed maximal $M_S$ which satisfies the perturbativity limit. However, this limit is not necessarily satisfied everywhere on the contours with fixed $m_1$ which we show in Fig. 4. It holds only on the parts of these contours which lie in the area covered by the contours in Fig. 3. We observe that the contours with fixed maximal $M_S$ show the existence of two main almost horizontal mountain ranges not far from each other and a lower almost vertical mountain range at low values of $m_{D1}$. The contours with fixed $m_1$, on the other hand, form a single horizontal mountain range and a secondary almost vertical mountain range at higher values of $m_{D2}$. At low values of $m_{D2}$ (and $m_{D1}$), we find that $M_1 < m_{\text{inf}}/2$ for a wide range of choices of $m_1$ and our scenario is irrelevant since the inflaton decays directly into RHNs. Also, the ratios $\hat{\zeta}_j/\hat{y}_j$ ($j = 1, 2$) acquire large values, which result in the $L_3$ asymmetry being washed out by dimension-five processes involving a $\hat{\zeta}$ coupling. The red/dark shaded areas in Figs. 3 and 4 are excluded because, for the maximal value of $M_S$, $M_1 < m_{\text{inf}}/2$ and $L_3$ is violated; the blue/lightly shaded areas, on the other hand, are excluded only because of $L_3$ violation. The value of the maximal $M_S$ in these areas is much smaller than the perturbativity bound, thus these areas are not considered acceptable in any case. We have also constructed the contours in the $m_{D1} - m_{D2}$ plane with fixed $M_S$ or $m_1$ for $m_{D3} = 10 \text{ GeV}$ and $m_{D3} = 20 \text{ GeV}$. We find that they reveal a very similar structure.

In order to understand the structure of these contour plots, let us consider some specific values for the three Dirac neutrino masses $m_{Di}$. For instance, let us fix
**FIG. 4:** The contours with fixed \( m_1 \) corresponding to the maximal \( M_S \) in the \( m_1^D - m_2^D \) plane for \( m_3^D = 30 \) GeV. The cyan, green, magenta, red and blue contours correspond, respectively, to \( m_1 = 3, 5, 10, 50 \) and 100 in units of \( 10^{-4} \) eV. In black and white, contours with higher \( m_1 \) appear to be darker. The red/dark shaded and blue/lightly shaded areas are as in Fig. 3.

\( m_1^D = 7 \) GeV, \( m_2^D = 30 \) GeV and take five characteristic choices of \( m_2^D \) (4, 4.35, 4.6, 4.95, 5.3 GeV). Then compute the value of \( M_S \) which is required in each case to obtain the correct \( n_B/s \) as a function of \( m_1 \). The result is depicted in the top left panels of Figs. 5-9 by a solid line. Moreover, we find that the constraint \( M_1 M_S < M_2^2 \) is satisfied to the right of the dashed line in these panels. In the other three panels in each of these figures, we show the corresponding values of \( M_i, i = 1, 2, 3 \), as functions of \( m_1 \). The shaded bands are excluded by the requirement that \( |M_1 - m_{\text{inf}}|/m_{\text{inf}} > 0.1 \).

To appreciate the meaning of these figures, we must first observe that, as our numerical findings show, the main contribution to the baryon asymmetry comes from the interference of vertex with self-energy diagrams, while the interference between self-energy diagrams is subdominant. The heaviest RHN mass \( M_3 \) is much larger than \( M_1, M_2 \) and \( m_{\text{inf}} \) as one can see from Figs. 5-9. So, the contribution to the baryon asymmetry from diagrams with \( \nu_3 \) exchange is suppressed. Moreover, in the interesting range of parameters where the maximal \( M_S \) is achieved, \( M_1 \) is nearer the inflaton pole than \( M_2 \) is. As a consequence, the dominant self-energy diagram is the one with \( \nu_1 \) exchange. It also turns out from our numerical study that the dominant vertex diagram in the interesting range of parameters is the one with \( \nu_2 \) exchange. So, the baryon asymmetry is dominated by the interference of the self-energy diagram with \( \nu_1 \) exchange with the vertex diagram with \( \nu_2 \) exchange.

From Figs. 5-9, we see that, for \( m_1 \to 0 \), the masses \( M_1 \) and \( M_2 \) acquire constant asymptotic values, while \( M_3 \) keeps increasing. In consequence, the kinematic factor in the dominant contribution to \( n_B/s \), which originates from the “stripped” diagrams, is constant in this limit. The prefactor originating from the product of coupling constants contains \( U_{c31} \) and \( U_{c32} \) which connect the \( \tilde{\nu}_3 \) state with the \( \nu_{1,2} \) mass eigenstates. We find that, as \( m_1 \) decreases, these elements of \( U^c \) first rise sharply and then approach their constant asymptotic values. Their rapid increase is compensated by an appropriate increase.
of $M_S$, which enters the prefactor through $|\zeta_1|$ and $|\zeta_2|$, so that the baryon asymmetry remains equal to its WMAP value. This explains the fact that, as $m_1$ decreases, $M_S$ generally exhibits a fast increase before entering its asymptotic plateau (see solid line in the top left panels of Figs. 5, 6). This phenomenon generally helps us to obtain larger maximal values of $M_S$.

From these figures, we also see that $M_2$ and $M_3$ (see two bottom panels) remain effectively unaltered as we vary $m_D^2$ for fixed $m_1^D$ and $m_2^D$. This explains also the fact that the constraint $M_3M_S < M^2$, as depicted in the top left panels (dashed line), is practically unaffected by changes in the value of $m_2^D$. The $M_1$ though is enhanced as $m_2^D$ increases. For $m_2^D = 4$ GeV, we see from Fig. 6 that the asymptotic value of $M_1$ (as $m_1 \to 0$) lies well below the inflaton pole. So, there is no enhancement of the dominant "stripped" self-energy diagram produced by this pole. As a consequence, the values of $M_S$ yielding the correct baryon asymmetry are relatively low. So, the maximal $M_S$, which is achieved at $M_3M_S = M^2$ in this case, is also quite low and corresponds to a low value of $m_1$ ($\sim 10^{-4}$ eV). This explains the fact that this particular case lies well below the lower horizontal mountain range in Fig. 4 and the horizontal mountain range in Fig. 7.

Let us now discuss how the situation changes as we increase $m_1^D$ with $m_1^D$ and $m_2^D$ fixed. For $m_2^D = 4.35$ GeV, Fig. 7 shows that the asymptotic $M_1$, which is larger than in the previous case, gets closer to the pole and thus the dominant "stripped" self-energy diagram is enhanced, leading to a larger maximal $M_S$. This is again achieved at $M_3M_S = M^2$ and corresponds to a somewhat bigger, but still quite small value of $m_1$. This case lies near the brow of the lower mountain range in Fig. 4 but still below the mountain range in Fig. 7. Increasing $m_2^D$ further to the value 4.6 GeV, we see from Fig. 8 that the asymptotic $M_1$ gets very close to the pole and, thus, $M_S$ is further enhanced. However, $M_1$ now enters into the band excluded by the requirement $|M_1 - m_{\text{inf}}|/m_{\text{inf}} > 0.1$, which in the top left panel extends to the right of the line $M_3M_S = M^2$. So, in this case, the maximal $M_S$ is achieved at the boundary of this excluded band and not at $M_3M_S = M^2$. It is thus quite low and corresponds to a considerably larger $m_1$. This case lies in the valley extending between the two horizontal mountain ranges in Fig. 8 and very near the brow of the horizontal mountain range in Fig. 6. In Fig. 8, we present the case $m_2^D = 4.95$ GeV. We see that the asymptotic $M_1$ now lies above the excluded band, but still close to the pole. The whole excluded band in the top left panel now lies to the right of the line $M_3M_S = M^2$, and thus the maximal $M_S$ is again achieved on this line and corresponds to a low $m_1$. Therefore, this case lies above the brow of the hori-
horizontal mountain range in Fig. 4. Actually, at some value of \( m_2^D \), the point at which the maximal \( M_S \) is achieved jumps from the right boundary of the excluded band to the line \( M_3M_S = M^2 \). So, \( m_1 \) drops suddenly to low values. This discontinuity explains the absence of the upper branches of the contours with \( m_1 = 10^{-2}, 5 \times 10^{-3} \) and \( 10^{-3} \) eV. The enhancement of the self-energy diagram due to the pole yields the sharp peak of \( M_S \) within the excluded (shaded) band. The tail of this enhancement leads to a large value for the maximal \( M_S \). So, this case is near the brow of the upper mountain range in Fig. 4. For \( m_2^D = 5.3 \) GeV, the asymptotic \( M_1 \) moves away from the pole and the excluded (shaded) band in the top left panel further to the right (see Fig. 6). Consequently, the maximal \( M_S \) is still achieved on the line \( M_3M_S = M^2 \), but drops to lower values. This case is well above the upper horizontal mountain range in Fig. 4 and the horizontal mountain range in Fig. 5.

The secondary vertical mountain range in Fig. 5 can be understood as follows. As \( m_2^D \) decreases for fixed and large values of \( m_2^D \), the mass \( M_1 \) also decreases. As a consequence, the band excluded by the requirement that \( |M_1 - m_{\text{inf}}|/m_{\text{inf}} > 0.1 \) moves to the left in the \( m_1 - M_S \) plane approaching the curve \( M_3M_S = M^2 \) and the maximal \( M_S \), which is achieved on this curve, increases since the inflaton pole gets nearer. At some small value of \( m_2^D \), the left boundary of this band touches the point corresponding to the maximal \( M_S \), which thus reaches its largest possible value for the chosen value of \( m_2^D \). This yields the brow of the vertical mountain range in Fig. 5. The corresponding value of \( m_1 \) is, however, still relatively small. For smaller values of \( m_2^D \), the band moves further to the left and then reaches a horizontal mountain range in Fig. 6. This sharp discontinuity in \( m_1 \) explains the absence of the right branch of the contour with \( m_1 = 10^{-3} \) eV. As the value of \( m_2^D \) decreases further, the excluded band shifts further to the left and the maximal \( M_S \) and the corresponding \( m_1 \) drop smoothly.

Despite the abundance of adjustable parameters in the non-SU(2)_R-symmetric case, it turns out not to be easy to achieve the observed baryon asymmetry. In particular, we find that, for central values of the neutrino mass-squared differences and mixing angles, the string scale \( M_S \) is restricted to be lower than about \( 10^{17} \) GeV. So, the standard weakly-coupled heterotic string scale \( (\approx 5 \times 10^{17} \) GeV) cannot be achieved in this case. However, lower string scales, such as the ones encountered here, can be easily obtained in other string models like the strongly-coupled heterotic string from M-theory (see e.g. Ref. [34]). Also, we find that it is difficult to generate an adequate baryon asymmetry with \( T_{\text{reh}} \sim 10^{18} \) GeV, which is the standard upper bound on \( T_{\text{reh}} \) from the gravitino constraint [35, 36]. We are thus obliged to allow higher reheat temperatures, which is though perfectly possible provided that the gravitino decay to photons and photinos is somewhat suppressed.

At the upper bound on \( M_S \) (\( \approx 10^{17} \) GeV), the parameters \( |\tilde{c}_i| (i = 1, 2) \) are about \( 5.63 \times 10^{-2} \) and may thus lead to processes observable in the future colliders. (For smaller values of \( M_S \), they are even larger.) Indeed, as mentioned in Sec. VI, the explicit R-parity violation in our model, required for leptogenesis, has some low-energy signatures coming from dimension-four effective scalar vertices. They may typically be the three-body slepton decay processes

\[ \tilde{l}_i \rightarrow h_1 h_2 h_3^*, \]

for \( i = 1, 2 \), which can easily be kinematically allowed. The magnitude of their effective coupling constants \( \tilde{c}_i \) is of order \( 10^{-3} \) near the largest possible value of \( M_S \), which is achieved at \( m_2^D \approx 3 \) GeV and \( m_2^D \approx 5 \) GeV for \( m_3^D = 30 \) GeV (see Fig. 7). The corresponding decay rates are then of order \( 10^{-8} \) GeV for mass of the decaying slepton of order 1 TeV.
We have seen that the maximal value of $M_S$, in most cases, is achieved at $M_3/M_S = M^2$, and the corresponding value of $m_1$ lies in the range $10^{-4} - 10^{-3}$ eV. However, for values of the Dirac neutrino masses in the valley between the two horizontal mountain ranges appearing in the $M_S$ contour plots in the $m_1^D$ - $m_2^D$ plane for fixed $m_3^D$ (see Fig. 3), the maximal $M_S$ is achieved at the boundary of the band excluded by the requirement $|M_1 - m_{\text{inf}}|/m_{\text{inf}} > 0.1$ and the corresponding $m_1$ is considerably larger reaching values of order $10^{-4}$ eV. This case, however, yields very low maximal values of $M_S$ and must, therefore, be excluded by the perturbativity requirement. In conclusion, we predict that, for acceptable values of $M_S$, the (in principle observable) smallest neutrino mass $m_1$ takes values between $10^{-4}$ and $10^{-3}$ eV. Thus our assumption of a hierarchical light neutrino spectrum is self-consistent.

B. SU(2)$_R$-symmetric case

In the SU(2)$_R$-symmetric case, we again put $\kappa = 10^{-4}$. The COBE value of the quadrupole anisotropy of the CMBR is now reproduced for $\lambda \simeq 3.45 \times 10^{-4}$ and $M \simeq 5.69 \times 10^{15}$ GeV, yielding $m_{\text{inf}} \simeq 8.05 \times 10^{11}$ GeV. The spectral index is again equal to unity for all practical purposes. In this case, $\tan \beta \simeq 55$ and the asymptotic values of $\hat{y}_i$ are 0.00014, 0.028 and 0.66 for $i = 1, 2$ and 3 respectively, as already mentioned. So, in the SU(2)$_R$-symmetric case, only $M_S$ and $m_1$ remain free.

To give a chance to this very restrictive case to possibly yield acceptable values of the baryon asymmetry, we allow all the neutrino oscillation parameters to vary within their 2$\sigma$ confidence interval given in Ref. [24] (for simplicity, we take the Dirac phase $\delta = 0$). Even then, we find that, when the restriction $\hat{\zeta}_3 = 0$ from preservation of the $B - 3L_3$ asymmetry is imposed, the resulting values of the baryon asymmetry are always some orders of magnitude smaller than the observed value for any reasonable values of $M_S$ (satisfying the perturbativity requirement) and any $m_1$ in the allowed range from WMAP data [13].

The main reason for this failure is the smallness of the off-diagonal elements of $U^c$. The existence of a CP asymmetry requires that $y_3$ vertices be connected to $\hat{\zeta}_1$ or $\hat{\zeta}_2$ vertices through $\nu^c$ internal lines, yielding products of the type $U^c_{3k}U^c_{3j}U^c_{3j}U^c_{3k}$ for fixed values of $j$ and $k$ ($j \neq k$). These products contain at least two off-diagonal elements of $U^c$. However, the strongly hierarchical values of $\hat{y}_i$ resulting from the SU(2)$_R$ symmetry imply that the off-diagonal elements of $U^c$ are always small. In addition, the requirement from the preservation of the $B - 3L_3$ asymmetry that $|U^c_{3j}| \lesssim 10^{-5}$ further restricts this product of matrix entries, such that the overall product of coupling constants entering the $L_3$ asymmetry per inflaton decay ($\epsilon_3$) is already less than $10^{-8}$, without considering any loop suppression factors. In contrast, in the non-SU(2)$_R$-symmetric case, the values of the off-diagonal elements of $U^c$ can be of order 0.1 or even unity.

X. CONCLUSIONS

We proposed a scenario of non-thermal leptogenesis following SUSY hybrid inflation, in the case where the light neutrinos acquire masses exclusively via the standard seesaw mechanism, i.e. through their coupling to heavy RHNs, and the decay of the inflaton to RHN superfields is kinematically blocked or (in the presence of a SU(2)$_R$ gauge symmetry) the RHNs with mass smaller than half the inflaton mass are too light to generate sufficient baryon asymmetry through their subsequent decay. The primordial lepton asymmetry is generated through the direct decay of the inflaton into light particles. We explored our scenario within the context of two simple SUSY GUT models, one without and one with SU(2)$_R$ gauge symmetry, which incorporate the standard version of SUSY hybrid inflation.

The $\mu$ problem is solved via a U(1) R-symmetry which forbids the existence of an explicit $\mu$ term, while allows a trilinear superpotential coupling of the gauge singlet inflaton superfield to the electroweak Higgs superfields. After the spontaneous breaking of the GUT gauge symmetry, this singlet inflaton acquires a suppressed VEV due to the soft SUSY-breaking terms. Its trilinear coupling to the Higgs superfields then yields a $\mu$ term of the right magnitude.

The main decay mode of the inflaton is to a pair of electroweak Higgs superfields via the same trilinear coupling. The initial lepton asymmetry is created in the subdominant decay of the inflaton to a lepton and an electroweak Higgs superfield via the interference of one-loop diagrams with exchange of different RHNs. The existence of these diagrams requires the presence of some specific superpotential couplings which explicitly violate the U(1) R-symmetry and R-parity. These couplings, however, do not affect the exact baryon number conservation in perturbation theory which is implied by the R-symmetry. Thus, the only way to generate baryons in these models is via a primordial leptogenesis (or via electroweak baryogenesis).

In our analysis, we took into account the constraints from neutrino masses and mixing. The requirement that the primordial lepton asymmetry not be erased by lepton-number-violating processes before the electroweak phase transition is a much more stringent constraint on the parameters of the theory. We showed that the model with SU(2)$_R$ gauge symmetry is too restrictive to be able to generate an adequate baryon asymmetry in accordance with these constraints and for natural values of the other parameters even if we allow the neutrino oscillation parameters to vary within their 2$\sigma$ confidence intervals. It is thus ruled out.

On the contrary, the non-SU(2)$_R$-symmetric model can be viable even with central values of the neutrino mass-squared differences and mixing angles. However, we find that this model is much more restrictive than the model studied in Ref. [11], which contained SU(2)$_L$ triplet superfields giving a second contribution to light neutrino
masses after that of the RHNs. Indeed, in order to generate the observed BAU, we had to take a larger reheat temperature, which is though perfectly acceptable if the gravitino decay to photons and photinos is somewhat suppressed, and a string scale somewhat smaller than the weakly-coupled heterotic string one. Such lower string scales are easily obtained in other string models such as the strongly-coupled heterotic string from M-theory.

We also find that the lightest neutrino mass eigenvalue, as the strongly-coupled heterotic string from M-theory. scales are easily obtained in other string models such

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led to erroneous results.


[36] This may be confirmed by considering that the negatively charged would-be Goldstone mode $h^-$ must have the same chemical potential as the $W^-$ in order to constitute its longitudinal mode. From the relation $h^- = W^- - h$, implied by the SU(2)$_L$ gauge interaction, we then conclude that $h = 0$.


