Cosmological Constraints on Newton’s Constant

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We present cosmological constraints on deviations of Newton’s constant at large scales, analyzing latest cosmic microwave background (CMB) anisotropies and primordial abundances of light elements synthesized by big bang nucleosynthesis (BBN). BBN limits the possible deviation at typical scales of BBN epoch, say at $10^9 \sim 10^{12}$ m, to lie between $-5\%$ and $+1\%$ of the experimental value, and CMB restricts the deviation at larger scales $10^7 \sim 10^9$ pc to be between $-26\%$ and $+66\%$ at the $2\sigma$ confidence level. The cosmological constraints are compared with the astronomical one from the evolution of isochrone of globular clusters.

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Newton’s law of gravitation has been extensively tested and verified in three length scales: the laboratory scales $r \lesssim 1$ m \cite{1}, the geophysical scales $r \approx 100$ m \cite{2}, and the astronomical scales $r \approx 10^8$ m \cite{3}. Such measurements nicely agree with the inverse square law within their experimental or observational uncertainty \cite{1,2}. In particular, the first two measurements at the laboratory and geophysical scales succeeded also in determining the experimental value $G_N$ of Newton’s constant, and the value determined at such terrestrial scales is applied for all phenomena from Planck scale to cosmological scale.

The astronomical measurements \cite{3}, mainly through planetary and satellite orbits, yield a strong constraint on the deviation from the inverse square law. However, it should be noted that the measurements can not give any information about the value of Newton’s constant $G$ itself without evaluating masses $M$ of interacting bodies, since constraint is possible only on $GM$. Therefore, the measurements can not exclude the possibility of different value of $G$ at astronomical and cosmological scales, if $G$ is almost constant at limited scales relevant to the measurements. In particular, we have only a poor knowledge at scales larger than the solar system, say $r \gtrsim 1$ pc $\approx 3 \times 10^{16}$ m. \cite{3}. Interesting trials to solve this problem were recently reported \cite{2,3}, in which the deviation of $G$ at Mpc scales is restricted by the power of the clustering of galaxies.

The possibility that Newton’s constant at laboratory scale, $G_N$, is different from that at very large scales, $G_\infty$, arises in many context. Historically, studies toward the problem of unifying gravity with the other fundamental forces suggested a departure from Newtonian gravity in the range $10 - 100$ m. \cite{3}. It is often assumed that such a correction can be represented by the addition of Yukawa term to the conventional gravitational potential: $V = -\frac{G(r)M}{r}$ for $G(r) = G_\infty (1 + \alpha e^{-r/\lambda})$, where $\alpha$ is the relative weight of the non-Newtonian term. In this expression, at cosmological distances $r$ satisfying $r \gg \lambda$, the exponential term vanishes, so that $G(r) = G_\infty$. On the other hand, for $r$ of experimental scales which satisfies $r \ll \lambda$, the exponential becomes unity and $G(r)$ recovers $G_N$, that is, $G_N = G_\infty (1 + \alpha)$. \cite{3}

Recently several types of higher-dimensional theories of gravity, motivated by superstring, have been proposed and many researchers pay great attention to the extra dimension scenarios. As a characteristic feature, all the theories lead to deviations from the conventional Newton’s law \cite{4,10}, since the theories allow graviton to propagate in higher-dimensional spacetime. Among them, an interesting idea was proposed by Dvali, Gabadadze, and Porrati. In the model the present accelerating expansion of the universe is attributed to leaking gravity into extra dimension \cite{11}. This idea reproduces the present cosmic acceleration without dark energy component, and consequently predicts the modification of Newton’s law at cosmological scales. Another interesting proposal is a braneworld model with Gauss-Bonnet term, which suggests $G_N = \frac{2\alpha}{3\phi}G_\infty$ with a model parameter $\phi$ \cite{12}.

These theoretical suggestions indicate that it is quite important to place possible constraints on the value of $G$ at astronomical and cosmological scales. In this paper, we take a simple parameterization of

$$G(r) = \xi G_N, \quad (1)$$

for a finite range of $r$ relevant to the measurement which we are considering. As such a measurement, we consider CMB anisotropies and primordial abundances created at BBN epoch, and put the constraints on the value of $\xi$ at two different scales relevant to these observations. The cosmological constraints are compared with an astronomical constraint determined from the isochrone of globular clusters.

The observed primordial light-element abundances constrain the value of $G$ during the BBN epoch from the time of weak reaction freezout ($t \sim 1$ sec, $T \sim 1$ MeV) to the freezeout of nuclear reactions ($t \sim 10^4$ sec, $T \sim$
10 keV). In this epoch, the length of cosmic horizon varies from $10^8$ m to $10^{12}$ m, and thus BBN can constrain Newton’s constant at these scales.

The primordial helium abundance is obtained by measuring extra galactic HII regions. We adopt range of $Y_p = 0.2452 \pm 0.0015 \ [13]$ for the helium abundance. The primordial deuterium is best determined from its absorption lines in high redshift Lyman α clouds along the lines of sight to background quasars. For deuterium there is a similar possibility for either a high or low value. For the present discussion, however, we shall adopt the generally accepted low value for the D/H abundance, D/H = $2.78^{+0.34}_{-0.38} \times 10^{-5} \ [14]$

The increase of Newton’s constant causes the increase of the universal expansion rate. This makes the neutron-to-proton ratio larger, because the weak reactions freeze-out at a higher temperature, and also because there is less time for neutrons to decay between the time of weak-out at a higher temperature, and also because there is a similar possibility for either a high or low value. For the present discussion, however, we shall adopt the generally accepted low value for the D/H abundance, D/H = $2.78^{+0.34}_{-0.38} \times 10^{-5} \ [14]$

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The upper limit of Newton’s constant comes from $^4$He upper bound and D/H upper bound. The lower limit comes from the lower bounds. We note that the constraint from $^7$Li is not consistent with those from $^4$He and D/H, even when we vary $\xi$. In the present analysis, however, we omit the constraint from $^7$Li abundance, since it involves an uncertainty more largely than the other primordial elements do. The BBN constraint thus obtained is $0.95 \leq \xi \leq 1.01$

The temperature fluctuations at recombination observed through the CMB anisotropies contain much information on many kinds of cosmological parameters and evolution of perturbations at a wide range of scales. Typically, the scale which can be explored by CMB observations currently available is from the horizon scale at present ($\sim 10$ Gpc) to $\sim 10$ Mpc in comoving coordinate. This shows that the scales relevant to CMB are from $10^2$ pc to $\sim$ Gpc, if we consider the evolution of perturbations from horizon crossing of each Fourier mode; for example, $10^2$ pc is the horizon scale at the time when the mode of $\sim 10$ Mpc in comoving coordinate enters the horizon. Thus it follows that CMB can constrain the value of $\xi$ at scales larger than $\sim 10^2$ pc. Here, in order to calculate CMB anisotropies in a consistent manner we assume that the scale dependence of Newton’s constant is very weak at the relevant scales, which is consistent with a simple parameterization of Eq. (1).

In order to obtain a constraint on $\xi$ from latest CMB anisotropy data sets, we generate CMB angular power spectra $C_\ell$ in a wide range of $\xi$ by using a Boltzmann code of CMBFAST [13]. It is well known, however, that in addition to $\xi$ there exist many other cosmological parameters relevant to CMB. Thus, we explore the likelihood in seven dimensional parameter space, i.e., $\Omega_b h^2$ (baryon density), $\Omega_c h^2$ (cold dark matter density), $h$ (Hubble parameter), $z_{re}$ (reionization redshift), $n_s$ (power spectrum index), $A_s$ (overall amplitude), and $\xi$. We then marginalize over nuisance parameters through the use of Markov Chain Monte Carlo technique [15].

![Fig. 1](image1.png) **FIG. 1:** Predicted BBN light-element abundances v.s. the baryon-to-photon ratio $\eta_{10}$. $^4$He, D and $^7$Li abundances are shown in the top, center and bottom panels, respectively. They are compared with the observationally inferred primordial abundances (horizontal lines). Plotted are models with $\xi = 1.01$ (solid) and $\xi = 0.95$ (dashed). In our analysis neutron life time is taken to be the average value of $\tau_n = 878.5$ sec [15] and $\tau_n = 885.7$ sec [16].

![Fig. 2](image2.png) **FIG. 2:** Constraints from the primordial abundances (lines) and CMB (contours) on $\eta_{10}$ and $\xi$. The shaded region denotes parameters allowed by BBN. The contours show the marginalized 1, 2σ limits on this parameter plane from fits to the CMB power spectrum. The solid and dashed contours correspond to the limits from WMAP data alone, and those from WMAP, CBI and ACBAR data sets, respectively.
The most distinguishable effects of changing Newton’s constant appear at the amplitude of the acoustic peaks in the CMB power spectrum as shown in Fig. 3. The main reason for this is that, as already found in [20], the visibility function, \( g(\tau) = \kappa \exp(-\kappa) \), changes with \( \xi \), where \( \kappa \) is the optical depth of the Thomson scattering. More specifically, increasing Newton’s constant makes the expansion of the universe faster at a given redshift, and makes it more difficult for photon and electron to recombine to form hydrogen atom. This leads to larger ionization fraction and broader visibility function at last scattering epochs, which damp the anisotropies at small scales due to canceling effect.

Second, in addition to the effect discussed above, we find that increasing Newton’s constant suppresses the second and higher acoustic peaks even larger since for the increase of \( \xi \) the scale of the first acoustic peak (\( \sim t_{\text{dec}} \propto \xi^{-1/2} \)) is shifted more largely than the diffusion scale for photons to spread through the random walk does, (\( \sim t_{\text{dec}}^{1/2} \propto \xi^{-1/4} \)). Thus, the shape of acoustic peaks can be used to constrain the variation of \( \xi \).

Figure 4 shows the marginalized likelihood of \( \xi \). We obtain from the figure that \( 0.74 \leq \xi \leq 1.66 \) by WMAP data alone [21], \( 0.75 \leq \xi \leq 1.74 \) by WMAP, CBI and ACBAR data sets [22, 23], at 95% confidence level.

Another constraint on the value of Newton’s constant can be obtained by analyzing the age of stars in globular clusters. The key idea is that increasing Newton’s constant causes stars to burn faster [24]. Thus, this allows us to constrain \( \xi \) at stellar scale (\( \sim 10^9 \) m), as we shall see below, by analyzing the timing of main sequence turn off.

Let us assume that luminosity of star depends on Newton’s constant \( G \) and helium abundance \( Y \), approximately as \( L \propto y(Y)g(G) \), where \( y \) and \( g \) are functions of \( Y \) and \( G \) [22]. Since helium production should be proportional to the luminosity, we have \( \frac{dY}{dt} \propto y(Y)g(G) \). A star which departs from the main sequence today (\( t_0 \)) should be considered to have \( Y \approx 1 \) at its center so that \( \int_{t_{\text{init}}}^{t_0} \frac{dY}{g(Y)} \propto g(G) \int_{t_{\text{init}}}^{t_0} dt \). We further assume that \( g(G) \propto G^\gamma \), where \( \gamma \approx 5.6 \) have been obtained from numerical simulation [27]. From the fact that the l.h.s. of the above equation does not depend on \( G \) and time, we have the relation,

\[
\tau_* = \xi^\gamma \int_{t_0}^{t_0 - \tau} dt = \tau \xi^\gamma .
\]

Here \( \tau_* \) is the apparent turn-off age, which should be obtained by analyzing HR diagram of a globular cluster with the standard value of \( G \), and \( \tau \) is the true age of the globular cluster. Thus, if information on the true age of globular cluster is available, the globular cluster can be used to constrain \( \xi \):

\[
\left( \frac{\tau_{\text{max}}}{\tau_*} \right)^{-\frac{1}{\gamma}} \approx \xi \approx \left( \frac{\tau_{\text{min}}}{\tau_*} \right)^{-\frac{1}{\gamma}} .
\]

If we take \( \tau_{\text{max}} = 15.8 \) (2 \( \sigma \) upper bound on the expansion age of the universe obtained by our analysis in Sec. III including the variation of \( \xi \)), and conservatively assume that \( \tau_{\text{min}} = 10 \) Gyr [27], we then obtain

\[
0.93 \leq \xi \leq 1.09 ,
\]

where we use \( \gamma = 5.6 \) and \( \tau_* = 12.9 \pm 2.9 \) Gyr, which is age of the galactic globular clusters [26].

All the higher-dimensional theories of gravity proposed recently allow Newton’s constant to be scale-dependent. In this paper, assuming the dependence is weak for horizon scales in BBN epoch (\( 10^8 \sim 10^{12} \) m) and also for those in CMB epoch (\( 10^2 \sim 10^9 \) pc), we place constraints on \( \xi = G/G_N \) at the cosmological scales. An important point is that the present analysis yields constraints on the value of \( G \) itself, while other astronomical tests of the inverse square law do so only on the value of \( GM \) including unknown mass \( M \) of interacting bodies.
found that the difference emerges at smaller scales. Thus, we can constrain to $0 < \xi \lesssim 1$. The value of $\xi$ is fixed to one at laboratory scale ($\xi < 0.01$), and then to confirm the necessity of the higher-dimensional theories of gravity.

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