Neutrino statistics and big bang nucleosynthesis

A.D. Dolgov\textsuperscript{1,2}, S.H. Hansen\textsuperscript{3}, A.Yu. Smirnov\textsuperscript{4,5}

\textsuperscript{1} Istituto Nazionale di Fisica Nucleare, Ferrara 40100, Italy
\textsuperscript{2} Institute of Theoretical and Experimental Physics, 117218, Moscow, Russia
\textsuperscript{3} Institute for Theoretical Physics, Univ. of Zurich, Winterthurerstrasse 190, CH-8057 Zurich, Switzerland
\textsuperscript{4} The Abdus Salam International Centre for Theoretical Physics, I-34100 Trieste, Italy
\textsuperscript{5} Institute for Nuclear Research, Russian Academy of Sciences, Moscow, Russia

Abstract. Neutrinos may possibly violate the spin-statistics theorem, and hence obey Bose statistics or mixed statistics despite having spin half. We find the generalized equilibrium distribution function of neutrinos which depends on a single fermi-bose parameter, $\kappa$, and interpolates continuously between the bosonic and fermionic distributions when $\kappa$ changes from -1 to +1. We consider modification of the Big Bang Nucleosynthesis (BBN) in the presence of bosonic or partly bosonic neutrinos. For pure bosonic neutrinos the abundances change (in comparison with the usual Fermi-Dirac case) by $-3.2\%$ for $^4\text{He}$ (which is equivalent to a decrease of the effective number of neutrinos by $\Delta N_\nu \approx -0.6$), $+2.6\%$ for $^2\text{H}$ and $-7\%$ for $^7\text{Li}$. These changes provide a better fit to the BBN data. Future BBN studies will be able to constrain the fermi-bose parameter to $\kappa > 0.5$, if no deviation from fermionic nature of neutrinos is found. We also evaluate the sensitivity of future CMB and LSS observations to the fermi-bose parameter.

E-mail: dolgov@fe.infn.it, hansen@physik.unizh.ch, smirnov@ictp.trieste.it
1. Introduction

Since neutrinos have spin one half they are believed to obey Fermi statistics. A serious argument in favor of this belief is an absence of a consistent quantum field theory of half-integer spin particles with any other statistics than the Fermi one. For electrons and nucleons this issue has been discussed earlier in ref. [1, 2, 3, 4], where it was shown that a possible violation of statistics for these particles is strongly restricted by experiment. On the other hand, direct experimental checks of Fermi statistics for neutrinos are absent, except for a study of the effects of purely bosonic neutrinos on Big Bang Nucleosynthesis (BBN) performed 10 years ago [5]. Recently the idea that neutrinos may possess Bose or mixed statistics reappeared in ref. [6] where phenomenological analysis of testable effects has been presented. A violation of the spin-statistics relation for neutrinos would lead to a number of observable effects in cosmology and astrophysics. In particular, bosonic neutrinos might compose all or a part of the cold cosmological dark matter (through bosonic condensate of neutrinos) and simultaneously provide some hot dark matter [6]. A change of neutrino statistics, would have an impact on the evolution of supernovae and on the spectra of supernova neutrinos. The presence of a cosmological neutrino condensate would enhance contributions of the Z-bursts to the flux of the ultra high energy (UHE) cosmic rays and lead to substantial refraction effects for neutrinos from remote sources [6].

As shown in [5] the change of the neutrino statistics from Fermi-Dirac (F-D) to Bose-Einstein (B-E) leads to a decrease of the $^4$He abundance produced during BBN by about 4%. The results of ref. [6] are in qualitative agreement with ref. [5], however, according to the estimates of ref. [6] the change of $^4$He is somewhat weaker.

Since the double beta decay excludes the possibility of pure bosonic neutrino [7], but still allows mixed statistics, we will study here an influence of the partially bosonic neutrinos on BBN. We introduce the fermi-bose parameter, $\kappa$, which describes the continuous transition from Fermi to Bose distributions in thermal equilibrium as $\kappa$ changes from $-1$ to $+1$, the boundary cases corresponding to purely fermionic and bosonic states with any value of $\kappa$ in between allowed. In particular, $\kappa = 0$ corresponds to Boltzmann statistics. We consider possible constraints on the parameter $\kappa$ from present and future BBN and cosmic microwave background (CMB) plus large scale structure (LSS) data.

2. The generalized distribution function

The form of the kinetic equation for particles with mixed statistics is not immediately evident. The statistics dependent factor $F[f]$ under the collision integral e.g. for the reaction $1 + 2 \leftrightarrow 3 + 4$ in the standard case of pure Bose or Fermi statistics has the form:

$$F = f_1(p_1)f_2(p_2)(1 \pm f_3(p_3))(1 \pm f_4(p_4))$$

$$- f_3(p_3)f_4(p_4)(1 \pm f_1(p_1))(1 \pm f_2(p_2)),$$

(1)
where the ± signs correspond to bosons or fermions.

Derivation of the kinetic equation in general depends on the operator of particle number density and the normalization of states with non-zero number of identical particles. The quantum creation-annihilation operators of neutrinos obeying mixed statistics seems natural to write in the form:

\[ a_k = a_f^k \cos \gamma + a_b^k \sin \gamma, \]
\[ a_k^+ = a_f^{k+} \cos \gamma + a_b^{k+} \sin \gamma, \]
(2)

where \( a_f^k \) and \( a_b^k \) are respectively the Fermi and Bose type operators annihilating states with momentum \( k \). However, the particle number operator in the standard form, \( n_k = a_k^+ a_k \), with \( a_k \) defined in (2) is not satisfactory if there are several identical particles with the same momenta. The emerging problems can be easily observed if one considers the matrix element of the particles number operator, \( n_k \), between the multiparticle states defined in the standard way, \( (a_k^N)^N|0\rangle \). This state is not an eigenstate of the operator \( n_k \). Moreover, it is unclear if there exists an operator of the particle number with the appropriate properties, in particular commuting with the free Hamiltonian. It may even be that one has to abandon the Hamiltonian approach for description of mixed statistics particles. Thus, at this stage we can only make a reasonable guess about the function \( F \).

One simple possibility which has the correct limiting behavior in the case of pure statistics is to suggest that the neutrino distribution function in the final states of any reaction enters \( F \) as

\[ g_\nu \equiv \cos^2 \delta (1 - f_\nu) + \sin^2 \delta (1 + f_\nu), \]
(3)

where \( \delta \) is the statistics mixing angle and \( f_\nu \) is the neutrino distribution function, which in equilibrium interpolates between the F-D and B-E statistics, so that \( f_\nu^{(\text{eq})} = f_{\text{FD}} \) for \( \delta = 0 \) and \( f_\nu^{(\text{eq})} = f_{\text{BE}} \) for \( \delta = \pi/2 \).

The natural assumption leading to eq. (3) is that the outgoing neutrino contributes partly with a Pauli blocking, \( 1 - f_\nu \), with weight \( \cos^2 \delta \), and partly with Bose enhancement, \( 1 + f_\nu \), with weight \( \sin^2 \delta \).

The angle \( \delta \) introduced above should somehow be connected to the angle \( \gamma \) in the operator mixture (2). The equality \( \delta = \gamma \) looks as a reasonable hypothesis. If this is true then the angle \( \gamma \) which parametrizes a violation of neutrino statistics in double beta decay can be constrained from BBN and vice versa. If however, the angles \( \gamma \) and \( \delta \) are different the bounds from BBN and 2\( \beta \)-decay are not directly related.

The distribution (3) can be rewritten as

\[ g_\nu \equiv 1 - \kappa f_\nu(\kappa), \]
(4)

where

\[ \kappa \equiv \cos^2 \delta - \sin^2 \delta = \cos 2\delta \]
(5)

which we call the fermi-bose parameter. Thus, we suggest that the distribution of neutrinos in the final state enters the factor \( F \) in the combination (4). For example, in
the case of elastic scattering $\nu_1 + l_1 \leftrightarrow \nu_2 + l_2$ the neutrino distribution functions with mixed statistics appear in the collision integral as

$$F = f_\nu(k_1)f_l(p_1)[1 - f_l(p_2)][1 - \kappa f_\nu(k_2)]$$

$$- f_\nu(k_2)f_l(p_2)[1 - f_l(p_1)][1 - \kappa f_\nu(k_1)].$$  \hspace{1cm} (6)

The same factor $(1 - \kappa f_\nu)$ appears (instead of $(1 - f_\nu)$) in any process involving mixed statistics neutrinos.

Considering the processes of neutrino scattering and production we have found that the factor $F(f_\nu)$, and consequently, the collision integral, vanish if

$$f_\nu = f^{(eq)}_\nu = [\exp(E/T) + \kappa]^{-1},$$  \hspace{1cm} (7)

while all other particle distributions are given by the standard equilibrium Bose or Fermi functions. That means that $f^{(eq)}_\nu$ given by eq. (7) is the equilibrium distribution function for the case of mixed statistics of neutrinos.

For $\kappa = +1$ the function $f^{(eq)}$ turns into the Fermi distribution, for $\kappa = -1$ it turns into the Bose equilibrium distribution, while for $\kappa = 0$, i.e. for equal mixture of Fermi and Bose statistics, it becomes the Boltzmann one.

In fig. 1 we present the equilibrium distributions (7) for different values of $\kappa$. According to this figure the distribution becomes softer with an increase of the bosonic fraction. The maximum number density shifts to smaller $E/T$, and the integrated number density increases.

For a positive $\kappa$ (when the fermionic component dominates) the distribution function $f_\nu$ is bounded from above by $f_\nu < 1/\kappa$.

If $\kappa$ is negative, then for a large lepton asymmetry a neutrino condensation would be possible. Indeed, for a given $\kappa$, the maximum allowed value of the chemical potential is

$$\mu^{(max)} = m_\nu - T \ln(-\kappa),$$  \hspace{1cm} (8)

this follows from the condition that $f_\nu$ should be non-negative. In particular, for the purely bosonic case we obtain the usual bound $\mu \leq m_\nu$.

If the neutrino charge asymmetry is so large that $\mu^{(max)}$ could not provide it, the neutrinos would form a Bose condensate with the equilibrium distribution function equal to

$$f^{(eq)} = \frac{1}{\exp(E - \mu^{(max)}/T + \kappa + C\delta(k)}.$$  \hspace{1cm} (9)

Here $k$ is the three momentum of the neutrino, and $C$ is a constant whose magnitude is determined by the value of the charge asymmetry of the neutrinos.

An interesting question to address is how unique is the distribution (7). Can one introduce other forms of the mixed statistics equilibrium distributions? An alternative to (3) could be

$$g_\nu \equiv \cos^2 \delta (1 - f_{FD}) + \sin^2 \delta (1 + f_{BE}).$$  \hspace{1cm} (10)
Neutrino statistics and BBN

Figure 1. The neutrino distribution function for different values of $\kappa$ ($-1, -0.5, 0, 0.5, 1$). Here $x = E/T$.

Such kinetics is equivalent to having two independent neutrino species, bosonic and fermionic, and, if equilibrium is established, it would lead to $24/7$ neutrino species (see next section). To avoid equilibrium the bound on $\delta$ in this case would be quite restrictive.

3. Effects on big bang nucleosynthesis

The effect of the change of neutrino statistics on BBN is related to two phenomena. First, the energy density of bosons in equilibrium ($\kappa = -1$) is larger than the energy density of fermions by the factor $8/7$. If all three neutrinos have B-E statistics, their larger energy density would correspond to an increase of the effective number of neutrino species by $\Delta N_\nu = 3/7$. The second, dominant, effect is an increase of the rate of the reactions of neutron-to-proton transformations due to the bosonic (anti blocking) factor $(1 - \kappa f_\nu)$. Because of the larger rate the freezing temperature of these reactions would be lower and consequently the frozen $n/p$-ratio would be smaller. It can be mimicked by a decrease of $N_\nu$. The first effect can be approximately described as $\Delta N_\nu \approx 0.2(1 - \kappa)$, while the second one is noticeably non-linear (exponential).

We performed the calculations of the abundances of light elements with the Kawano nucleosynthesis code [8] which was modified to include the effects of mixed statistics of neutrinos described by the distribution (7). This code [8] is accurate enough for calculations of the relative changes of the abundances, while for the absolute values of
Neutrino statistics and BBN

Figure 2. Upper panel: the change in the effective number of neutrino degrees of freedom found from the change of the $^4$He abundance as a function of the effective fermi-bose parameter $\kappa$. Lower panel: the relative change of the primordial abundances of deuterium, helium-4, and lithium-7, as functions of the effective fermi-bose coefficient $\kappa$. We take $\eta = n_B/n_\gamma = 6.5 \cdot 10^{-10}$.

The results of the computations are shown in figs. 2 and 3. In the upper panel of fig. 2 we present the change of the effective number of neutrino species, $\delta N_\nu$, as a function of $\kappa$, which is equivalent to a decrease of the $^4$He primordial abundance. If the neutrinos have a purely bosonic distribution ($\kappa = -1$), the effect is similar to having $\Delta N_\nu \approx -0.57$.

However, the effect of modified statistics cannot be described by a simple change in $N_\nu$ if other light elements are included. In the lower panel of fig. 1 the relative changes of the abundances of $^2$H, $^4$He, and $^7$Li with $\kappa$ are shown. As expected the mass fraction of $^4$He drops down and for pure bosonic neutrinos we get the relative decrease about

$$\frac{^4\text{He}(\kappa = -1)}{^4\text{He}(\kappa = 1)} - 1 = -3.2\%.$$ (11)
Neutrino statistics and BBN

This is slightly smaller than the value $-3.7\%$ found in [5].

The amount of $^{2}\text{H}$ goes up with decrease of $\kappa$. A higher deuterium abundance can be explained by a slower conversion rate of deuterium to heavier elements due to fewer neutrons and higher expansion rate at BBN epoch when $T \approx 0.8 \cdot 10^{9}\text{K}$. In the pure bosonic case the increase is about 2.6%. The $^{7}\text{Li}$ abundance decreases with $\kappa$, and for $\kappa = -1$ the decrease is about 7%.

![Figure 3](image.png)

**Figure 3.** Upper panel: the ratios of abundances of different elements in the cases of purely bosonic neutrinos with respect to the standard fermionic case as functions of the baryon number density, $\eta$. The vertically hatched (cyan) region shows the WMAP $2\sigma$ determination of $\eta$. Lower panel: the absolute abundance of $^{4}\text{He}$ as a function of $\eta$ for the purely bosonic, Boltzmann, and fermionic neutrino distributions, corresponding to $\kappa = -1, 0, +1$ respectively. The two skew hatched regions show the observation of primordial helium from ref. [12] (lower, yellow) and ref. [13] (upper, magenta), which marginally overlap at $1\sigma$.

Let us confront the absolute values of the abundances for partly bosonic neutrinos with observational results. We will use the relative changes presented in fig. 2 and the central values of the abundances calculated for usual F-D neutrinos in ref. [9]: $Y_\text{p} = 0.2481$, $X_\text{H}/X_H = 2.44 \cdot 10^{-5}$, and $X_\text{7Li}/X_H = 4.9 \cdot 10^{-10}$. Other codes
Neutrino statistics and BBN

may give even slightly larger values for the helium abundance (G. Steigman, private communication). We are here (and in the figures) using the symbol \( X \) for the abundance of \( D \) or \( Li \) ratios to \( H \) by number, not by mass. At \( \kappa = -1 \) we find for \( ^4 \text{He} \): \( Y_p = 0.240 \), which makes much better agreement with the value extracted from observations (for a review of the latter see e.g. [11]). Different helium observations yield different results, e.g., the ref. [12] finds \( Y = 0.238 \pm 0.002 \), and ref. [13] finds \( Y = 0.2421 \pm 0.0021 \) (1\( \sigma \), only statistical error-bars). These results are shown in figure 2 as the skew hatched regions, where the upper (magenta) is the results of ref. [13] and the lower (yellow) is the results of ref. [12]. Whether the existing helium observations are accurate or slightly systematically shifted will be tested with future CMB observations [14].

The amount of \( ^2 \text{H} \) rises at most to \( X_{^2 \text{H}} / X_H = 2.5 \cdot 10^{-5} \), and the agreement between BBN and WMAP data remains good, bearing in mind the observational uncertainties. Primordial \( ^7 \text{Li} \) drops down to \( X_{^7 \text{Li}} / X_H = 4.55 \cdot 10^{-10} \), again slightly diminishing the disagreement between theory and observations.

We see that at the present time BBN does not exclude even a pure bosonic nature of all three neutrinos. Furthermore, the agreement between the value of the baryonic mass density, \( \eta \), inferred from CMBR and the predicted abundances of \( ^4 \text{He} \), \( ^2 \text{H} \), and \( ^7 \text{Li} \) becomes even better. In other words, in the standard BBN model there is an indication of disagreement between observations of \( ^4 \text{He} \) and \( ^2 \text{H} \) - they correspond to different values of \( \eta \) with the observed abundances of \( ^4 \text{He} \) indicating a smaller value of \( \eta \) than the one given by CMBR, while \( ^2 \text{H} \) agrees with CMBR. Motivated by these results the value of \( \Delta N_\nu = -0.7 \pm 0.35 \) was suggested in ref. [15]. In the case of predominantly bosonic neutrinos, as discussed above, the discrepancy between \( ^2 \text{H} \), \( ^4 \text{He} \), and CMBR disappears.

When the problem of large systematic uncertainties of primordial helium determinations will be resolved and the statistical error-bars will dominate the error-budget, one can expect to measure \( N_\nu \) with an accuracy at the level of 0.1. This would exclude \( \kappa < 0.5 \), if an agreement with the standard BBN values is found. Otherwise, if the discrepancy between \( ^4 \text{He} \) and \( ^2 \text{H} \) remains it may be considered as an indication to the mixed statistics of neutrinos.

Our results change only slightly with variation of the baryon number density \( \eta \), as seen in fig. 2. The upper panel shows the ratio of abundances for purely bosonic to purely fermionic neutrinos. The changes are always of the order a few percent for the 3 abundances considered. The results are in good agreement with ref. [5]. The vertically hatched (cyan) region shows the 2\( \sigma \) WMAP result. The lower panel shows the absolute value of the \( ^4 \text{He} \) abundance as a function of \( \eta \), for purely bosonic, Boltzmann, and purely fermionic neutrino distribution functions. For other values of \( \kappa \), the result will be in between those lines, and can be obtained using curves of fig. 2. The skew hatched (yellow) region shows the range of observed values of the helium abundance from refs. [12, 13], which marginally overlap at 1\( \sigma \).

It is well known that CMB can be used to constrain the number of relativistic degrees of freedom at the time of photon decoupling (see e.g. [16]). For CMB and LSS, in
contrast to BBN, the presence of the bosonic neutrinos increases the number of degrees of freedom from 0 to \(3/7 = 0.43\) as \(\kappa\) goes from +1 to −1. The present bounds from CMB and LSS data are insensitive to such changes and too weak to constrain the fermi-bose parameter (see [18] for references). The Planck experiment is forecast to constrain the relativistic degrees of freedom to the level \(\delta N \approx 0.24\) [17] at 1σ. This means that Planck alone will be able to measure \(\kappa\) with a precision of about \(\Delta \kappa \approx 1\). In particular, a pure bosonic distribution function for neutrinos can be excluded at about 2σ level. An “ambitious” future experiment (see details in ref. [19], and for earlier predictions see [20]) will constrain \(\Delta N\) to about 0.02, which corresponds to a determination of \(\kappa\) with precision \(\Delta \kappa \approx 0.1\) at 1σ.

It is known from cosmological considerations that the mass of fermionic hot neutrinos are bounded from above by approximately 1 eV, for recent studies see e.g. [21, 22]. This bound is applicable to any hot dark matter particle, independently of their statistics, which have the same number density as that of neutrinos. For particles which have different number densities, or freeze out at different times, this number changes somewhat [21]. Since large scale structure basically constrains the quantity

\[
\Omega_{\nu} h^2 = \frac{\sum m_i n_i}{93 eV n_{th}}
\]

where \(n_{th}\) is the number density of a thermal fermionic neutrino, then bosonic neutrinos will have their masses constrained a factor 4/3 weaker than fermionic neutrinos. We thus see that cosmological probes of neutrino masses remain roughly as strong as always, in comparison to terrestrial tritium or double beta decay experiments.

In addition to the thermal neutrino component bosonic neutrinos might condense in the early universe and have much larger number density than thermal relics, as argued in ref. [6]. However, the cosmological upper bound on their mass would remain practically the same because the latter is valid only for hot thermal relics which suppress structure formation at small scales.

The higher number density of bosonic neutrinos will imply a marginally later freeze out, and hence a larger sharing of the entropy from the annihilating electrons than in the standard scenario [23, 24, 25]. This effect is, however, very small. More general non-thermal neutrino spectra and the effect on freeze out is studied in [19].

Let us comment on a possibility of large neutrino condensate in the case of partly bosonic neutrinos. Such a condensate would contribute as cold dark matter in the Universe (and show up in the CMB and LSS analysis). The condensate is formed when the lepton asymmetry is larger than that which could be ensured by the maximal possible chemical potential. In the case of pure bosonic neutrinos the chemical potential is restricted by the neutrino mass and is therefore negligible, especially at the BBN epoch. In contrast, in the case of partly bosonic neutrinos the maximal potential given by (8), or \(\xi = \mu/T = m_{\nu}/T - \ln(-\kappa)\), can be large. So, in the case of partly bosonic neutrinos, the formation of the condensate would imply a large chemical potential, which could destroy the excellent agreement with BBN. Due to mixing between the
active neutrinos the chemical potentials should be equal for all 3 neutrino species at the time of BBN [26, 27]. Then using the strong bound on the leptonic asymmetry in the electron neutrinos we find the bound $\kappa < -0.9$: For $\kappa > -0.9$ such a chemical potential for the electron neutrinos will significantly underproduce helium, leading to a disagreement with observations. For negative chemical potential $^4He$ would be strongly overproduced leading to essentially the same bound, $\kappa < -0.9$.

That is, neutrinos should be almost purely bosonic to produce the condensate and satisfy the BBN bound. On the other hand, almost purely bosonic neutrinos are excluded (disfavored) by the double beta decay [7]: the mixing angle $\theta$ at the level $\sin \theta \sim 0.8$ is still allowed, however, the angle $\theta$ is not necessarily equal to $\delta$ introduced above. Notice further that the relation of $\kappa$ with the fermi-bose parameter relevant for the $\beta\beta$ -decay is not clear, as discussed in section 2.

Anyway, an improvement of the BBN bound on $\kappa$ can exclude a possibility of the neutrino condensate which might contribute substantially to the cold dark matter in the Universe.

4. Conclusions

We find the equilibrium distribution function for partially bosonic neutrinos which depends on a single fermi-bose parameter, $\kappa$. The change of this parameter from +1 to $-1$ corresponds to a continuous transition between Fermi and Bose distributions.

We have considered the influence of bosonic or partially bosonic neutrinos on BBN. In the extreme case of completely or predominantly bosonic neutrinos the primordial abundances change in comparison with the usual F-D cases in the following way: $^4He$ decreases by 3.2%, $^2H$ increases by 2.6% and $^7Li$ decreases by 7%. The agreement between theory and observations becomes noticeably better.

Future determinations of $^4He$ will allow to exclude values of fermi-bose parameter $\kappa < 0.5$, if agreement with the standard case is found. The BBN bounds on $\kappa$ can be compatible with those obtained from the analysis of two neutrino double beta decay [7].

Future CMB+LSS observations can constrain or observe this parameter, possibly to the level $\Delta \kappa \approx 0.1$, potentially providing indications of a violation of the Pauli exclusion principle.

Acknowledgments

SHH thanks the Tomalla foundation for financial support.

References

Neutrino statistics and BBN

[8] Kawano L, Fermilab-Pub-92/04-A
[9] Serpico P D *et al*, 2004 JCAP **0412** 010