Updated results on the Monte Carlo Simulation of the SPD/PRS Pulseshape

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Abstract

In this note a full Monte Carlo Simulation of the LHCb SPD/PRS subdetectors tile signal temporal shape is presented. Being this simulation too slow a fast Monte Carlo simulation is introduced to tune the free parameters from the full MC simulation. The pulseshape information is needed to be introduced in the general Monte Carlo Simulation (Gauss) and it is extracted from this fast simulation. The fast simulation is also applied for ECAL and HCAL to compare it to test-beam data.

1 Introduction

LHCb [1, 2] Calorimeters [3] Preshower (PRS) and Scintillator Pad Detector (SPD) consist of a plane of scintillator tiles separated by a 1.4 cm thick lead converter divided in three zones: inner, middle and outer. They are used at L0 trigger to recognise and correct the energy of electromagnetic showers (PRS) and to differentiate between electrons and photons (SPD). A charged particle that crosses a scintillator tile [4] excites the atoms next to its path and these emit light to get back to their fundamental state. This light is then collected by the WLS fibre [5] inside the scintillator tile and lead to the photomultiplier [6]. The PMT transforms this light into electronic signal that is processed by the very front end electronics [7, 8]. The time distribution of this electronic signal [9] has a non negligible fraction that lays outside of the 25 ns window (gate) of integration time (spill-over effect). This makes necessary to make simulations of this signal to introduce it in the Monte Carlo simulation of the LHCb experiment: SICBMC (see [10]) before and now Gauss.

At first, a stand alone fully detailed Monte Carlo simulation of the photons, propagating through the scintillator and the fibre until the PMT, has been implemented. This simulation reproduces step-by-step the propagation of photons and takes into account all interactions at fundamental level of the photons with the media through which they propagate. These interactions are geometrical (reflections and limit angles), as well as related to the media properties (emission times and mean free paths).

As not all of the full MC simulation parameters are known (the reflectivity\(^\text{11}\) with the pad walls and the reemission time inside the fibre), they have to be tuned to experimental data (cosmic ray tests and test-beam data). But this full MC simulation is quite slow (\(~ 5\) hours to simulate \(10^4\) photons) to be run for several parameters, in order to have the results compared to the experimental data. That’s why a second fast Monte Carlo simulation has been developed that mimics the pulseshape results of the full MC simulation.

With this fast MC simulation the full MC parameters are obtained by tuning them with a cosmic ray test data. It is then used to simulate the pulseshape of the SPD/PRS pads with the settings that they will have in the experiment, and introduce its results in the general MC simulation of the experiment. Moreover,

\(^{11}\text{the probability of not being lost in a reflection}\)
the fast MC simulation allows a re-tuning every time that there is a change in the experimental setup.

The same techniques used to simulate the SPD/PRS pulshape can be applied to ECAL and HCAL as is shown in this note.

2 Full Monte Carlo Simulation

In order to simulate the SPD/PRS tiles time response four different times have to be taken into account:

- **Scintillator decay time.** Once the scintillator atoms have been excited by a charged particle they take some time to emit the light. It follows a 2.1 ns decay exponential distribution.

- **Propagation time inside the scintillator** from its emission until it is captured. See below.

- **WLS Fibre decay time.** When a photon is captured by the scintillating core of the fibre it takes an exponential decay distributed time of about 10 ns mean time to re-emit the photon (with a higher wave length). This dominates the decay shape of the total time distribution.

- **Propagation time inside the fibre** until it arrives to the PMT. See below.

2.1 Propagation Time Inside the Scintillator

Photons are created randomly in the tile’s volume (except inside the fibre) and they are given a direction. Once the photon is created it is followed in steps of 1 ps along its trajectory until it touches the fibre. The Y11 fibre trapping efficiency is 5.4% so there is a 94.6% probability that the photon is not captured, gets reflected and follows its path until it touches again the fibre and maybe is captured. Before this happens, the photon can be lost in a reflection (with a low probability) depending on the angle with the wall or be reabsorbed by the scintillator (its mean free path is 3.8 m).

In order to know when does the photon meet the fibre the mathematical equation of the fibre surface is needed. The fiber disposition inside the scintillator pad can be approximated to a helix.

In order to take into account the scintillator mean free path the simulated photon should be captured before a randomly generated time following a decay exponential of 3.8 m (divided by the speed of light in the medium) of mean.

2.2 Propagation Time Inside the Fibre

Once a photon is captured, it is reemitted inside the fibre scintillating core (88% of the total fibre diameter) and, as with the scintillator, its passage through the fibre is simulated by 1 ps steps. When a photon arrives to a fibre wall, if its angle
with the fibre surface normal is greater than the effective limit angle $\beta_L$, it gets reflected.

As with the propagation in the scintillator the fibre mean free path is taken into account imposing that the photon has to arrive to the end of the fibre before a randomly generated time following a decay exponential of 3.35 m (divided again by the speed of light in the medium) of mean.

If any clear fibre is simulated, the same longitudinal to the fibre axis component of the photon is kept, but the mean free path is changed to $\sim 30$ m.

3 Fast Monte Carlo Simulation

In order to simulate the SPD/PRS tiles time response the same four different times as in the full MC simulation are generated following the distributions that will be explained below. As explained in the introduction these distributions try to imitate as closely as possible the time response distributions obtained from the full MC simulation. Once having generated these four times they have to be summed up to obtain the total time spent by the photon in the scintillator and the fibre. The four times are, as before:

- Scintillator decay time.
- Propagation time inside the scintillator, whose unknown parameter is the loss probability in a reflection.
- WLS fibre decay time.
- Propagation time inside the fibre, whose variable parameter is the reemission mean decay time.

The scintillator decay time and the WLS fibre decay time are just generated as in the full MC simulation.

3.1 Propagation Time Inside the Scintillator

To obtain the time distribution of photons from their emission to the moment they are captured by the fibre, for the three different sizes of tiles (inner, middle and outer), the time distributions from the full Monte Carlo simulation have been looked at.

As the reflectivity in the tiles is not well known, the shape of the distribution of times inside the scintillator is got depending on the reflectivity. This distributions are well fitted to a decaying exponential. The full MC simulation is run in a range of $90\% - 100\%$ reflectivity taking into account the mean free path in the scintillator and the resulting decay times ($\tau$) are shown in figure 1. They can be fitted by

$$\tau = \frac{1}{A + B \cdot r} \quad (1)$$

being $r$ the reflectivity. The different values of $A$ and $B$ for the different tiles sizes are shown in table 1.
<table>
<thead>
<tr>
<th>Pad Size ($cm^2$)</th>
<th>$A$ ($ns^{-1}$)</th>
<th>$B$ ($ns^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 4$</td>
<td>$11.34 \pm 0.04$</td>
<td>$-11.14 \pm 0.04$</td>
</tr>
<tr>
<td>$6 \times 6$</td>
<td>$9.58 \pm 0.03$</td>
<td>$-9.43 \pm 0.03$</td>
</tr>
<tr>
<td>$12 \times 12$</td>
<td>$7.39 \pm 0.03$</td>
<td>$-7.29 \pm 0.03$</td>
</tr>
</tbody>
</table>

Table 1: Fit values for the dependence in the reflectivity of the decaying exponential distribution of the time spent in the scintillator.

Figure 1: Fitted decay times of the simulated time inside the scintillator distribution for (a) $4 \times 4$ $cm^2$ pad (b) $6 \times 6$ $cm^2$ pad (c) $12 \times 12$ $cm^2$ pad. The bigger is the reflectivity, the longer last the photons and the bigger is the decay time.

The reflectivity is one of the parameters that have been tuned comparing with cosmic rays tests data.

### 3.2 Propagation Time Inside the Fibre

To generate the propagation time inside the fibre to go from its reemission point to the PMT the following method is used:

- **Step 1)** As a first step, using the known analytical expression of the propagation time distribution of photons through an homogeneous straight fibre (not exiting it), a random $\cos \theta$ is generated\(^2\).

- **Step 2)** Since photons are only reemitted inside the scintillating core and not all of them who would remain in the straight fibre do it with a bent fibre, the distribution of those that are lost with the full MC simulation is found and fitted.

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\(^2\) $\theta$ is the local polar angle and $\theta = 0$ is the fibre longitudinal axis direction.
Step 3) Then, three different times have to be considered: the propagation time inside a straight segment of fibre if the photon is reemitted in it, the time needed to cover the fibre loops and the time to cover the straight segment of fibre until the PMT.

Step 4) Finally the mean free path in the fibre is also taken into account.

Step 1

As explained above, it can be determined analytically which is the time distribution of photons emitted in one end (uniformly in its surface) of a straight fibre that propagate until the other end. In a straight fibre this time divided by the fibre length ($T_f$; in time units) corresponds to $1/\cos \theta$, being $\theta$ the photon direction angle with respect to the fibre axis.

If the photon is emitted at $(r, \alpha)$ in the fibre section (in polar coordinates) and in the direction $(\theta, \phi)$ and $\phi^{13}$ the condition to remain in the fibre is:

$$\cos \beta_L > \sin \theta \sqrt{1 - \left(\frac{r \sin(\phi - \alpha)}{r_f}\right)^2}$$

(2)

$\beta_L$ is the fibre effective limit angle (63.3°) and $r_f$ is the total fibre radius (0.5 mm).

This can be calculated taking the equation of the photon trajectory before it gets reflected in the fibre wall, and the fibre wall’s equation:

$$\frac{x_{wb} - r \cos \alpha}{\sin \theta \cos \phi} = \frac{y_{wb} - r \sin \alpha}{\sin \theta \sin \phi} = \frac{z_{wb}}{\cos \theta}$$

(3)

$$x_{wall}^2 + y_{wall}^2 = r_f^2$$

The reflection point is then calculated, as well as the scalar product of the normal vector to the wall and the photon’s vector, to find the cosine of the reflection angle. This angle should be greater than the effective limit angle for the photon to remain in the fibre, thus the cosine of the reflection angle should be less than the cosine of the effective limit angle. If a photon remains in the fibre after the first reflection, it will remain for the rest of reflections, due to the symmetry of the straight fibre.

In this way the time probability density function of not lost photons can be calculated by convolution of the following uniform distributions.

$$\rho(t) = N \int_0^{T_f} \frac{2r}{r_f}dr \int_0^{2\pi} \frac{d\alpha}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^1 d(\cos \theta) \delta(t - \frac{T_f}{\cos \theta}) \times$$

$$\times \theta \left(\cos \beta_L - \sin \theta \sqrt{1 - \left(\frac{r \sin(\phi - \alpha)}{r_f}\right)^2}\right)$$

(4)

$^{13}$The azimuthal angle
Figure 2: The thin line represents the time distribution of photons, if all of them remained in the fibre (it is $T/t^2$) and the thick one is $\rho(t/T)$. Before $1/\sin \beta_L$ they coincide, that means that all photons with $\cos \theta < \sin \beta_L$ remain in the fibre.

\[
\rho = \begin{cases} 
0 & \text{, for } t < T \\
N \frac{T}{t^2} & \text{, for } T \leq t < \frac{T}{\sin \beta_L} \\
N \frac{T}{t^2} \left(1 - \frac{2}{\pi} \left( \arcsin A + A \sqrt{1 - A^2} \right) \right) & \text{, for } \frac{T}{\sin \beta_L} \leq t
\end{cases}
\]

where $N$ is a normalisation factor and $T$ is the length of the fibre divided by the speed of light in it, that is, the time that a photon would need to cover the fibre length in a straight line, parallel to the fibre axis. In figure 2 the function is shown and in figure 3 a check comparing with the full simulated straight fibre can be made.

The same distribution function could be found for photons generated only in the scintillating fibre core (as it is done further on), but the fit of the full MC simulation distributions is simpler starting from equation 4.

**Step 2**

As photons are reemitted inside the scintillating core of the fibre and as the fibre in the pad is not straight but bent, some of the photons that would have arrived
Figure 3: The histogram has been obtained with a full MC simulation of the propagation of photons through a straight fibre and the dotted line is $\rho(t/T)$ normalised to the number of simulated photons.
Figure 4: The left plot shows the $\cos \theta$ distribution of missing photons divided by the straight fibre $\cos \theta$ distribution for $\cos \theta > \sin \beta_L$ and the right one shows the same for $\cos \theta < \sin \beta_L$. Both for the $4 \times 4$ pad. The line is the fitted with $f_{\text{lost}}(1/\cos \theta)$ to the PMT if the fibre was straight are lost. With the full MC simulation of the curved fibre it has been calculated that the loss is:

- $30.2 \pm 0.1\%$ for the $12 \times 12 \text{ cm}^2$ pad.
- $34.5 \pm 0.1\%$ for the $6 \times 6 \text{ cm}^2$ pad.
- $38.8 \pm 0.1\%$ for the $4 \times 4 \text{ cm}^2$ pad.

It has been observed that these numbers do not depend in the number of loops that the photon covers. The photons that are not lost in the first reflections with the fibre walls are not lost any more.

The $4 \times 4$ pad $1/\cos \theta$ distribution of these lost photons divided by the known time distribution of a straight fibre can be seen in figure 4 as an example. The lost photons distribution divided by the straight fibre distribution can be fitted by the function $f_{\text{lost}}(1/\cos \theta)$ in equation 5.

$$f_{\text{lost}}(1/\cos \theta) = \begin{cases} G \left( \frac{1}{\cos \theta}, F_1, \frac{1}{\sin \beta_L}, \sigma_1 \right) & \text{, for } \cos \theta > \sin \beta_L \\ G \left( \frac{1}{\cos \theta}, F_1, \frac{1}{\sin \beta_L}, \sigma_1 \right) + \Lambda \left( \frac{1}{\cos \theta}, F_2, \mu_2, \sigma_2 \right) + 1 - \left( \frac{t_0 + \cos \theta}{t_0 + \sin \beta_L} \right)^\lambda & \text{, for } \cos \theta \leq \sin \beta_L \end{cases}$$

(5)

where $G(x, F, \mu, \sigma)$ is a gaussian distribution and $\Lambda(x, F, \mu, \sigma)$ is a landau distribution and $F_1$, $\sigma_1$, $F_2$, $\mu_2$, $\sigma_2$, $t_0$ and $\lambda$ are the parameters that have been fitted. The results of the fit for every pad size can be seen in table 2.
Table 2: Values for the function to fit the missing photon distribution due to the fibre curve.

<table>
<thead>
<tr>
<th>Pad Size (cm$^2$)</th>
<th>$F_1$</th>
<th>$\sigma_1$</th>
<th>$F_2$</th>
<th>$\mu_2$</th>
<th>$\sigma_2$</th>
<th>$t_0$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 4</td>
<td>0.58</td>
<td>0.0177</td>
<td>1.53</td>
<td>1.168</td>
<td>0.0159</td>
<td>-0.270</td>
<td>-4.34</td>
</tr>
<tr>
<td>6 x 6</td>
<td>0.47</td>
<td>0.0120</td>
<td>1.72</td>
<td>1.157</td>
<td>0.0137</td>
<td>0.0524</td>
<td>-5.18</td>
</tr>
<tr>
<td>12 x 12</td>
<td>0.36</td>
<td>0.0061</td>
<td>1.51</td>
<td>1.153</td>
<td>0.0117</td>
<td>0.535</td>
<td>-6.32</td>
</tr>
</tbody>
</table>

The angular distribution function of photons remaining in a curved fibre is then:

$$\rho_{\text{bent}}(1/\cos \theta) = \rho_{\text{straight}}(1/\cos \theta) \times (1 - f_{\text{lost}}(1/\cos \theta)) \quad (6)$$

**Step 3**

Note that these distribution functions depend on $\cos \theta$ (where $\theta$ is the local angle with the fibre longitudinal axis at the emission point) that corresponds to $t/T$ when the fibre is straight, but the time a photon needs to cover a segment of bent fibre is not $1/\cos \theta$. Moreover, when a photon enters a straight segment after having passed a curved one it should not have the same $\cos \theta$ with which it was created.

![Curve Time Error](image1.png) ![Back Time Error](image2.png)

Figure 5: Relative error taking $T/\cos \theta$ instead of the time needed to go along a curved zone of a fibre (a) and to go along a straight zone having passed the curved one (b) for a curve of radius 1.85 cm (that is the 4 x 4 cm$^2$ pad fibre helix radius).

Figures 5 show the relative error committed when taking $1/\cos \theta$ instead of the time needed to go along a half loop of a fibre and to go along a straight zone.
having passed the half loop. So, to generate the propagation time inside the SPD pad fibre this three times have to be taken into account:

- $t_S$ is $1 / \cos \theta$, the time the photon would need to cover $T = 1 \text{ ns}$ of straight fibre. It is used in the straight segments of fibre in which the photon has been reemitted. This is the time that is generated with the distributions explained above.
- $t_C$ is the time that a photon needs to cover $T = 1 \text{ ns}$ of curved fibre. It depends on the radius of the helix. It is used in the curved segments of fibre either if the photon has been created in it or if it has entered the curved zone from a straight segment.
- $t_B$ is the time that a photon needs to cover $T = 1 \text{ ns}$ of straight fibre after having passed through a curved zone.

See next subsection for an explanation of how is $t_C$ obtained from a distribution function parametrised by $\cos \theta$ and how $t_B$ is obtained from another distribution function parametrised by $\cos \theta$, having integrated the dependence on $t_C$.

**Fibre Times Obtention**

In this subsection the way to calculate $t_C$ and $t_B$ from $\cos \theta$ (section 4.2.2) is explained. First of all the two relative time differences have to be defined:

$$
\epsilon_C = t_C \cos \theta - 1
$$

$$
\epsilon_B = t_B \cos \theta - 1
$$

![Graphs](image)

Figure 6: $\epsilon_C$ for a $4 \times 4$ (a) $\cos \theta > (0.99 + 0.01 \sin \beta_L)$ (b) $0.01 + 0.99 \sin \beta_L > \cos \theta > \sin \beta_L$ (c) $\cos \theta < \sin \beta_L$.

Figure 6 shows $\epsilon_C$ for three different ranges of $\cos \theta$. The distributions of $\epsilon_C$ and $\epsilon_B$ parametrised in function of $\cos \theta$ have to be found. It can be seen that
for $e_C$ it can be done fitting double gaussian distributions (equation 7) for small ranges of $\cos \theta$.

$$\rho(1/\cos \theta) = \frac{P}{\sqrt{2\pi \sigma_1}} e^{-\frac{(1/\cos \theta - \mu_1)^2}{2\sigma_1^2}} + \frac{1 - P}{\sqrt{2\pi \sigma_2}} e^{-\frac{(1/\cos \theta - \mu_2)^2}{2\sigma_2^2}}$$  \hspace{1cm} (7)

$P$, $\mu_1$, $\sigma_1$, $\mu_2$ and $\sigma_2$ depend on $1/\cos \theta$. The dependence of these parameters is characterised by:

- They follow a linear function from $\cos \theta = 1$ to a certain $1/\cos \theta = t_0$ that depends on the helix radius.
- From $1/\cos \theta = t_0$ to $1/\cos \theta = 1/\sin \beta_L$ the dependence becomes vague because the two gaussians are overlapped.
- From $1/\cos \theta = 1/\sin \beta_L$ the parameters become nearly constant.

So, each parameter is fitted by a straight line until $1/\cos \theta = t_0$ and then another straight line is imposed from its value in $1/\cos \theta = t_0$ to the constant value in $1/\cos \theta = 1/\sin \beta_L$.

Figure 7: $e_B$ for (a) $1 < t_S < 1/(0.99 + 0.01 \sin \beta_L)$ (b) $1/(0.01 + 0.99 \sin \beta_L) < t_S < 1/\sin \beta_L$ (c) $t_S > 1/\sin \beta_L$ and $e'_B$ with $f = 1.7$ for (d) $1 < t_S < 1/(0.99 + 0.01 \sin \beta_L)$ (e) $1/(0.01 + 0.99 \sin \beta_L) < t_S < 1/\sin \beta_L$ (f) $t_S > 1/\sin \beta_L$.

Figure 7 shows $e_B$ and $e'_B$ for three different ranges of $1/\cos \theta$. To find the $e_B$ distributions it has to be taken into account that $t_B$ not only depends on $1/\cos \theta$. 

11
but also on $t_C$. By using the variable $e'_B = e_B - f \times e_C$ the dependence on $t_C$ is integrated and a one dimensional function parametrised by $1 / \cos \theta$ can be fitted ($f$ is a factor that permits the $e'_B$ distribution to be fitted as a simple gaussian). For all helix radius and $1 / \cos \theta$ ranges 1.7 has been seen to be a satisfactory factor (this can be seen in figure 7 too), but for the $12 \times 12$ pad radius from $\cos \theta = 1$ to $1 / \cos \theta = t_0$. For $1 / \cos \theta = 1$ the best factor $f$ is 0.8 and from this value a straight line is used until $1 / \cos \theta = t_0$, where it reaches 1.7.

As with $e_C$ the distribution parameters depend on $t_S$. The dependence of the mean and sigma is found similarly to the previous cases: two straight lines and a constant. In the case of the sigma the second straight line does not go from $t_0$ to $t_S = 1 / \sin \beta_L$, the limits are two different values $t_1$ and $t_2$ that also depend on the helix radius.

When simulating pulses $e_C$ and $e'_B$ are generated randomly following their distributions dependent on $t_S$ and then the other two times are obtained.

$$t_C = t_S (1 + e_C)$$

$$t_B = t_S (1 + e'_B + f \times e_C)$$

### 3.3 Fast MC Simulation Procedure

First the scintillator is simulated as explained in previous sections: a photon emission in the scintillator decay time is generated randomly, as well as the time from its emission to its capture by the fibre, which depends on the reflectivity of the pad walls which is not known (see subsection 4.2.1). Then the time for the photon's reemission is also generated, with a variable decay time that comparison with data will determine, as well as the pad walls reflectivity.

This photon reemitted inside the fibre may be created in a straight zone of the fibre or in the helix zone. If it is created in a straight zone and goes out of the pad (case 1) no correction is needed, $t_S = 1 / \cos \theta$ is just used. If it goes toward the helix (case 2) then $t_S$ (before the helix), $t_C$ (in the helix) and $t_B$ (from the helix to the PMT) are needed. If it is created in the helix (case 3) $t_C$ and $t_B$ are needed (but $1 / \cos \theta$ has to be generated all the same because this is the one whose distribution function is available; the other two are calculated from this one).

To generate a time inside the fibre a point in all the length of the fibre inside the pad is selected randomly uniformly distributed from one edge to the other. In the full MC simulation it has been observed this approximation can be made. Then the fibre end to which the photon will go is selected randomly too (half probability to one end and half to the other). A $t_S$ following $\rho_{\text{end}}(t_S)$ (equation 6) is generated and from it $t_C$ and $t_B$ are also generated. In this way $T$ is calculated as the fibre length that the photon has to cover until the PMT (divided by the speed of light in the medium) and is divided in three parts corresponding to $t_S$, $t_C$ and $t_B$:

- $T_S$ is the distance (in time units) in which $t_S$ is valid. It is the distance to the PMT in case 1, the distance to the helix in case 2 and zero in case 3.
Fast-Full MC Comparison

Figure 8: Comparison between the full (solid line) and fast (dashed line) Monte Carlo simulation for reflectivity 0.91 and fibre decay time 7.1 ns.

- $T_C$ is the curved fibre distance that the photon has to cover. It is zero in case 1, $7\pi R$ (being $R$ the helix radius) in case 2 and the distance to the straight part of the fibre in case 3.

- $T_B$ is the straight fibre distance when the photon comes from a bent segment. It is zero in case 1 and the distance to the PMT in case 2 and 3.

Then the total propagation time spent inside the fibre is calculated adding $t = t_S \times T_S + t_C \times T_C + t_B \times T_B$. To take into account the effect of the mean free path, a random time following a decay exponential is generated and should be greater than $t$, otherwise another $t$ will have to be generated.

The total time from its emission in the scintillator to its arrival to the PMT is then calculated by adding all of the four times (scintillator decay time, time in the scintillator, fibre decay time and time in the fibre; this last time is the sum of $t_S \times T_S + t_C \times T_C + t_B \times T_B$). A comparison between the results of the full Monte Carlo program and its fast version, introduced in this thesis, is made in figure 8. The fast simulation mimics very well the full simulation and is $\sim 6 \times 10^3$ times faster.
4 Extension to ECAL & HCAL

Using the same technique as for PRS/SPD the pulsecture of ECAL and HCAL can be simulated, but the geometries are different. Moreover to have the most signal within the first 25 ns window the signal clipping is performed. This consists in subtracting a fraction of the signal 12.1 ns before to the current signal with a retarding coaxial cable in such a way that the signal tail is suppressed and nearly all of the signal falls into a 25 ns window. This can be done in ECAL and HCAL because their PMT’s receive around two orders of magnitude more photoelectrons than SPD or PRS.

4.1 ECAL

An ECAL cell consists in a module of 66 layers, each one made of a lead layer (2 mm thick), reflecting paper (120 μm thick) and scintillator (4 mm thick). At the downstream end of a module the PMT receives the light emitted in all the scintillators. The collection of the emitted light is performed by Y11 WLS fibres that cross longitudinally the module. When the fibres reach the upstream part of the module they are bent 180° with a bending radius of 1.5 cm and cross the module again until the PMT (outer and middle regions) or mirrored at the edge in inner region because such a small radius bending could not be performed. In this way, when a photon is captured by the fibre it can go directly to the PMT or the other way, through the bent segment of fibre or being reflected at the upstream edge, depending on the region.

As with SPD/PRS the simulation of the time a photon needs to arrive to the PMT is divided into four different times:

- The emission time in the scintillator (1),
- the spent time in the scintillator until the capture by the fibre (2),
- the remission time in the fibre (3) and
- the propagation time until the PMT (4).

The two emission times (1) and (3) are the same as with SPD/PRS. Just the two propagation times (2) and (3) need are specific of ECAL.

In order to obtain the time spent in the scintillator until the capture by the fibre a full MC simulation of the scintillator tile is performed like with SPD/PRS. The reflectivity of the tile walls used in this full MC simulation is 92%, the obtained for SPD/PRS after comparing to experimental data (See subsection 4.4.2). These time distribution results are well fitted by a double decaying exponential (equation 8) as is shown in figure 9 and table 3 summarises the values obtained for the three cell sizes.

\[
p_{\text{tile}}(t) = P \frac{e^{-t/\tau_1}}{\tau_1} + \frac{1 - P}{\tau_2} \frac{e^{-t/\tau_2}}{\tau_2}
\]  

(8)

In order to obtain the propagation time in the fibre the two possible ways to arrive to the PMT have to be considered. There is half probability to go one way and half the other. If the photon goes the straight way, a similar distribution
Figure 9: Fitted decay times of the simulated spent time inside the scintillator distribution for (a) $4 \times 4 \text{ cm}^2$ (b) $6 \times 6 \text{ cm}^2$ (c) $12 \times 12 \text{ cm}^2$ ECAL tile.

<table>
<thead>
<tr>
<th>Cell Size $(\text{cm}^2)$</th>
<th>Fibres per Cell</th>
<th>Dist. between Fibres $(\text{mm})$</th>
<th>Bending Radius $(\text{mm})$</th>
<th>$\tau_1$ (ns)</th>
<th>$\tau_2$ (ns)</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 4$</td>
<td>16</td>
<td>10.10</td>
<td>--</td>
<td>0.234</td>
<td>0.660</td>
<td>0.589</td>
</tr>
<tr>
<td>$6 \times 6$</td>
<td>36</td>
<td>10.10</td>
<td>15.15</td>
<td>0.261</td>
<td>0.888</td>
<td>0.580</td>
</tr>
<tr>
<td>$12 \times 12$</td>
<td>64</td>
<td>15.25</td>
<td>15.25</td>
<td>0.297</td>
<td>1.345</td>
<td>0.489</td>
</tr>
</tbody>
</table>

Table 3: Scintillator and fibre geometry values and distribution function parameters for the time spent in the scintillator. It can be observed that the mean time mostly depends on the distance between fibres inside the scintillator tiles.

to the straight fibre propagation time distribution explained in section 4.2.2 is applied. In this case there is not the problem of the bent fibre, but the photons are still created in the scintillating core and not homogeneously in all the fibre section. This distribution can be also calculated analytically as shown in equation 9

\[ \rho(t) = N \int_{0}^{r_f} \frac{2r}{(fr_f)^2} dr \int_{0}^{2\pi} \frac{2\alpha}{2\pi} d\alpha \int_{0}^{2\pi} \frac{d\phi}{2\pi} \int_{0}^{1} d(cos \theta) \delta(t - \frac{T}{cos \theta}) \times \]

\[ \times \theta \left( \cos \beta_L - \sin \theta \sqrt{1 - \left( \frac{r \sin(\phi - \alpha)}{r_f} \right)^2} \right) = \]

9
<table>
<thead>
<tr>
<th>Radius</th>
<th>Lost %</th>
<th>$F_1$</th>
<th>$\sigma_1$</th>
<th>$F_2$</th>
<th>$\mu_2$</th>
<th>$\sigma_2$</th>
<th>$t_0$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 cm</td>
<td>41.3 ± 0.1</td>
<td>0.627</td>
<td>0.0208</td>
<td>1.41</td>
<td>1.17</td>
<td>0.0151</td>
<td>-0.546</td>
<td>-3.35</td>
</tr>
</tbody>
</table>

Table 4: Fit values for the function to fit the missing due to the fibre curve photon distribution for outer and middle ECAL modules.

$$
= \begin{cases} 
0 & , \text{for } t < T \\
N \frac{T}{T^2} & , \text{for } T \leq t < \frac{T}{\sin \beta_L} \\
N \frac{T}{T^2} \left(1 - \frac{2}{\pi} \left(\arcsin \frac{A}{2} + \frac{A}{2} \sqrt{1 - \left(\frac{A}{2}\right)^2}\right)\right) & , \text{for } \frac{T}{\sin \beta_L} \leq t < t_{lim} \\
0 & , \text{for } t_{lim} \leq t 
\end{cases}
$$

$$
t_{lim} = \frac{1}{\sqrt{1 - \cos^2 \beta_L}}
$$

where $A$ is the same as with equation 4 and $f$ is the fraction of the fibre radius corresponding to the scintillating core. Note that this distribution is zero from a certain time value that depends on $f(t_{lim})$ and that if $f$ tends to one, $t_{lim} \to \infty$ and we recover equation 4. Equation 9 is not used for SPD and PRS because the fit of $f_{lost}(1/\cos \theta)$ resulted more easy starting from equation 4.

If the photon goes the long way the missing photons distribution $f_{lost}(1/\cos \theta)$ is obtained the same way as with PRS/SPD, but with the corresponding fibre radius shown in table 3. Like with PRS/SPD this missing photons distribution for bent fibres is of the form of equation of 5. Table 4 is the equivalent to 2 for ECAL. In this case the $t_S$, $t_C$ and $t_B$ are also generated as explained in subsection 3.3. When simulating fibres in an inner module a reflection efficiency of 90% is assumed in the fibre edge and only the distribution in equation 9 is used.

Depending on the position of the tile in the beam direction, the length of fibre is calculated and then the time in the fibre. To accept the generated photon this time should be smaller than the photon’s life time after the reemission, which follows the decay exponential with the fibre attenuation length.

Finally, as with SPD/PRS, the four times are summed up and the pulseshape distribution is obtained.

### 4.2 HCAL

The disposition of the HCAL scintillator tiles and absorber plates is different to ECAL’s. The plates are placed in the $ZY$ plane, parallel to the beam. So there are only six tile positions in $Z$. For every tile the scintillator light is collected by the fibres glued at the upper and/or lower edge of the tile. As in the two previous cases four times have to be generated, which is much easier than before due to the much simpler geometry of HCAL:

- Emission time by the scintillator, which is the same as with SPD/PRS/ECAL.
• Time spent in the scintillator which is easy to simulate because the photon propagation just finds the fibre in one or two edges of the tile.

• Reemission time by the fibre, the same as with SPD/PRS/ECAL.

• Propagation time inside the fibre. The straight fibre propagation time distribution function can be used here without corrections because, instead of bending the fibre, the upstream edge of the fibre has a mirror with a 90\% reflectivity.

The procedure to simulate the time in the scintillator consists in generating randomly in the tile volume photons in a random direction. The initial position is generated following an uniform distribution in the transversal directions to the beam and a decaying exponential of 21 cm decay length for the direction parallel to the beam, as this is the typical energy deposition distribution function for HCAL. Once the initial position and direction are generated the number of times the photon will touch the fibre until it is captured (depending on the fibre trapping efficiency) is calculated. With this value the number of reflections with the tile walls is calculated and the probability to survive them all too. If the photon survives the time has to be less than its life time. This is generated with a decaying exponential according to the scintillator mean free path.

To generate the time in the fibre, the length from the capture point to the PMT is calculated, depending on the way the photon takes when it is reemitted (upstream, reflecting in the fibre edge's mirror, or downstream, directly to the PMT). In any case the straight fibre propagation time distribution function in equation 9 is used. If it goes upstream it has to survive the reflection with a 90\% probability.

5 Comparison to Data

5.1 Cosmic Rays Tests in Clermont-Ferrand

Cosmic rays tests have taken place in the LPC (Laboratoire de Physique Corpusculaire) with two PRS modules: one outer module and one inner module, with 16 pads each, with a 80 cm length of fibre out of the scintillator and no clear fibre.

The way to register the signal is shown in figure 10. The pad under study is put between two scintillator tiles that act as trigger: when signal is detected in both it means a cosmic ray has crossed the three of them and the signal in the scintillator is recorded. Then the pre-analysis of the saved cosmic rays is made as follows:

• The integrated signal curve is obtained and the time at which the cosmic ray signal begins ($t_0$) is calculated looking at the point where the straight line tangent to the integrated curve, in the point where it reaches 10\% of its total height, is zero.

• Then the signal of the cosmic rays as a function of time is summed up into a histogram for each pad. In figure 11 the signal decay time, obtained fitting
an exponential function, is plotted for different pads of the two sizes. The mean is a slope of 9.6 ns for $4 \times 4$ pads and 10.8 ns for $12 \times 12$ pads.

- Another calculated quantity is the fraction of signal that falls into the 25 ns after $t_0$. Figure 12 shows the integrated signal for different pads. These fractions can be seen in figure 13 and averaging are 85% for $4 \times 4$ pads and 81% for $12 \times 12$ pads.

5.2 Simulation Parameters Tuning

From these cosmic rays tests two simulation parameters can be extracted: the reflectivity in the scintillator pad walls and the reemission decay time in the fibre. The conditions in the tests have been reproduced, this is, a $4 \times 4$ pad and a $12 \times 12$ pad with a length of 80 cm outside of the pad that go directly to the photomultiplier. The effect of the photomultiplier is simulated by adding an “asymmetric gaussian” $S_{\text{PMT}}(t)$ as indicated in equation 10 with $\sigma_1 = 0.73$ ns and $\sigma_2 = 1.28$ ns to simulate the rise time and the decay time of the PMT signal for a single photoelectron. The values of $\sigma_1$ and $\sigma_2$ are set to get a proper width of the signal around the maximum.

\[
S_{\text{PMT}}(t) = \begin{cases} 
\frac{e^{-t^2/2\sigma_1^2}}{\sqrt{2\pi}\sigma_1} & \text{for } t < 0, \\
\frac{e^{-t^2/2\sigma_2^2}}{\sqrt{2\pi}\sigma_2} & \text{for } t > 0.
\end{cases} \tag{10}
\]

The resulting reemission decay time may not be the one given (7.1 ns), as this absorbs effects that have not been considered in the simulation, like the possibility that a photon may be reemitted inside the fibre within the absorption spectrum, and so it would be reabsorbed and emitted again. Here the reemission time should count double. There is also the possibility of a second reemission decay
Figure 11: Negative exponential slope after the signal maximum for the 16 pads of the two modules. In one of the 12 x 12 pads a discrepancy can be observed, it may be due to a fibre crack.

time that happens with less probability. These two effects would not depend on the pad size\textsuperscript{14}. From all effects not considered in the simulation, that are absorbed by the fibre decay time, there is one that depends on the pad size. This is the effect of the fibre cracking due to the bending of the fibre. The smaller the bending radius is, the bigger the cracking could be and the bigger is its effect. But it has also to be taken into account that if the bent segment of fibre through which photons propagate is longer, the effect will be bigger. The same effect can be produced by the bending of WLS and clear fibres outside the pad.

The parameters tuning has been performed by comparing the time histograms obtained with the simulation with the histograms obtained by the cosmic rays test. These histograms are in 200 ns windows divided in 500 bins, so the width of every bin is 0.4 ns. The simulation histograms are created with the same number of bins and bin width, and they are adjusted over the analysed data histogram in such way that their maximum bins coincide and they have the same height. After comparing the simulation histogram over the data histogram the

\textsuperscript{14}In fact, if the absorption probability before the second reemission were very small the photon could travel some distance before this and then the effect would depend on the fiber length, and thus on the pad size.
Figure 12: Integrated signal function.

The corresponding $\chi^2$ has been used to find the simulation parameters that adjust as closely as possible to the data histograms:

$$\chi^2(\tau, r) = \sum_{k=k_{\text{max}}-30}^{k_{\text{max}}+40} (B_i(k) - B_{\tau,r}(k))^2$$  \hspace{1cm} (11)

where $k_{\text{max}}$ is the index of the maximum data histogram bin, $B_i(k)$ are the bin values, $i$ is the index of the pad (there are 16 for the tested inner module and 15 valid for the outer) and $B_{\tau,r}(k)$ are the values of the pulse shape simulation histograms that depend on $\tau$ (the fibre decay time) and $r$ (the pad walls reflectivity). Since the data histograms are obtained by means of an oscilloscope, and its resolution does not change in the time axis, the error of the data histogram bins are taken to be constant, and so it is not worth introducing them dividing, as it is usual, in the $\chi^2$. The $\chi^2$ sum is performed from 30 bins before the histogram maximum and 40 after. This is because this fit should be made from after the signal has begun to the point before the oscillations observed in figure 14. In figure 15 is shown an example of the $\chi^2$ typical dependence on the reflectivity, for a given decay time in the fibre.

The results obtained for the 12×12 pads are that the best adjusting is achieved with $\tau = 9.58 \pm 0.02 \text{ ns}$ and $r = 91.9 \pm 0.8\%$. These are obtained by averaging over the best $\tau$’s and $r$’s for the different 15 pads.
Figure 13: Signal fraction inside the 25 ns after \( t_0 \) for the 16 pads of the two modules. As in the previous figure there is a discrepancy in one of the 12 \( \times \) 12 pads.

In the case of 4 \( \times \) 4 pads the reflectivity has very small influence in the \( \chi^2 \) for reflectivities less than 94\%. Thus the reflectivity is fixed to \( \sim 92\% \) (12 \( \times \) 12's best reflectivity) and the optimal \( \tau \) is looked for, that results to be \( 8.62 \pm 0.06 \) ns.

In figure 14 an example of a 12 \( \times \) 12 pad and a 4 \( \times \) 4 pad cosmic rays test histograms and the corresponding simulation histograms with the optimal parameters can be seen.

Note that the 12 \( \times \) 12 \( \tau \) is more away than the 4 \( \times \) 4 \( \tau \) from the 7.1 ns given by the manufacturer. This means that the contribution to the cracking effect of the bent distance to cover is larger than the contribution of the bending radius, at least for this range of distances and radii.

As histograms have to be produced to be introduced with this simulation and these parameters in the experiment’s Monte Carlo simulation, the same parameters for the 6 \( \times \) 6 pad are needed. As there is no available data for this pad size, an interpolation to obtain the \( \tau \) has to be made (the reflectivity is assumed to be the same). The simplest interpolation is the straight line (\( \tau \sim A + BR \)), given that only two points are available.

Note that this linear interpolation is only valid from \( R_{4\times4} \) to \( R_{12\times12} \). Applying the interpolation with \( B = 26.3 \pm 1.7 \) ns/m; 8.13 \pm 0.09 ns a value of \( \tau_{6\times6} = \)
Figure 14: The histogram is the cosmic rays test histogram, while the line is the simulation shape for the optimal parameters for (a) a $12 \times 12$ pad and (b) a $4 \times 4$ pad.

$8.87 \pm 0.10$ ns is obtained.

5.3 ECAL and HCAL Simulation Comparison to Test-Beam Data

Here, for completeness, the simulation for ECAL and HCAL is compared with test-beam data, even though it is not being used currently in the experiment MC simulation (Gauss).

5.3.1 ECAL

Accordingly to the SPD/PRS simulation the fibre decay time absorbs the fibre cracking effect, apart from other effects already explained, that may be quite large for a fibre bending of 15 $mm$ as in ECAL outer modules. So a different decay time has to be put for photons that go the direct way to the PMT and for those that go through the bent segment. There may be some fibre cracking effect when fibres are put together in a bundle to fit in the PMT. In figure 17 is shown the average pulseshape for a fibre decay time of 8.5 ns for photons taking the direct way to the PMT and 10 ns for those going the long way. The signal decay slope is in these conditions $\sim 10$ ns.

In figure 18 several $50 GeV$ electrons shower pulseshapes in an outer ECAL module are shown. With the average energy deposition of a $50 GeV$ electron in every scintillator tile of outer module, obtained with GEANT3, shown in figure 16. The mean of all of the shower exponential slopes in figure 18 gives a decay slope of $\sim 10.8 \pm 0.5$ ns. To make a better comparison more statistics from test-beam data is needed.
Figure 15: Example of the $\chi^2$ typical dependence on the reflectivity.

Figure 16: Longitudinal 50 GeV electron shower profile in ECAL module.

5.3.2 HCAL

The test beam data shown in figure 19 represent the pulseshape after signal clipping of the six different (by their position in a HCAL submodule position) scintillator tiles. To obtain it the tested module has been rotated 90° in such a way that the beam is directed perpendicular to the tile $YZ$ surface. The beam points at one side of the middle of the tile, as in the middle there is a hole, needed for assembling.

This setup has been simulated with the HCAL fast Monte Carlo. To simulate the clipping the subtracted fraction of 12.1 ns previous signal is needed. This is calculated by simulating a typical HCAL shower pulseshape, with the approximation that the energy deposition has the shape of a decaying exponential of 20 cm of decay length. Given the decay slope ($\tau$) of this pulseshape, the clipping
factor is $\exp(-12.1ns/\tau)$. In figure 20 an analog plot to 19 is shown with a fibre decay time of 8 ns (as there is no bent fibre here the decay time is the same for both photons going upstream or downstream) and they look similar. For more precision, the pulselshape without clipping would be needed.

6 Implementation at LHCb Software

The purpose of the obtention of the parameters in the fast Monte Carlo simulation was to simulate the experiment’s conditions and get some timing histograms that are used in the general LHCb MC Simulation program Gauss.

These histograms are used to simulate the spill-over effect in which the signal coming from a bunch crossing affects the data taken in the next bunch crossings.

The way to simulate the spill-over in every SPD and PRS cell is the following. Once a certain energy is deposited in a certain cell during a certain event, it is divided into several 25 ns window depositions according to the histogram (see figure 21) provided by the fast MC simulation. This histogram $X$ axis is time and the bins are 1 ns wide. Every bin height is the probability of a photon arriving within the 1 ns of the bin. Every deposit of energy is divided in different 25 ns window sub-deposits.

In the following phase these sub-deposits are used for digitisation with the program Boole. The energy deposits that are not in the current window are used to simulate the spill-over, with minimum bias events. To the current window deposit of an event in the cell situated at $(x,y)$ is added (sometimes, this is

Figure 17: ECAL 50 GeV electron shower pulselshape simulation.
decided randomly, because there are empty bunch crossings) the deposit in the
\((-x, -y)\) cell (to avoid correlations) at the previous window in the next spill-over
event, at the next window for the previous spill-over event and/or at the second
next window for the second previous spill-over event. With this the total energy
deposition from all events is simulated and then digitisation can proceed.

For ECAL and HCAL, as there is the signal clipping, only two numbers are
used: the current and the previous window fractions of photons that arrive to
the PMT. These are extracted from test-beam data.

To generate the necessary histograms, pads of the three sizes have been sim-
ulated, with the tuned parameters, with 80 cm of WLS fibre outside of the pad
and 3 m of clear fibres. An example of the resulting histograms can be seen in
figure 21.
Figure 19: Test-beam HCAL pulreshape for the six different tiles positions, with the beam perpendicular to the HCAL tiles.

Figure 20: HCAL pulreshape simulation, as in figure 19.
Figure 21: Histograms used in Gauss for the SPD/PRS (a) 4 × 4 cm² pad (b) 6 × 6 cm² pad (c) 12 × 12 cm² pad.

References

[6] E. Grauges et al., Test of multi-anode photomultiplier tubes for the LHCb scintillator pad detector, accepted for publication by NIMA ref. 42870