A MEASUREMENT OF THE RATIOS OF THE RATES

$K_L^0 \rightarrow 3\pi^0/K_L^0 \rightarrow \pi^+\pi^-\pi^0, K_L^0 \rightarrow 3\pi^0/K_L^0 \rightarrow \text{all charged}$

AND $K_L^0 \rightarrow \pi\mu\nu/K_L^0 \rightarrow \pi e\nu$

AND TESTS OF THE $\Delta I = 1/2$ RULE

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1. INTRODUCTION

In the Summer of 1966 the CERN heavy liquid bubble chamber was exposed to a neutral beam for the purpose of studying the decay modes of K_L^0 mesons. The chamber was filled with heavy freon (CF_3Br , radiation length 11 cm) in order to provide a high conversion efficiency for gamma-rays. The magnetic field strength was 27 kilogauss.

An aluminium pipe of 4 cm internal diameter was installed in the chamber in order to transport the large K_L^0 and accompanying neutron flux in vacuum. A mirror attached to the back plate of the chamber permitted detection and measurement of gamma rays converted in the region shadowed by the pipe.

The K_L^0 flux was such as to give on the average 1.5 K_L^0 decays per photograph.

In the present paper we report direct measurements of the following ratios of decay rates:

$$R = \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \pi^+ \pi^- \pi^0}$$

$$R' = \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \text{all charged modes}}$$

and

$$r = \frac{K_L^0 \rightarrow \pi\mu\nu}{K_L^0 \rightarrow \pi e\nu}.$$

This programme was motivated by several factors. In particular:

- i) The quantity R is of direct physical interest and is sensitive to any admixture of an $I = 3$ final state with the dominant $I = 1$ state.
- ii) The quantity r can be compared with the corresponding quantity for K^+ decay and hence check the leptonic $\Delta I = \frac{1}{2}$ rule.

- iii) Accurate values of R and R' can be combined with existing precise information on the K_L^0 lifetime and the branching ratios for charged decay modes of K_L^0 to obtain more exact values of absolute rates for all the K_L^0 decay modes. This permits better tests of the $\Delta I = \frac{1}{2}$ rule for leptonic and non-leptonic decays by comparing with corresponding K^+ decay rates.
- iv) Since in several experiments the $K_L^0 \rightarrow 3\pi^0$ rate is used as a normalization in the measurement of the rate for the CP violating decay mode $K_L^0 \rightarrow 2\pi^0$, it seemed very desirable to improve the precision of the $3\pi^0$ rate.

The three laboratories have carried out separate analyses using different scanning, selection and analysis procedures. This enables intercomparison of the results to provide a valuable check against the possible presence of unknown systematic errors. These analyses are described in turn below. The results are then compared, combined and discussed.

2. A MEASUREMENT OF THE RATIO $R = K_L^0 \rightarrow 3\pi^0 / K_L^0 \rightarrow \pi^+ \pi^- \pi^0$ (CERN)

The results reported are based on a study of about 40,000 photographs scanned for the following two topologies:

- a) N converted gamma rays ($2 \leq N \leq 6$) coming from a single point within a 50 cm long fiducial volume in the vacuum pipe.
- b) Two charged non-electron tracks leaving the vacuum pipe, compatible with having opposite signs of charge, accompanied by one or two gamma rays converted in freon. The charged tracks and gamma rays were again required to look compatible with a common origin inside the fiducial volume.

These two topologies correspond to $3\pi^0$ and $\pi^+ \pi^- \pi^0$ candidates, respectively. The scanning rules provided for the rejection of photographs in which more than one decay occurred in or near the fiducial volume.

Since the scanning rules were framed with the aim of collecting all possible events, the $\pi^+\pi^-\pi^0$ candidates found in the scanning contained a substantial ($\sim 15\%$) background. This consisted principally of events in which the gammas were either bremsstrahlung of one of the charged tracks (if the decay mode was really $\pi\mu\nu$) or produced in a secondary interaction of a charged track. It was removed by careful study of the events by physicists, aided by the results of measurement and subsequent geometrical and kinematical reconstruction¹⁾.

On the other hand the $3\pi^0$ candidates contained essentially no background since their topology was sufficiently unique to identify them clearly at the scanning table.

The number of $\pi^+\pi^-\pi^0$ events thus obtained still required correction to allow for various losses. The observed frequency of gamma-ray conversion in the pipe wall indicated that $8.5 \pm 3\%$ of events had been rejected for that reason. From the observed gamma-detection efficiency, 1% had been lost because neither of the gammas converted. Study of pion interactions in the liquid and the observed number of otherwise good candidates lacking one of the charged secondaries showed that $3 \pm 1\%$ had been lost because of absorption of a pion in the pipe wall. Since an event was rejected if a pion left the pipe outside a certain allowed region, the fiducial volume was in effect smaller for configurations in which a pion emerged far from the decay point. The required correction ($5.5 \pm 1\%$) was determined by appropriately weighting each event.

Observation of otherwise good $\pi^+\pi^-\pi^0$ candidates with three apparently associated gamma rays, and of a slight difference between the $\pi^+\pi^-$ mass distributions of events with one and two gammas, showed that $2.5 \pm 0.8\%$ of the accepted $\pi^+\pi^-\pi^0$ events were really $\pi\mu\nu$ events with an accidentally associated gamma and should be removed.

Finally, the numbers of $3\pi^0$ and $\pi^+\pi^-\pi^0$ events were increased by $9 \pm 2\%$ and $2 \pm 1\%$, respectively, to correct for scanning inefficiency and reduced by $5 \pm 1\%$ and $2 \pm 1\%$, respectively, to correct in a statistical way for unmeasured events lying outside the fiducial volume.

The final numbers of events after applying the above corrections were as follows:

$$3\pi^0 \text{ events : } 1010 \pm 39$$

$$\pi^+\pi^-\pi^0 \text{ events : } 563 \pm 33$$

From these numbers the value of R is

$$R = \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \pi^+\pi^-\pi^0} = 1.80 \pm 0.13$$

3. A MEASUREMENT OF THE K_L^0 BRANCHING RATIOS (ORSAY)

The results reported here are based on a study of 36,000 pictures scanned twice for single K_L^0 decays occurring in the second quarter of the beam path (~ 25 cm) through the chamber. Photographs containing other decay products anywhere in the visible volume were rejected.

At the scanning table, events were accepted which fell into the following four categories:

- a) $3\pi^0$ events defined by 3, 4, 5 or 6 materialized gamma rays.
- b) $\pi^+\pi^-\pi^0$ events defined either by two non-electron tracks plus one or two gammas or by one non-electron track plus two gammas.
- c) $\pi e \nu$ events defined by one non-electron track plus one electron track.
- d) $\pi \mu \nu$ events defined by two non-electron tracks.

It was required in all cases that the charged tracks and gamma rays should be compatible with a common origin in the fiducial volume.

Events with decay points outside the fiducial volume were rejected. For the $3\pi^0$, $\pi e \nu$ and $\pi \mu \nu$ events, the decay point was reconstructed at the scanning table by using the lines of flight of the gammas in the first case or by extrapolating the first charged track back in the other two cases. The distribution along the beam direction of these roughly reconstructed decay points was perfectly uniform both within

and near the edges of the fiducial volume. This confirms the validity of the procedure. For the $\pi^+\pi^-\pi^0$ events, the decay point was reconstructed more accurately by measurement.

The above four categories were selected as being those for which the identification of the decay mode at the scanning table was generally clear. Other events due to the decay modes considered, in which half or more of the decay products were not seen, were rejected and later corrected for. Upper limits to the systematic errors due to misclassification of events were determined by studying the reproducibility of the selection and classification of events at the scanning table.

The corrections for loss of $\pi\mu\nu$ and $\pi e\nu$ events with a missing track were estimated both by Monte Carlo calculation and by studying photographs containing just one electron or non-electron track, with consistent results.

The correction for loss of $\pi^+\pi^-\pi^0$ events with only one non-electron track and one gamma ($1\pi1\gamma$ events) was obtained from the observed relative numbers of $2\pi2\gamma$, $2\pi1\gamma$ and $1\pi1\gamma$ events:

$$N(1\pi1\gamma) = N(2\pi1\gamma) \times [N(1\pi2\gamma)/N(2\pi2\gamma)] .$$

The relative number of $2\pi2\gamma$ and $2\pi1\gamma$ events determines the gamma-ray detection efficiency and hence the correction for loss of $\pi^+\pi^-\pi^0$ events observed as $2\pi0\gamma$ events. Since the $2\pi0\gamma$ events were in fact classified as $\pi\mu\nu$ events, the number of the latter requires a corresponding correction.

Table 1 gives the values of all the corrections made and the final numbers of events, from which we obtain the ratios:

$$\begin{aligned} R &= \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \pi^+\pi^-\pi^0} &= 1.62 \pm 0.13 \\ R' &= \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \text{all charged modes}} &= 0.251 \pm 0.014 \\ r &= \frac{K_L^0 \rightarrow \pi\mu\nu}{K_L^0 \rightarrow \pi e\nu} &= 0.71 \pm 0.05 \end{aligned}$$

Table 1

Mode	Correction for fiducial volume cut	Correction for wrong identifications	Correction for missing particle	Correction for scanning inefficiency	Final number
$3\pi^0$	$0 \pm 1\%$	none	none	$4 \pm 1\%$	549
$\pi^+ \pi^- \pi^0$	$0 \pm 2\%$	$0 \pm 2.2\%$	$1\pi 1\gamma$ events: $+(1 \pm 0.8)\%$ $2\pi 0\gamma$ events $+(1 \pm 1)\%$	$4 \pm 1\%$	339
$\pi e \nu$	$0 \pm 1\%$	$0 \pm 2\%$	1π events and $1e$ events $+(4 \pm 0.5)\%$	$4 \pm 1\%$	1075
$\pi \mu \nu$	$0 \pm 1\%$	$0 \pm 2.5\%$	1π events and 1μ events $+(4 \pm 0.5)\%$ $2\pi 0\gamma$ events $-(0.4 \pm 0.4)\%$	$4 \pm 1\%$	770

4. A MEASUREMENT OF THE RATIO $R' = K_L^0 \rightarrow 3\pi^0/K_L^0 \rightarrow$ ALL CHARGED MODES
(ECOLE POLYTECHNIQUE)

We report here a direct determination of R' by a method which required no geometrical reconstruction of events.

The result is based on a study of about 40,000 photographs scanned for the two topologies

- a) N materialized γ rays ($2 \leq N \leq 6$) coming from a single point within a 27 cm fiducial volume in the vacuum pipe (the second quarter of the beam path through the chamber).
- b) One or two non-electron tracks, the most upstream one of which leaves the pipe within the fiducial volume.

These topologies correspond to the $3\pi^0$ mode and all charged modes, respectively. All tracks leaving the pipe were classified as either electron tracks, non-electron tracks or ambiguous tracks. The scanning rules were such that the presence of an additional gamma-ray origin ($N \geq 3$) or non-electron track anywhere in the visible volume of the chamber caused rejection of the photograph. However, the presence of one or two additional ambiguous or electron tracks caused rejection only if one of them left the pipe in a small forbidden region at its upstream end. Because the magnetic field in the chamber is uniform, the 27 cm long fiducial volume for the exit point of the first non-electron track corresponds to a 27 cm long fiducial volume for the decay point.

No measurements were necessary except for noting on the scanning table:

- a) for the $3\pi^0$ events, the position of the intersection point of the lines of flight of the gamma rays.
- b) for the charged track events, the nature, sign of charge, and position of the exit point for each charged track.

The number of charged track events observed required correction to (i) remove events where the two non-electron tracks were due to different decays, (ii) replace the $\pi\nu$ events lost because the electron left the pipe in the forbidden region, (iii) remove events with single non-electron

tracks in which the first non-electron track had in fact left the pipe before entering the visible volume, (iv) replace events lost because of absorption of mesons in the pipe wall, and (v) to replace events lost because of classification of tracks not due to electrons as ambiguous tracks.

Statistical correction factors were determined for all of these effects. For correction (i) we used the fact that if the two non-electron tracks originate from separate decays their signs of charge and their exit points are uncorrelated. For the geometrical corrections (ii) and (iii) we used the observed distributions of the separation of exit points for the various classes of events. Correction (iv) was estimated by studying the meson interactions observed in the liquid near the pipe. Finally, correction (v) was seen to be negligible.

All of these corrections were found to be small ($< 10\%$ in total) and the uncertainties in the corrections ($\sim 1\%$) were small compared with the statistical error.

The resulting corrected numbers of events were:

- a) $3\pi^0$ events: 444
- b) charged track events: 1668

The scanning efficiencies were found to be

- a) for $3\pi^0$ events: 0.85 ± 0.04
- b) for charged track events: 0.885 ± 0.015 .

From these numbers, the value of R' is found to be:

$$R' = \frac{K_L^0 \rightarrow 3\pi^0}{K_L^0 \rightarrow \text{all charged modes}} = 0.277 \pm 0.021 .$$

5. COMBINATION OF RESULTS

The results from CERN, Orsay and Ecole Polytechnique are summarized in Table 2.

Table 2

Ratio	CERN	Orsay	E.P.
$R = \frac{3\pi^0}{\pi^+\pi^-\pi^0}$	1.80 ± 0.13	1.62 ± 0.13	(1.72 ± 0.16)
$R' = \frac{3\pi^0}{\text{all charged}}$	(0.290 ± 0.023)	0.251 ± 0.014	0.277 ± 0.021
$r = \frac{\pi\mu\nu}{\pi e\nu}$		0.71 ± 0.05	

The numbers given in brackets were obtained using the value of the branching ratio

$$R'' = \frac{K_L^0 \rightarrow \pi^+\pi^-\pi^0}{K_L^0 \rightarrow \text{all charged modes}} = 0.161 \pm 0.005$$

determined by Hopkins et al.²⁾.

The CERN value of R , the Ecole Polytechnique value of R' and the relative numbers of $3\pi^0$, $\pi^+\pi^-\pi^0$ and $\pi e\nu$ events from Orsay have been combined together with the above data of Hopkins et al. in a least squares fit (3 degrees of freedom, $\chi^2 = 2.9$) yielding the following results:

$$R = 1.66 \pm 0.08$$

$$R' = 0.264 \pm 0.010$$

$$R'' = 0.159 \pm 0.0045 .$$

6. APPLICATION TO TESTS OF THE $\Delta I = 1/2$ RULE

If only the $\Delta I = 1/2$ or $3/2$ amplitudes enter into the $K \rightarrow 3\pi$ decay, the ratio R is predicted to be $3/2$ multiplied by an appropriate correction for the difference in available phase space due to the difference in mass between π^+ and π^0 . If this correction is taken to be just the ratio of

Dalitz plot areas for $3\pi^0$ and $\pi^+\pi^-\pi^0$ decays, the predicted value of R is 1.83. A more sophisticated calculation, by Devlin³⁾, taking into account the linear and quadratic dependence of the matrix elements on the Dalitz plot variables, predicts a value of 1.71. Our result is not inconsistent with either prediction.

While the value of r is somewhat lower than the average of previously published values for K^0 decay, it is in reasonable agreement with the corresponding value for K^+ decay, 0.60 ± 0.02 ⁴⁾, and thus with the leptonic $\Delta I = \frac{1}{2}$ rule.

The above data can be combined with the relatively recent and precise measurement of the K_L^0 lifetime

$$\tau_L = (5.15 \pm 0.14) \times 10^{-8} \text{ sec}$$

by Devlin et al.⁵⁾ to provide rather precise values of the decay rates of the various K_L^0 decay channels. The results, shown in Table 3, can be applied to further tests of the $\Delta I = \frac{1}{2}$ rule by comparing relevant K_L^0 and K^+ decay rates using the averages of Rosenfeld et al.⁶⁾ for the latter.

Table 3

Decay mode	Branching ratio	Rate
$K_L^0 \rightarrow 3\pi^0$	0.209 ± 0.011	$(4.05 \pm 0.23) \times 10^6 \text{ sec}^{-1}$
$K_L^0 \rightarrow \pi^+\pi^-\pi^0$	0.126 ± 0.004	$(2.44 \pm 0.10) \times 10^6 \text{ sec}^{-1}$
$K_L^0 \rightarrow \pi\ell\nu$	0.665 ± 0.012	$(12.91 \pm 0.42) \times 10^6 \text{ sec}^{-1}$

On comparing the leptonic rates we find

$$\frac{\Gamma(K^0 \rightarrow \pi\ell\nu)}{\Gamma(K^+ \rightarrow \pi\ell\nu)} = \frac{12.91 \pm 0.42}{6.65 \pm 0.17} = 1.94 \pm 0.08.$$

The value predicted by the $\Delta I = \frac{1}{2}$ rule is 2.024, where the difference from 2.0 is the mass difference effect recently calculated by Brene⁷⁾. The ratio of experimental to theoretical value is therefore 0.96 ± 0.04

On comparing the 3π rates, we find

$$\frac{\Gamma(K_L^0 \rightarrow 3\pi^0)}{\Gamma(K^+ \rightarrow \pi^+\pi^+\pi^-)} = \frac{4.05 \pm 0.25}{4.51 \pm 0.03} = 0.90 \pm 0.06$$

where the value predicted by the $\Delta I = \frac{1}{2}$ rule is 1.11. The ratio of experimental to theoretical value is therefore 0.81 ± 0.05 , indicating a substantial violation of the non-leptonic $\Delta I = \frac{1}{2}$ rule. This result is in agreement with the analysis of Devlin and Barshay⁸⁾.

While other 3π rates can also be compared, Devlin³⁾ has shown that the above ratio is the least subject to uncertainties in calculating corrections due to mass differences. This is because both decay modes lead to three pions of equal mass and there is therefore no ambiguity in either mode as to which point is to be taken as the centre of the Dalitz plot.

As Willis has pointed out⁹⁾, these tests by comparing K_L^0 and K^+ decay rates are not really independent since they depend in the same way on the K_L^0 lifetime value. Indeed, if both the leptonic and non-leptonic $\Delta I = \frac{1}{2}$ rules were valid, the K_L^0 lifetime would be expected to be $(4.70 \pm 0.09) \times 10^{-8}$ sec, about three standard deviations below the measured value. From the data discussed in this paper, the violation appears to be exclusively, or at least principally, in the 3π modes rather than the leptonic modes. Both selection rules can be rescued if and only if the K_L^0 lifetime measurement turns out to be significantly in error.

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