Direct CP Violation in Hadronic B Decays *

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Abstract

There are different approaches for the hadronic B decay calculations, recently. In this paper, we upgrade three of them, namely factorization, QCD factorization and the perturbative QCD approach based on $k_T$ factorization, by using new parameters and full wave functions. Although they get similar results for many of the branching ratios, the direct CP asymmetries predicted by them are different, which can be tested by recent experimental measurements of B factories.

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I. INTRODUCTION

The hadronic B decays are important for testing the standard model (SM), and also for uncovering the signal of new physics. Understanding non-leptonic B meson decays is crucial for the CKM matrix elements measurements and CP violation detection. Especially the direct CP violation is expected to be measured in hadronic B decays. Recently, both Belle and BaBar claimed to find direct CP asymmetry in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \pi^+K^-$ decays \cite{1}. It is the start of direct CP measurement in B physics.

In the theoretical side, there are much more complication in direct CP asymmetry predictions. The direct CP violation in hadronic decays requires at least two decay amplitudes with different weak phase and strong phase. In the standard model, the weak phase comes from the so called CKM matrix, which is well defined. Although the weak phase from SM is clean, the strong phase requires hadronic matrix element calculation, which is usually model dependent.

The simplest case is two-body hadronic B meson decays, for which Bauer, Stech and Wirbel (BSW) proposed the naive factorization assumption (FA) in their pioneering work \cite{2}. Considerable progress, including generalized FA \cite{3,4} and QCD-improved FA (QCDF) \cite{5}, has been made. On the other hand, technique to analyze hard exclusive hadronic scattering was developed by Brodsky and Lepage \cite{6} based on collinear factorization theorem in perturbative QCD (PQCD). A modified framework based on $k_T$ factorization theorem was then given in \cite{7,8}, and extended to exclusive B meson decays in \cite{9,10,11}.

The predictions of branching ratios agree well with experiments in most cases, thus it is difficult to tell from experiments that which method is better than others. However, the strong phase, which is important for the CP violation prediction, is quite sensitive to various approaches. The mechanism of this strong phase is quite different for various method, and give quite different results. The recent experimental results \cite{1} can make a test for the validity of these approaches.

In this paper, we will first introduce the factorization approach in next section. The QCD factorization and improved PQCD approach based on $k_T$ factorization are then introduced in section \textsection III and section \textsection IV respectively. In section \textsection V we upgrade the numerical results of these approaches using newest parameters. We compare the three major approaches to show the difference of direct CP asymmetry predicted by them. Finally the summary is
II. NAIVE AND GENERALIZED FACTORIZATION APPROACH

The calculation of non-leptonic decays involves the short-distance Lagrangian and the calculation of hadronic matrix elements which are model dependent. The short-distance QCD corrected Lagrangian is calculated to next-to-leading order [12]. In 1987, Bauer, Stech and Wirbel first calculate the hadronic $B$ and $D$ decays using the naive factorization approach [2]. In their factorization method, the hadronic matrix element is expressed as a product of two factors $\langle h_1 h_2 | \mathcal{H}_{\text{eff}} | B \rangle = \langle h_1 | J_1 | B \rangle \langle h_2 | J_2 | 0 \rangle$. The first factor is proportional to the $B \rightarrow h_1$ form factor, while the second one is proportional to the decay constant of $h_2$ meson. All the perturbative channel dependent part is described by the Wilson coefficients of four quark operators.

These effective four-quark operators are induced by the weak interaction in the quark level, which gives the short distance contribution to the non-leptonic decays of $B$ mesons. The effective Hamiltonian for the charmless non-leptonic $B$ decays is [12]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{q'q} V_{q'q}^* \left( \sum_{i=1}^{10} C_i O_i + C_g O_g \right) \right],$$

where $q = d, s$ and $V_{q'q}$ denotes the CKM factors. The operators $O_1, O_2$ are tree level current operators. The operators $O_3, \ldots, O_6$ are QCD penguin operators. $O_7, \ldots, O_{10}$ arise from electroweak penguin diagrams, which are suppressed by $\alpha/\alpha_s$. Only $O_9$ has a sizable value whose major contribution arises from the $Z$ penguin. The $C_i$'s are the Wilson coefficients of four quark operators with QCD corrections.

Although this is a very simple method, later experiments show that many of the decay branching ratios explained well by the FA [13], especially for the color enhanced decays class I, III and class IV [3]. When charmless $B$ meson decays are considered, efforts have been made to generalize the FA approach [3, 4]. To explain the non-factorizable dominated class II and class V decays, phenomenological parameter $N_c^{\text{eff}}$ is introduced. Most of the branching ratios of hadronic $B$ decays agree well with experiments by $N_c^{\text{eff}} = 2$ [3, 4].

An effort is also made to predict the CP violation parameters in different hadronic $B$ decays [14]. Since direct CP violation requires a strong phase difference between decay amplitudes in addition to a weak phase difference, the precision of strong phase calculation presented.
FIG. 1: The perturbative charm quark loop diagram generating strong phases in FA and QCDF.

is essential. In naive FA, the hadronization is described by the form factor only, therefore, no strong phase is given.

In generalized FA \[14\], the next-to-leading order Wilson coefficients contain strong phases generated by the charm quark loop, where charm quark can be on shell. The diagram shown in Fig.1 is also called BSS mechanism \[15\]. The size of this kind of strong phase is sensitive to the momentum of the gluon connecting to the charm quark loop. However, in FA, this momentum is not well defined, since all hadronic dynamics are defined only by form factors. In ref. \[14\], the authors use \(k^2 = \frac{m_b^2}{2} \pm 2\text{GeV}^2\) to give the CP asymmetry parameters of many channels.

III. QCD FACTORIZATION APPROACH

In 1999, Beneke, Buchalla, Neubert, and Sachrajda (BBNS) proposed a formalism for two-body charmless \(B\) meson decays \[5\]. In this approach, they expand the hadronic matrix element by the heavy b quark mass and \(\alpha_s\)

\[
\langle \pi \pi | O_i | B \rangle = \langle \pi | j_1 | B \rangle \langle \pi | j_2 | 0 \rangle \left[ 1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b) \right].
\]

(2)

Here \(O_i\) is a local operator in the effective Hamiltonian and \(j_{1,2}\) are bilinear quark currents. By neglecting the power corrections of \(\mathcal{O}(\Lambda_{QCD})\), one need only calculate the order \(\alpha_s\) corrections including the vertex corrections for the four quark operators and the non-factorizable diagrams. These diagrams are shown in Fig.2. The first 6 diagrams have already been included in the generalized FA approach as next-to-leading order QCD corrections to local
four quark operators. What new are the last two non-factorizable diagrams, which has a hard gluon line connecting the four quark operator and the spectator quark.

They claimed that factorizable contributions, for example, the form factor $F^{B\pi}$ in the $B \rightarrow \pi\pi$ decays, are dominated by long distance contributions. Hence, it is treated in the same way as FA, and expressed as products of Wilson coefficients and form factor $F^{B\pi}$. The non-factorizable contributions calculated perturbatively, are written as the convolutions of hard amplitudes with three ($B, \pi, \pi$) meson wave functions. Annihilation diagrams are neglected as in FA. Values of form factors at maximal recoil $q^2 = m^2_{\pi}$ and non-perturbative meson wave functions are all treated as input parameters. It is easy to see from eq.(2), that this equation is only applicable for those color enhanced decay modes, where the factorizable contribution dominates the final results. While for the color suppressed modes, where the non-factorizable contributions are not small, the expansion of eq.(2) is not right, since the large non-factorizable contribution is grouped into the next-to leading order term of eq.(2).

The numerical results show that the theory and experiments agree well for those class I and IV decays, which are color enhanced and dominated by the factorizable contribution. This also agrees with the FA result, since the dominant part in eq.(2) is the same as the FA. The success of QCD factorization is that one can calculate the sub-leading $O(\alpha_s)$ non-factorizable contribution (second term in eq.(2)) using perturbative QCD. While in the FA, one has to input a free parameter $N^c_{\text{eff}}$ to accommodate the non-factorizable contribution.

In the QCD factorization calculations, people found that there exist endpoint divergence
in the annihilation diagram calculations. Logarithm divergence occurred at twist 2 contribution, and linear divergence exists in twist 3 contribution. If not symmetric wave function, like $K^{(*)}$ meson, there is also soft divergence in the non-factorizable diagrams. It is very difficult to treat these singularity in the BBNS approach. A cut-off is introduced to regulate the divergence, thus makes the QCD factorization approach prediction parameter dependent especially in the annihilation diagrams.

As for the strong phase in BBNS, like in FA, it mainly comes from the BSS mechanism. Here the momentum of the inner gluon is well defined, since wave functions are introduced. However, it predicts too small strong phase, because of the small gluon momentum. Hence, small direct CP asymmetry is predicted. There is also another source of strong phase from the annihilation diagrams, but strongly depends on the cut-off parameter. The strong phase in QCD factorization can be large due to this cut-off.

In a word, the QCD factorization approach is at least one step forward from Naive Factorization approach. It gives systematic prediction of sub-leading non-factorizable contribution for the class I and class IV decays, which are dominated by the factorizable contribution. Problem remained is the endpoint singularity in higher order calculations, but may be solved with Sudakov form factors like PQCD approach.

IV. FORMALISM OF PQCD APPROACH

In this section, we will introduce the idea of PQCD approach. The three scale PQCD factorization theorem has been developed for non-leptonic heavy meson decays, based on the formalism by Brodsky and Lepage, and Botts and Sterman. In the non-leptonic two body B decays, the $B$ meson is heavy, sitting at rest. It decays into two light mesons with large momenta. There must be a hard gluon to kick the light spectator quark (with small momentum) in the $B$ meson to form a fast moving light meson. So the dominant diagram in this theoretical picture is that one hard gluon from the spectator quark connecting with the other quarks in the four quark operator of the weak interaction. Unlike the usual factorization approach, the hard part of the PQCD approach consists of six quarks rather than four. We thus call it six-quark operators or six-quark effective theory. There are also infrared (soft and collinear) gluon exchanges between quarks. Summing over those leading soft contributions gives a Sudakov form factor, which suppresses the soft
contribution. Therefore, it makes the PQCD reliable in calculating the non-leptonic decays. With the Sudakov resummation, we can include the leading double logarithms for all loop diagrams, in association with the infrared contribution.

There are three different scales in the B meson non-leptonic decays $M_W$, $m_b$ and $1/b$. The first scale describe the intrinsic electroweak decay of $B$ meson, through charged current. The second scale $m_b$ denote the scale of energy release in the decay. Since the $b$ quark decay scale $m_b$ is much smaller than the electroweak scale $M_W$, the QCD corrections to the four quark operators are non-negligible, which are usually summed by the renormalization group equation. This has already been done to the leading logarithm and next-to-leading order for years [12]. The third scale $1/b$ involved in the $B$ meson exclusive decays is usually called the factorization scale, with $b$ the conjugate variable of parton transverse momenta. The dynamics below $1/b$ scale is regarded as being completely non-perturbative, and can be parameterized into meson wave functions. The meson wave functions are not calculable in PQCD. But they are universal, channel independent. We can determine them from experiments, and they are constrained by QCD sum rules and Lattice QCD calculations. Above the scale $1/b$, the physics is channel dependent. We can use perturbation theory to calculate channel by channel.

With all the large logarithms resummed, the remaining finite contributions are absorbed into a perturbative $b$ quark decay sub-amplitude $H(t)$. Therefore the three scale factorization formula is given by the typical expression,

$$C(t) \times H(t) \times \Phi(x) \times \exp \left[ -s(P,b) - 2 \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right],$$

where $C(t)$ are the corresponding Wilson coefficients, $\Phi(x)$ are the meson wave functions and the variable $t$ denotes the largest mass scale of hard process $H$, that is, six-quark effective theory. The quark anomalous dimension $\gamma_q = -\alpha_s/\pi$ describes the evolution from scale $t$ to $1/b$. Since logarithm corrections have been summed by renormalization group equations, the above factorization formula does not depend on the renormalization scale $\mu$ explicitly.

As shown above, in the PQCD approach, we keep the $k_T$ dependence of the wave function. In fact, the approximation of neglecting the transverse momentum can only be done at the non-endpoint region, since $k_T \ll k^+$ is qualified at that region. At the endpoint, $k^+ \to 0$, $k_T$ is not small any longer, neglecting $k_T$ is a very bad approximation. By, keeping the $k_T$ dependence, there is no endpoint divergence as occurred in the QCD factorization approach,
while the numerical result does not change at other region. Furthermore, the Sudakov form factors suppress the endpoint region of the wave functions.

The main input parameters in PQCD are the meson wave functions. It is not a surprise that the final results are sensitive to the meson wave functions. Fortunately, there are many channels involve the same meson, and the meson wave functions should be process independent. In all the calculations of PQCD approach, we follow the rule, and we find that they can explain most of the measured branching ratios of B decays. For example: $B \to \pi \pi$ decays, $B \to \pi \rho$, $B \to \pi \omega$ decays, $B \to K \pi$ decays, $B \to K K$ decays, $B \to K \eta$ decays, $B \to K \phi$ decays etc.

We emphasize that non-factorizable (Fig. 3(c)(d)) and annihilation diagrams (Fig. 3(e-h)) are indeed sub-leading in the PQCD formalism as $M_B \to \infty$. This can be easily observed from the hard functions in appendices of ref. [17, 18]. When $M_B$ increases, the $B$ meson wave function enhances contributions to factorizable diagrams. However, annihilation amplitudes, being independent of $B$ meson wave function, are relatively insensitive to the variation of $M_B$. Hence, factorizable contributions become dominant and annihilation contributions are sub-leading in the $M_B \to \infty$ limit [22]. Although the non-factorizable and annihilation diagrams are sub-leading for the branching ratio in color enhanced decays, they provide the main source of strong phase, by inner quark or gluon on mass shell. The BSS mechanism should also be present in the PQCD approach. However this mechanism of strong phase is
TABLE I: Direct CP asymmetries calculated in FA [14], QCDF [5] and PQCD [17, 18] for $B \to \pi \pi$ and $B \to K \pi$ decays together with the averaged experimental results at percentage.

<table>
<thead>
<tr>
<th>Quantities</th>
<th>FA</th>
<th>QCDF</th>
<th>PQCD</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \pi^+\pi^-$</td>
<td>$-5 \pm 3$</td>
<td>$-6 \pm 12$</td>
<td>$+30 \pm 20$</td>
<td>$+37 \pm 11$</td>
</tr>
<tr>
<td>$B^0 \to \pi^+K^-$</td>
<td>$+10 \pm 3$</td>
<td>$+5 \pm 9$</td>
<td>$-17 \pm 5$</td>
<td>$-10.9 \pm 1.9$</td>
</tr>
<tr>
<td>$B^+ \to K^0\pi^+$</td>
<td>$+1.7 \pm 0.1$</td>
<td>$+1 \pm 1$</td>
<td>$-1.0 \pm 0.5$</td>
<td>$-2.0 \pm 3.4$</td>
</tr>
<tr>
<td>$B^+ \to K^+\pi^0$</td>
<td>$+8 \pm 2$</td>
<td>$+7 \pm 9$</td>
<td>$-13 \pm 4$</td>
<td>$+4 \pm 4$</td>
</tr>
</tbody>
</table>

small, since it is at next-to-leading order $O(\alpha_s)$ corrections.

V. NUMERICAL RESULTS AND DISCUSSION

In numerical analysis, we use recently updated decay constants and form factors for the FA and QCDF approaches. The branching ratios changes a little bit, since they are sensitive to these parameters. But the direct CP asymmetry changes very little. In the PQCD approach, we use the full set of light meson wave functions, including two twist 3 distribution amplitudes, where only one used in the previous papers [17, 18]. The threshold resummation for the endpoint of the hard part calculation is also newly included [23]. The numerical results for the branching ratios and CP asymmetries change only a little bit.

As discussed in the previous section, the strong phase generated from PQCD approach is quite different from the FA and QCDF approaches. The direct CP asymmetry in SM is proportional to the sine of the strong phase difference of two amplitudes. Therefore the direct CP asymmetry will be different if strong phase is different. The predicted CP asymmetry by the three methods are shown in table I. It is easy to see that the FA [14] and QCDF [5] results are quite close to each other, since the mechanism of strong phase is the same for them.

Recently the two B factories measure some channels with non-zero direct CP asymmetry [1], which are shown in table II [24]. It is claimed that direct CP has been found in $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^+K^-$ decays with more than 4$\sigma$ signal. It is easy to see that our PQCD results of direct CP asymmetry [17] agree with the experiments, especially for the experimentally well measured channels $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^+K^-$ decays. Although FA and QCDF are not yet ruled out by experiments, but the experiments at least tell us that the dominant strong
phase should come from the mechanism of PQCD not QCDF. Charm quark loop mechanism, which gives the central value of strong phase in QCDF, is argued next-to leading order in PQCD. This argument is now proved by B factories experiments.

In summary, branching ratios of both FA and QCDF depend on the values of form factors. The strong phase generated from these approaches is dominantly from perturbative BSS mechanism. In the PQCD approach, dominant by short distance contribution, the form factors are sensitive to meson wave functions. By including the $k_T$ dependence and Sudakov suppression, there is no endpoint divergence. In the PQCD formalism, annihilation amplitudes are of the same order as non-factorizable ones in powers of $1/M_B$, which are both $O(1/(M_B \Lambda_{QCD}))$. The strong phase comes mainly from the annihilation and non-factorizable diagrams in PQCD approach, which is quite different from the FA and QCDF approaches. The experimentally measured direct CP asymmetry implies that PQCD gives at least the dominant strong phase than other approaches. This will be further tested by experiments.

1615 (1997).


