CONFINEMENT OF SUPERNOVA EXPLOSIONS IN A COLLAPSING CLOUD

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ABSTRACT

We analyze the confining effect of cloud collapse on an expanding supernova shockfront. We solve the differential equation for the forces on the shockfront due to ram pressure, supernova energy, and gravity. We find that the expansion of the shockfront is slowed and in fact reversed by the collapsing cloud. Including radiative losses and a potential time lag between supernova explosion and cloud collapse shows that the expansion is reversed at smaller distances as compared to the non-radiative case. We also consider the case of multiple supernova explosions at the center of a collapsing cloud. For instance, if we scale our self-similar solution to a single supernova of energy $10^{51}$ ergs occurring when a cloud of initial density $10^2$ H/cm$^3$ has collapsed by 50%, we find that the shockfront is confined to $\sim 15$ pc in $\sim 1$ Myrs. Our calculations are pertinent to the observed unusually compact non-thermal radio emission in blue compact dwarf galaxies (BCDs). More generally, we demonstrate the potential of a collapsing cloud to confine supernovae, thereby explaining how dwarf galaxies would exist beyond their first generation of star formation.

Subject headings: galaxies: individual (SBS 0335-052, Henize 2-10) – galaxies: dwarf – supernova remnant – galaxies: starburst – stars: dwarf nova

1. INTRODUCTION

Blue Compact Dwarf galaxies (BCDs) are observed to be experiencing intense star formation in a spatially compact region. The timescale for this star formation episode is only a few million years. Massive stars have short lifetimes and thus supernova (SN) explosions may occur concurrently in regions of cloud collapse. It is quite probable that this occurs in blue compact dwarf galaxies. In this paper, we study the confining effect of cloud collapse on an expanding supernova shockfront.

The forces acting on the shockfront include those due to supernova energy, ram pressure and gravity. When a supernova shockfront expands into a stationary cloud (Sedov 1946, Taylor 1950, Bisnovatyi-Kogan & Silich 1995), the accreted mass does not contribute to a change in the momentum flux. In a region of cloud collapse, not only is mass accreted at a faster rate, it also contributes to a change in the momentum flux. This additional ram pressure leads to the confinement of a supernova shockfront in a collapsing cloud. The potential of ram pressure to confine stellar winds of a noncentral OB star to generate a steady-flow situation has been shown (Dopita 1981). We demonstrate that ram pressure can also confine expanding supernova shockfronts with pressures as high as $\sim 10^{-9}$ dyne/cm$^2$ as compared to stellar wind pressure which is $\sim 10^{-12}$ dyne/cm$^2$ (Dopita et al 1981). Our self-similar solution describes the time evolution of a central supernova explosion in a collapsing cloud.

We start with a discussion of the gravitational collapse of a cloud with negligible pressure. The gravitational force on a differential spherical shell with initial radius $R_0$ and radius $R(t)$, is

$$dM(R) \frac{d^2R}{dt^2} = -\frac{GM(R_0)}{R_2} dM(R),$$

where $G$ is the universal gravitational constant and $M(R_0)$ is the initial mass enclosed by this shell. $M(R_0) = 4\pi R_0^2 \rho_0/3$, where $\rho_0$ is the initial density of the cloud which is assumed to be uniform. Solving this equation, assuming the initial velocity is zero, gives

$$\frac{dR}{dt} = -\left[2GM(R_0) \left(\frac{1}{R} - \frac{1}{R_0}\right)\right]^{1/2}.$$

The free-fall time of the cloud is

$$t_{ff} = \int_0^{R_0} \frac{dR}{-dR/dt} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \approx 5.2 \times 10^6 \sqrt{n_2 \text{yr}} \left(n_2 \text{H/cm}^3\right)^{-1/2},$$

where $n_2$ is density in units of $10^2$ H/cm$^3$. The free-fall time is independent of the initial radius. Thus, the collapse is homologous as shown in Figure 1.

The velocity of the shell hitting the shock front contributes to the ram pressure that slows down the front. We normalize the time and radius as $\tau = t/t_{ff}$ and $\overline{R} = R/R_0$. Hence we obtain

$$\tau = \frac{2}{\pi} \int_{\overline{R}}^1 dr' \left(\frac{r'}{1-r'}\right)^{1/2}.$$

This integral gives a cumbersome expression for $\overline{R}(\tau)$. Therefore, we approximate the dependence as

$$\overline{R} \approx (1-\tau^2)^{1/2} \equiv f(\tau).$$
Thus the velocity of the cloud at the shockfront radius, \( R_{sh} \), is

\[
V_{cl}(\tau) = -\frac{\pi R_{sh}[1 - f(\tau)]^{1/2}}{2 t_{ff}[f(\tau)]^{3/2}}.
\]  

(6)

Accretion of mass just outside the shockfront acts to increase in density. Mass conservation gives

\[
\Delta M = 4\pi R_{0}^{2} \Delta R_{0} \rho_{0} = 4\pi R^{2} \Delta R \rho(t).
\]

(7)

Using \( R = R_{0} f(\tau) \) and \( \Delta R = \Delta R_{0} f(\tau) \) gives

\[
\rho(\tau) = \frac{\rho_{0}}{[f(\tau)]^{3}}.
\]

(8)

There are three forces experienced by the shock front: (i) the energy released by the supernova explosion \( E_{SN} \) inside the shell gives an outward directed pressure \( p = (\gamma - 1)E_{SN}/(4\pi R_{sh}^{3}) \) or force \( 4\pi R_{sh}^{2} p \). We use \( \gamma = 5/3 \). (ii) The ram pressure of the cloud hitting the shockfront results in an inward directed force (or pressure). When the direction of motion of the external medium and the shockfront are the same, the ram pressure acts only if the speed of the medium is greater than that of the shockfront. This is accounted for by a Heaviside function, \( H \), of the difference between the shockfront velocity and cloud velocity (with \( H(>0) = 1 \) and \( H(<0) = 0 \)). (iii) The gravitational force is of course inward.

The ambient pressure of the interstellar medium is \( \sim 10^{-10} \) dynes/cm\(^2\) for a density \( \sim 100 \) H/cm\(^3\) and temperature \( \sim 10^{4} \) K. For a typical speed \( \sim 100 \) km/s, the ram pressure is \( \sim 10^{-8} \) dynes/cm\(^2\). For this reason, we neglect the ambient pressure of the interstellar medium.

The rate at which the SN shockfront accretes mass is proportional to the relative velocity between the cloud and the shockfront. Thus,

\[
\frac{dM_{sh}}{dt} = 4\pi R_{sh}^{2} \rho \ U,
\]

(9)

where

\[
U = \frac{dR_{sh}}{dt} - V_{cl}.
\]

(10)

The accreted mass initially moved at the cloud velocity. It thus contributes to a net change in the momentum flux. For the case where the SN explosion and cloud collapse begin at the same time we have

\[
\frac{d}{dt} \left( \frac{dR_{sh}}{dt} \right) = 4\pi R_{sh}^{2} \left[ \frac{E_{SN}}{2\pi R_{sh}^{3}} + \rho H(U) \ U \ V_{cl} \right] - \frac{GM_{sh}^{2}}{2R_{sh}^{2}}
\]

(11)

We normalize time by the free-fall time and the shockfront radius by the Sedov-Taylor radius at the free-fall time (within a factor of 1.24); that is, \( R_{sh} = R_{sh}/R_{ST}(t_{ff}) \), where

\[
R_{ST}(t_{ff}) \equiv ((\gamma - 1)E_{SN} t_{ff}/\rho_{0})^{1/5} \approx 52 \text{ pc} \ (E_{51})^{1/5} (n_{2})^{-2/5}.
\]

(12)

We normalize mass by \( M_{sh} = M_{sh}/[4\pi \rho_{0} R_{ST}^{3}(t_{ff})/3] \).

We define a ‘momentum’ variable

\[
\mathcal{P} \equiv \frac{M_{sh} dR_{sh}}{dt}.
\]

(13)

Rewriting equation (11) and equation (13) in terms of the normalized parameters gives

\[
\frac{dM_{sh}}{dt} = 3\pi R_{sh}^{2} \left( \frac{dR_{sh}}{dt} + \frac{\pi R_{sh} (1 - f)^{1/2}}{2 f^{3/2}} \right).
\]

(14)

\[
\frac{d\mathcal{P}}{dt} = \frac{9}{4\pi R_{sh}} \times \frac{3\pi R_{sh}^{3} (1 - f)^{1/2} H(U) \ U \ \pi R_{sh}^{2}}{2 f^{3/2}} \frac{\pi^{2} M_{sh}^{2}}{16 R_{sh}^{2}},
\]

(15)

where

\[
U = \frac{\mathcal{P}}{M_{sh}} + \frac{\pi R_{sh} (1 - f)^{1/2}}{2 f^{3/2}}.
\]

(16)

Thus, we have a system of three non-linear coupled first order equations. For the initial conditions we assume at an early time, \( R_{sh} \) and \( dR_{sh}/dt \) are given by the standard Sedov-Taylor solution for a stationary cloud. We use the initial condition that at \( \tau = 10^{-6}, R_{sh} = 0.0049, M_{sh} = 1.2 \times 10^{-7} \) and \( \mathcal{P} = 0.00024 \). Figure 2 shows \( R_{sh}(\tau) \) and the corresponding Sedov-Taylor solution. The infalling cloud at first slows the shock expansion and later reverses its motion.

### 3. Influence of Radiative Losses

In the above analysis we assumed that the energy within the SN shock was constant, but this is valid only for relatively short times. We next take into account the radiative losses using the pressure driven snowplow model of Cioffi, McKee, and Bertshinger (1988). We find that the supernova is confined to a much smaller region in a shorter time. We use the mean pressure \( \mathcal{P}_{cl} \) including radiative losses derived by Cioffi et al. (1988) which can be written as

\[
\mathcal{P}_{cl} = \frac{\alpha E_{SN}^{5/3} R_{sh}^{-k/9}}{\rho_{0}^{10/9} n_{2}^{4/7}}.
\]

(17)

where magnitude of \( \alpha \) is \( 2.3 \times 10^{-17} \) and \( \eta_{M} \) is metallicity in units of solar metallicity. Thus \( \mathcal{P}_{cl} \) replaces our earlier pressure \( p = E_{SN}/(2\pi R_{sh}^{3}) \). Next, we assume that the metallicity is solar metallicity and we use equation (17) for density as a function of time. The differential equation including the radiative losses is

\[
\frac{d}{dt} \left( M_{sh} \frac{dR_{sh}}{dt} \right) = 4\pi R_{sh}^{2} \left( \mathcal{P}_{cl} - \rho H(U) \ U \ V_{cl} \right) - \frac{GM_{sh}^{2}}{2R_{sh}^{2}}.
\]

(18)

We continue to normalize time by the free-fall time. However, we now normalize the radius by

\[
R_{cl} = \left( \frac{3\alpha E_{SN}^{-5/3} R_{sh}^{-k/9}}{\rho_{0}^{10/9} n_{2}^{4/7}} \right)^{1/7} \approx 11.8 \text{pc} \ (E_{51})^{5/21} (n_{2})^{-26/63}.
\]

(19)

We again obtain a self-similar differential equation and we solve it using the initial condition that the shockfront follows the Sedov solution at \( \tau = 10^{-6} \). We use this initial condition because the transition from the Sedov solution to the pressure driven snowplow radiative shockfront occurs at

\[
t_{PD} = 1.33 \times 10^{4} \text{ yr} \ (E_{51})^{3/4} (n_{2})^{-7/4}.
\]

(20)
The SN explosion may occur at a time $\Delta t$ after the cloud collapse has begun. The velocity and density of the shells hitting the shock front is then larger. We take the zero of time $t$ or $\tau = t/t_{ff}$ to be the time when the supernova explodes so that only equation (18) needs to be modified and replaced by $f(\tau) = \frac{R}{(1 - (\tau + \Delta \tau)^2)^{1/2}}$, where $\Delta \tau = \Delta t/t_{ff}$ is termed the “lag.”

Figure 3 shows sample results for different lags which shows that the shock expansion is more strongly decelerated the longer the lag. For no time lag, the shockfront is confined to $R_{sh} = 1.31$ and $\tau = 0.51$. In comparison, for 50% time lag, the shockfront is confined to $R_{sh} = 1.33$ and $\tau = 0.23$ and for 80% time lag, the shockfront is confined to $R_{sh} = 0.75$ and $\tau = 0.09$.

4. MULTIPLE SUPERNOVA EXPLOSIONS

We now consider a more complete physical picture. Suppose that there is a localized clump of density $10^4$ H/cm$^3$ in a cloud of density $10^2$ H/cm$^3$. Suppose that the ambient outer cloud and the inner clump begin to free-fall. The clump collapses in $\sim 0.5$ Myr and forms supernovae in another 2–3.5 Myrs (lifetime of $30$–$24 M_\odot$ star). The free-fall time of the cloud is $\sim 5$ Myrs and thus, the time lag between when the supernova explodes and when the cloud begins to collapse ranges from $0.5 t_{ff}$ to $0.8 t_{ff}$.

When the localized clump collapses, the initial mass function is such that many low mass (hm) stars form at the same time as a few high mass stars. The distribution is $dn/dM \propto M^{-2.35}$ (Salpeter 1955). If we take the high mass (hm) threshold at $> 20 M_\odot$, then $M_{lm} \sim 200 N_{hm} M_\odot$. The mass of the low mass stars contributes to the gravitational force in addition to the forces described in equation (18). The right-hand side of this equation becomes

$$4 \pi R_{sh}^2 \left[ \rho \frac{\nabla (U)}{V} - \frac{GM_{sh}}{2 R_{sh}^2} - \frac{GM_{sh} M_{lm}}{R_{sh}^2} \right].$$

We find however that the gravitational force contribution of the low mass stars does not significantly alter the trajectory of the SN shockfront.

5. DISCUSSION

Regarding the stability of the shockfront, notice that the Rayleigh-Taylor instability occurs in a stratified region where the effective gravity points in the direction of the less dense medium. In the case of an expanding supernova shockfront, the effective gravity in the frame of the shockfront is the shockfront acceleration. Our results for the trajectory are all concave down implying that effective gravity points radially outward and thus, away from the relatively rarer medium. Hence, the shockfront is Rayleigh-Taylor stable.

We conclude that ram pressure from a collapsing cloud reduces the shockfront velocity of one radiative supernova explosion (occurring in a localized region when the ambient cloud has collapsed by 50%) to zero at a normalized radius of 1.33 and a normalized time of 0.23. These fractions scale by the energy of the supernova and the density of the interstellar medium.

Observations of nonthermal radio emission in galaxies suggest whether or not the supernovae are confined. For example, a 22 GHz map of M82 extending over a few hundred parsecs is estimated to have 40 unconfined supernovae (Golla et al 1996). An 18 cm nonthermal radio emission map of VIIZw19 extending over 310 pc is estimated to have 2500 unconfined supernovae remnants (Beck et al 2004). Two examples of apparently confined emission, SBS-0335 and Henize 2-10, are described below.

In a model of the nonthermal radio emission from SBS-0335 the emission is confined to 17 pc (Hunt et al. 2004) within a 520 pc region (Thuan et al. 1997) where star formation occurs in six super-star clusters with ages $\leq 25$ Myrs. If a supernova explosion of energy $10^{51}$ ergs occurs after a cloud of initial density $10^2$ H/cm$^3$ has collapsed by 50%, it is confined to $\sim 15$ pc in $\sim 1$ Myr. If this were a supernova exploding in a stationary cloud, it would expand beyond $\sim 20$ pc in as little as 0.4 Myrs. This would destroy the cloud and the case of a young supernova remnant in a stationary cloud is considerably less probable than a supernova confined in a collapsing cloud.

Very Large Array Imaging of another BCD, Henize 2-10, indicates a < 8 pc region of 1 mJy radio sources in the central 5” starburst region. Henize 2-10 has HH regions of sizes between 3 pc and 8 pc and densities between 1500 and 5000 H/cm$^3$ (Kobulnicky, Johnson 1999). If we hypothesize that a cloud of an average initial density of 3000 H/cm$^3$ has collapsed by 50% when 10 supernovae explode, we find that the supernovae are confined to $\sim 7$ pc in $\sim 0.2$ Myr. Thus, cloud collapse successfully confines supernova explosions and can account for observed compact nonthermal radio emission. This simple model can help understand continuous or second/third generation star formation since it suggests why the cloud is not devastated by first generation supernovae.

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Fig. 1.— The collapse of cloud shells with different initial radii are shown as solid lines. The radius is normalized by the initial radius of the outermost cloud shell and the time is normalized by the free-fall time. The dotted line indicates the approximation of the trajectory used here. The approximation has a maximum deviation of 5.3% and a mean deviation of 2.6%.
Fig. 2.— Shock wave expansion $R_{sh}(\tau)$ in a collapsing cloud and the standard Sedov-Taylor expansion in a stationary medium. Time $\tau$ is normalized by the free-fall time and the radius is normalized by the Sedov-Taylor radius at the free-fall time. The shock expansion is slowed down and subsequently turned around by the collapsing cloud.
Fig. 3.— Shockfront radii including influence of the radiative losses. We also show the shockfront radius for a time lag of 50% and 80% of the free-fall time between supernova explosion and cloud collapse. Time zero is taken as the instant the supernova exploded.
Fig. 4.— Evolution of the shockfront radius for single and multiple SN explosions.