The FWHM of local pulses and the corresponding power-law index of gamma-ray burst FRED pulses

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ABSTRACT

The FWHM of gamma-ray burst (GRB) pulses is known to be related with energy by a power-law. We wonder if the power-law index $\alpha$ is related with the corresponding local pulse width $FWHM_0$. Seven FRED (fast rise and exponential decay) pulse GRBs are employed to study this issue, where six of them were interpreted recently by the relativistic curvature effect (the Doppler effect of fire-balls) and the corresponding local pulses were intensely studied. A regression analysis shows an anti-correlation between $\log \alpha$ and $\log FWHM_0$ with a slope of $-0.37 \pm 0.13$. This suggests that, for the class of the GRB pulses which are consequences of the curvature effect, the difference of the local pulse width might lead to the variation of the power law index, where the smaller the width the larger the value of $\alpha$. Since the number of sources employed in this analysis is small, our result is only a preliminary one which needs to be confirmed by larger samples.

Subject headings: gamma rays: bursts

1. Introduction

Gamma-ray bursts (GRBs) were discovered thirty-eight years ago by chance, and now are recognized as the most luminous known objects in the Universe (Fishman 2001). Since then, many observations of the objects have been made, which have amassed a great deal of information. Owing to their brief and random appearance in the gamma-ray region, their study had become very difficult since their discovery. Although the progress has been
made in GRB research, GRBs remain one of the most inexplicable astrophysical phenomena observed today (Kouveliotou 1997).

Temporal and spectral characteristics of prompt emission of gamma-ray burst pulses have been intensely studied since they could constrain the energizing and emission mechanisms (Ryde et al. 2002). The correlation between GRB spectral and temporal properties have been investigated by several research groups. It was first noted by Norris et al. (1986) that GRB pulses exhibit a hard-to-soft spectral evolution, and associated with it the pulses are seen to be narrower at higher energies than they are at lower bands which were confirmed by later works (see Fishman et al. 1992; Link, Epstein, & Priedhorsky 1993). By using the average autocorrelation function and the average pulse width, Fenimore et al. (1995) showed that the average pulse width has a power-law dependence on energy with an index of about -0.4 (the range of it is from -0.37 to -0.46, depending on how it is measured). This is the first quantitative relationship between temporal and spectral structure in gamma-ray bursts. Norris et al. (1996) found that average raw pulse shape dependence on energy is approximately power law, with an index of -0.40, consistent with the autocorrelation analysis of Fenimore et al. (1995). Furthermore, Nemiroff (2000) brought forward that over the energy range 100 keV-1 MeV in GRB 930214c (BATSE trigger 2193) the temporal scale factors between a pulse measured at different energies are related to that energy by a power law. The corresponding power-law indexes found by Feroci et al. (2001) for GRB 990704 and by Piro et al. (1998) for GRB 960720 are −0.45 and −0.46 ± 0.10, respectively. Costa (1999) also found that the power-law index for GRB 960720 is −0.46, the same to Piro et al. (1998). The spectral lag as a function of energy was examined for individual pulses in GRBs (Norris et al. 2000), which confirmed the earlier result of Fenimore et al. (1995). In a recent study (Crew et al. 2003), the power-law relationship between the duration of GRB 021211 and energy further confirmed the earlier result. The anti-correlation between pulse widths and gamma-ray energy have also been investigated by many other authors (e.g., Tavani 1997; Wang et al. 2000; Beloborodov et al. 2000; Guidorzi et al. 2003; Sakamoto et al. 2004; Dado et al. 2004).

Norris et al. (1995) found an anti-correlation between \( T_{90} \) and peak intensity, while a positive correlation between \( T_{90} \) and total fluence was shown in Lee & Petrosian (1997), and a positive correlation between peak energy and variability was found by Lloyd-Ronning & Ramirez-Ruiz (2002). A correlation between luminosity and variability for BATSE bursts with known redshifts was revealed by Fenimore & Ramirez-Ruiz (2000). Ramirez-Ruiz & Fenimore (2000) found a quantitative relationship between pulse amplitude and pulse width: the smaller amplitude peaks tend to be wider, with the pulse width following a power law with an index of about -2.8 (the range of it is from -2.8 to -3.0, depending on how it is measured). The anti-correlation between the pulse amplitude and pulse width was also revealed by Lee
et al. (2000). $T_{90}$ being correlated with peak heights (Lestrade 1994) and peak energy being correlated with peak flux (Mallozzi et al. 1995) were other reported relationships.

It was suggested that, the most likely radiation progress in GRBs is synchrotron emission in the standard fireball scenario (see Katz 1994; Sari, Narayan, & Piran 1996). The power-law dependence has led to the suggestion that this effect could be attributed to synchrotron radiation (see Piran 1999). Kazanas, Titarchuk, & Hua (1998) proposed that synchrotron cooling could well account for the effect (see also Chiang 1998; Dermer 1998; and Wang et al. 2000). Fenimore et al. (1995) showed that synchrotron emission can give rise to the correlation $t_{\text{syn}}(E) \propto E^{-0.5}$ between GRB spectral and temporal properties, which is consistent with the observed correlation $\Delta \tau \propto E^{-0.45 \pm 0.05}$. Cohen et al. (1997) put forward that the power-law relationship between pulse width and energy with the index of $-0.4$ is in reasonable agreement with expectations for a population of electrons losing energy by synchrotron radiation, for which an exponent of $-1/2$ is predicted. It was suspected that a simple relativistic mechanism might be at work in producing this relationship (Nemiroff 2000). In deed, it was shown recently in Qin et al. (2004; hereafter Paper I) and Qin et al. (2005) that the Doppler effect of a relativistically expanding fireball surface (the so-called relativistic curvature effect) could lead to a power law relationship between the pulse width and energy for FRED (fast rise and exponential decay) pulses, regardless the real forms of the rest frame radiation and the local (or intrinsic) pulse involved. The same effect was also observed by Shen et al. (2005).

In this paper, we investigate if local pulses are related with the power law relationship (in other words, we wonder how the local pulse width is related with the index of the power law observed). In section 2, we choose several GRBs with each of them comprising a single FRED pulse to calculate the corresponding data. Relationship between the index and the local pulse width is explored in section 3. Conclusions are presented in section 4.

2. Sources and data

To study how local pulses affect the index of the power law between the pulse width and energy, we focus on FRED pulse bursts. As revealed recently by many authors, the observed FRED structure of pulses could be interpreted by the relativistic curvature effect when the observed plasma moves relativistically towards us and appears to be locally isotropic (see, e.g., Fenimore et al. 1996; Ryde & Petrosian 2002; Kocevski et al. 2003; Paper I; Shen et al. 2005). If this interpretation is correct, FRED pulses would form in nature a class identified by the GRB temporal structure. In this way, it would not be great surprise to us if quantities associated with the pulses are correlated with each other.
As illustrated in Paper I, Shen et al. (2005), and Qin & Lu (2005), the local pulse width is essential to produce the observed pulse shape due to the curvature effect. Accordingly, those FRED pulses with their local pulses having been intensely studied become our first choice. We find six bursts studied in Paper I belonging to this kind. They are GRB 910721 (#563), GRB 920925 (#1956), GRB 930612 (#2387), GRB 941026 (#3257), GRB 951019 (#3875) and GRB 951102B (#3892).

Light curve data for which the background counts have been subtracted are available in the BATSE website (http://cossc.gsfc.nasa.gov/batse/batseburst/sixtyfour_ms/bckgnd.fits.html). The signal data are taken within the zone \([t_{\text{min}}, t_{\text{max}}]\), where \(t_{\text{max}} - t_{\text{min}} = 2T_{90}\), and \(t_{\text{min}}\) is at \(T_{90}/2\) previous to the start of \(T_{90}\).

There might be many different methods to estimate the pulse width. The theme of all possible methods is to find the central values of the scattering data. In other words, one always manages to find the real values of the data that are assumed to be get rid of the chaos arising from the influence of the background as well as other statistical errors. Owing to the fact that the light curve function of Kocevski et al. (2003) (the KRL function; equation [22] of Kocevski et al. 2003) could well describe the observed light curves of FRED pulses (see also Qin & Lu 2005), we simply employ this function to fit the four channel light curves of the six bursts, where parameters of the function associated with different channels are allowed to be different for the same burst. In order to allow the fitting curves shifting along the time axis so that the time coordinate of the light curve data is unnecessary to be resettled, we introduce an extra parameter \(t_0\) to the KRL function, where \(t\) should be replaced by \(t - t_0\) and \(t_m\) should be replaced by \(t_m - t_0\). Thus, we have five free parameters \((f_m, t_m, r, d, t_0)\) in our fit, instead of four. The widths of the four channel light curves are then estimated from the corresponding fitting curves, where the errors are determined by the uncertainties of the fitting parameters via the error transfer formula.

We perform the fit with the software of ORIGIN, where the fitting parameters as well as their uncertainties are available. Illustrated in Figure 1 are the fits to the four channel light curves of GRB 951019 (#3875). For GRB 941026 (#3257) and GRB 951102B (#3892), the widths in channel 4 are not available since the signal in that channel is too weak to be detected. The estimated values of the FWHM of the observed light curves of the six bursts calculated with the fitting curves (determined by the fitting parameters) are listed in Table 1.

Assuming that the widths of pulses are related with energies by a power law, we calculate the indexes with the estimated values of the observed pulse widths of the six sources. The results are presented in Table 2.
There are several local pulses discussed in Paper I, some of which are as follows:

1. The local pulse with an exponential decay

\[ \tilde{I}(\tau_{\theta}) = I_0 \exp\left(-\frac{\tau_{\theta} - \tau_{\theta,\min}}{\tau_{\theta,\min}}\right) \quad (\tau_{\theta,\min} \leq \tau_{\theta}) \quad (1) \]

2. The local pulse with a power-law rise and a power-law decay

\[ \tilde{I}(\tau_{\theta}) = I_0 \left\{ \begin{array}{ll}
(\tau_{\theta,0} - \tau_{\theta,\min})^\mu & (\tau_{\theta,\min} \leq \tau_{\theta} \leq \tau_{\theta,0}) \\
(1 - \frac{\tau_{\theta,\max} - \tau_{\theta,0}}{\tau_{\theta,\max} - \tau_{\theta,\min}})^\mu & (\tau_{\theta,0} < \tau_{\theta} \leq \tau_{\theta,\max})
\end{array} \right. \quad (2) \]

3. The local pulse with a power-law rise

\[ \tilde{I}(\tau_{\theta}) = I_0 \left( \frac{\tau_{\theta} - \tau_{\theta,\min}}{\tau_{\theta,\max} - \tau_{\theta,\min}} \right)^\mu \quad (\tau_{\theta,\min} \leq \tau_{\theta} \leq \tau_{\theta,\max}) \quad (3) \]

In Paper I, local pulse (1) was adopted to account for the light curves of GRB 910721 and GRB 930612 when the curvature effect was considered, for GRB 941026 and GRB 951102B local pulse (2) with \( \mu = 1 \) was taken, while or GRB 920925 and GRB 951019 local pulse (3) with \( \mu = 1 \) was assumed. After smoothing the signal data, they got a very good fit to these sources (see the \( \chi^2 \) values listed in Table 2 of Paper I), which suggests that the assumption that the observed light curves could arise from the local pulses adopted when taking into account the curvature effect is acceptable.

According to the local pulse parameters listed in Table 2 of Paper I, we get from equations (1)-(3) the widths of the corresponding local pulses (note that \( \tau_{\theta,\min} = 0 \) was adopted in Paper I), which are listed in Table 2 in this paper as well.

### 3. Relationship analysis

Relation between the index of the power law, \( \alpha \), and the FWHM of the local pulses, \( FWHM_0 \), is displayed in Figure 2. A linear correlation between \( \log \alpha \) and \( \log FWHM_0 \) could be observed.

We wonder if sources other than those selected in Paper I are in agreement with this trend. As a FRED pulse source, GRB 930214c (#2193) was previously intensely studied (Nemiroff 2000). We include this burst in our study. As done in the case of the six bursts, we once more employ the KRL function to fit the four channel light curves of this source, and in the same way, parameters of the function associated with different channels are allowed.
to be different. Also, the fit is performed with the software of ORIGIN. The widths of the 4 channels of this burst are estimated with the fitting parameters, which are listed in Table 1 as well. From these widths we get the power law index of this source under the assumption that the widths are related with energies by a power law. The estimated value of the index is presented in Table 2.

To obtain the local pulse width of this burst, we follow what were done by Qin et al. in Paper I for the six GRBs. One can find the details of the analysis in the mentioned paper, which are omitted in the following. Briefly stating, we fit the count rate of the third channel of GRB 930214c (#2193) with equation (21) of Paper I, where local pulse (1) in this paper (which is local pulse [83] in Paper I) is adopted. Relations (e.g., \( t = t_1 \tau + t_0 \), see Paper I for a detailed explanation) and functions (e.g., DB3) and the corresponding parameters taken for the fit are exactly those adopted in Paper I in the case of GRB 910721. The fit yields: \( \sigma = 1.70, \chi^2 = 0.416 \) for the data smoothed with DB3 wavelet in the level of the first-class decomposition, \( \chi^2 = 0.819 \) for the data without smoothing, and other free parameters (they are not related to the local pulse width). (Owing to the limited space provided, the figure showing the fit is omitted.) We find for GRB 930214c (#2193) that the reduced \( \chi^2 \) associated with the fit is reasonable, which suggests that the light curve of this burst could indeed be accounted for by the relativistic curvature effect.

The data point of \((\alpha, FWHM_0)\) for GRB 930214c (#2193) is also plotted in Figure 2, which is in agreement with the trend mentioned above (see Figure 2).

A linear correlation analysis of the data of the seven bursts yields: \( \log \alpha = (-0.43 \pm 0.06) + (-0.38 \pm 0.11) \log FWHM_0 \) \((r = -0.84, N = 7)\). However, it should be noticed that the number of the sources concerned is small. In this case, the result of the correlation analysis might obviously depend on some lonely located data points (see GRB 920925 and GRB 930214c in Figure 2). According to Isobe et al. (1990) and Feigelson & Babu (1992), the true regression coefficient uncertainty in samples of small size would be underestimated when the usual standard formulas are applied. Thus, resampling procedures such as the jackknife or bootstrap should be used to evaluate regression uncertainties in these cases. We thus try to use the bootstrap method to estimate the regression coefficient uncertainties. Applying the bootstrap error analysis we get indeed a larger slope uncertainty: \( \log \alpha = (-0.43 \pm 0.08) + (-0.37 \pm 0.13) \log FWHM_0 \). This is what we should hold.
4. Conclusions

In this paper, we investigate the relationship between the power-law index $\alpha$ and the FWHM of local pulses, $FWHM_0$, of seven FRED pulse GRBs. Our analysis shows that there exists a linear relationship with a slope of $-0.37 \pm 0.13$ between $\log \alpha$ and $\log FWHM_0$ for the bursts. This suggests that different widths of local pulses could lead to different values of the power law indexes, with the larger the former the smaller absolute value the latter, for FRED pulse bursts (at least for those which were previously interpreted by the relativistic curvature effect). If this relationship could be confirmed, the distribution of the local pulse width would be an important factor that leads to the variation of the index observed in GRB samples (this might likely be true if the sample contains only FRED pulses).

Of the seven GRBs, local pulses of six were previously intensely studied and that of the other one is explored in this paper. As the number of the sources involved is small, the result is only a preliminary one, which is not at all conclusive in terms of statistics. However, a trend in the relationship is explicitly illustrated in our analysis, although the analysis is qualitative rather than quantitative. A large sample of FRED pulses is thus required to check statistically if this conclusion could hold.

Say frankly, the cause of this relationship is currently unclear. Since the seven bursts studied here are single FRED pulse sources which were assumed to suffer from the relativistic curvature effect, we suspect that it might be this effect that gives birth to the relationship. A theoretical analysis on this issue is necessary.

Besides the curvature effect, there might be other factors that can affect the value of the power law index. One would be the variation of the rest frame emission mechanism, which was revealed in Qin et al. (2005). For example, different rest frame spectra or different speeds of the rest frame spectral softening could lead to different values of the power law index. This also requires a further investigation.

Our thanks are given to Dr. Robert Nemiroff for providing us helpful suggestions which make the paper significantly improved. This work was supported by the Special Funds for Major State Basic Research Projects (“973”) and National Natural Science Foundation of China (No. 10273019).

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Kouveliotou, C. 1997, AAS, 191, 3601


Fig. 1.— Illustration of the fit with the KRL function to the four channel light curves of GRB 951019.
Fig. 2.— Plot of the power law index versus the FWHM of local pulses. The solid line is the linear regression line of the data.
Table 1. The FWHM of the observed light curves of various channels estimated with the KRL function for GRB 910721(#563), GRB 920925(#1956), GRB 930214c(#2193), GRB 930612(#2387), GRB 941026(#3257), GRB 951019(#3875) and GRB 951102B(#3892), respectively.

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Table 2. Estimated values of the power law index of the 7 bursts and the FWHM of the corresponding local pulses.

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