Instability of human societies as a result of conformity

A. L. Efros
University of Utah, Salt Lake City UT, 84112 USA

P. Désesquelles
Institut de Physique Nucléaire, 15 rue Georges Clémenceau, F91406 Orsay France.

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We introduce a new model that mimics the strong and sudden effects induced by conformity in tightly interacting human societies. Such effects range from mere crowd phenomena to dramatic political turmoil. The model is a modified version of the Ising Hamiltonian. We have studied the properties of this Hamiltonian using both a Metropolis simulation and analytical derivations. Our study shows that increasing the value of the conformity parameter, results in a first order phase transition. As a result a majority of people begin honestly to support the idea that may contradict the moral principles of a normal human beings though each individual would support the moral principle without tight interaction with the society. Thus, above some critical level of conformity our society occurs to be instable with respect to ideas that might be doubtful. Our model includes, in a simplified way, human diversity with respect to loyalty to the moral principles.

Keywords: conformity, opinion dynamics, social impact.

INTRODUCTION

It is commonly observed that strongly interacting groups of people may show very abrupt and unexpected behavior or opinion changes. These "transitions" may even lead people to behave against their inmost sense of good and evil. This is seen in many crowd phenomena, less harmfully, in fashion phenomena, or, more dramatically, in the setting off of civil wars often leading to despotic regimes. Of course, these phenomena may have very different reasons, but we propose a very simple model that shows the same features and which may give some insights on the mechanisms of these regime reversals. Our hope is based upon similarities in the manifestations of different non-democratic regimes which suggest that similar universal mechanisms may have played a role.

All those regimes had powerful and merciless mechanisms of repression against people who were either enemies of the regime or merely neutral to it. However, one could argue that (see, for example, [1]) the stability of those regimes was not based only on repression. In some cases, the honest belief of the majority of the population in the official ideas, or, at least the fact that people behave as if it was the case, could play a major role. For example, as soon as this faithfulness became weak, the Soviet Union fell and the security forces were unable to stop this process. In many cases these ideologies drastically contradict the moral principles of a normal human being. It is reasonable to think that those principles are shared by a vast majority of human beings. The experience of the 20th century thus shows that under certain conditions these principles fail and the behavior of people seems to be controlled by some collective phenomena. Once more, we do not pretend that the model we have developed explains these historical events. Our modest goal is to show its analogies with human situations and to show how conformity can lead to unexpected extreme and paradoxical situations.

These sociological or historical phenomena are often sudden, turmoil is drastic, and its apparent cause seems insignificant. For a physicist, this problem resembles a phase transition in a system consisting of interacting elements. In the system under study these elements are people and the most important part of their interaction is conformity. Conformity implies the ability, or the propensity, of people to change their opinion under influence of the opinions of surrounding people. The conformity may be measured quantitatively by a simple experiment of the following type. A few pieces of paper each with its own number are demonstrated to a group of people. Each piece is either white or black. After the pieces are removed the experimentalist asks what is the color of piece #2. In fact, only one person in a group is being tested in this experiment; the rest of the group are also experimentalists, though the tested person is unaware of that. The tested person is asked the question after all the experimentalist’s team presents the same wrong answer one after another. This experiment shows how difficult is to say that white is white after all other members of a group claimed that the white piece is black. The majority of people are unable to remain true to their conviction and will give a wrong answer (when the tested persons are alone, they almost always give the right answer). As far as we know, these studies began in the middle 50th. Nowadays they flourish mostly in the purpose of marketing. Now everybody can take a free test to estimate roughly his/her level of conformity using Asch’s Conformity Study [2]. The role of conformity in a society has been considered previously [3, 4].
We do not wish to imply that conformity itself is something bad. This is one of human properties which allows social life. However, we study below how conformity may lead to instabilities in human societies.

**STATISTICAL APPROACH**

Statistical physics is widely used for studying opinion dynamics in society in the framework of different models. There are good reviews of the field. Our paper is devoted to a different aspect of the problem, and we use here rather the Gibbs ensemble than dynamical equations.

Our rough model considers the opinion of the society with respect to only one statement. We assume that all previous experience of the society including the inherited ideas of good and bad dictates a negative opinion with respect to that statement. However there is a conformity interaction between overlapping groups of the society and also there are some individuals who, in spite of everything, have a positive opinion with respect to the statement. Their positive opinion is a little bit stronger than the negative opinion of other people. Following the terminology of Mobilia we shall call these individuals “zealots”. The number of zealots is not fixed and is determined from the thermal equilibrium. We interpret the phase transition in this system as social instability. At small conformity the majority has a negative opinion while at high conformity the opinion of the majority becomes strongly positive, i.e. the majority of people become zealots.

The Hamiltonian of the model reads:

$$ H = \sum_i \phi_i q_i - \frac{1}{2} \sum_{i \neq j} V_{i,j} q_i q_j. \quad (1) $$

For simplicity it is formulated on a two-dimensional lattice. Variables $q_i$ may take values $-1/2$ or $1/2 + \alpha$. They represent the opinion of the person on a site $i$ with respect to a given statement. In terms of the previous terminology zealots have positive opinions and value $\alpha > 0$. This parameter describes some extra strength of the positive opinion, the negative opinion being the "normal" one (whether because it corresponds to the true color of the piece of paper, or to the opinion dictated by the "universal" notion of good and evil...). In the absence of conformity ($V = 0$), at low temperatures all the opinions are negative (within thermal fluctuations).

The random values $\phi_i$ are positive with a Gaussian distribution $F(\phi) = \sqrt{2/(\sqrt{\pi} A)} \exp(-\phi^2/2 A^2)$. They describe the diversity of the individuals with respect to the statement under consideration. Indeed, considering only the first term in Eq. (1), one finds that the energy $H$ has a minimum when all values of $q_i$ are negative. The second term describes interaction and conformity. It has a minimum when all $q_i$ have the same sign, in other words when all people think in the same way. Moreover, at any positive value of $\alpha$, it prefers all people to be zealots. For simplicity we consider only the interactions with the four nearest neighbors on a square lattice and $V_{i,j} = V$.

Eq. (1) can be represented as an Hamiltonian of the Ising model in some fictitious magnetic field. Introducing the new variables $S_i = q_i - \alpha/2$ one gets:

$$ H = \sum_i S_i h_i - \frac{1}{2} \sum_{i \neq j} V_{i,j} S_i S_j + C, \quad (2) $$

where:

$$ h_i = \phi_i - \sum_j V_{i,j} \alpha/2 \quad (3) $$

is a random “magnetic field” at a site $i$, $C$ is is irrelevant constant, and the values of $S_i$ are $\pm 1/2(1 + \alpha)$.

**FIRST ORDER PHASE TRANSITION**

We now show that our system has a first order phase transition. At zero temperature it can be derived exactly. To find the transition point we compare the energies of the two states. In the state I all $q_i = -1/2$. It has the energy per site:

$$ E_I = -\frac{1}{2} \langle \phi \rangle - \frac{V z}{8}. \quad (4) $$

In the state II all $q_i = 1/2 + \alpha$ which means all people are zealots. It has an energy per site:

$$ E_{II} = \left( \frac{1}{2} + \alpha \right) \langle \phi \rangle - \frac{V z}{2} \left( \frac{1}{2} + \alpha \right)^2. \quad (5) $$

Here $\langle \phi \rangle = \int_0^\infty \phi F(\phi) d\phi$ and $z = 4$ is the number of nearest neighbors taking part in the interaction with a given site. When calculating the interaction per site we must take $V/2$ since this contribution belongs to two sites. It is easily found that:

$$ \langle \phi \rangle = \sqrt{\frac{2}{\pi} A}. \quad (6) $$

One can see that, at small interaction $\alpha z V/4 \ll 1$, the state I has the lower energy while in the opposite case the state II has the lower energy and represents the ground state of the system. This is a typical situation for a first
order phase transition. It happens at \( E_I = E_{II} \). Using Eqs. (11,13) one finds that the transition occurs at:

\[
\left(\frac{V}{A}\right)_t \approx \frac{1.6}{\alpha z}
\]

(7)

and the energy in the transition point is given by the equation:

\[
H_I = -\frac{N V z}{8} (1 + 2\alpha)
\]

(8)

where \( N \) is the number of sites in the system. Note that this result can be obtained from the condition \( \langle h \rangle = 0 \), where \( h \) is given by Eq. (3).

If conformity is small \( \left(\frac{V}{A}\right) < \left(\frac{V}{A}\right)_t \), the total energy can be found from Eqs. (11,13). It has the form:

\[
H_I = -\frac{1}{2} V N \left[ \sqrt{\frac{2}{\pi}} \left(\frac{V}{A}\right)^{-1} + \frac{z}{4} \right].
\]

(9)

The total energy at large conformity can be found from Eqs. (11,13). One obtains:

\[
H_{II} = V N \left( \frac{1}{2} + \alpha \right) \left[ \sqrt{\frac{2}{\pi}} \left(\frac{V}{A}\right)^{-1} - \left(\frac{1}{2} + \alpha\right) \frac{z}{2} \right].
\]

(10)

**MONTE-CARLO SIMULATION AT FINITE TEMPERATURE**

A Metropolis [11] code has been written for the simulation of the Hamiltonian (Eq. (1)) on a 2D \((L \times L = N)\) square periodical lattice. At each Monte-Carlo step, a site \( k \) of the lattice is chosen at random. The value of \( q_k \) of the site is flipped with a probability \( P_k = \text{Min}(1, \exp -\delta H_k/k_B T) \) where \( \delta H_k = \epsilon_k \delta q_k \) is the energy cost of flipping. Here:

\[
\epsilon_k = \phi_k - \sum_j V_{kj} q_j.
\]

(11)

It is well known that, after an initial number of Monte-Carlo steps that are necessary for relaxation, the averaging of any function of the \( q_i \) over the subsequent set of distribution of \( q_i \), obtained by this way, is equivalent to averaging over the Gibbs ensemble. Depending on the conditions, the operation is performed from one hundred to one million times per site. The simulation has been run for different values of the independent parameters \( T, \frac{V}{A}, \alpha \), starting from different initial distributions of the \( q_i \). In what follows we use, instead of \( T \), a dimensionless temperature, so that \( T \rightarrow k_B T/V \).

The main results of the simulation are shown in Fig. 1. The time averaged value \( \langle q \rangle \) is plotted against \( \frac{V}{A} \) at various values of the temperature at \( \alpha = 0.1 \). One can see that, at low temperatures and low conformity the point of view of the population with respect to the statement under study is negative. At strong conformity \( \left(\frac{V}{A} \rightarrow \infty\right) \), all members of the population become zealots. At low temperatures one can see a wide hysteresis. At higher temperatures it disappears and all curves intersect at one point at \( \frac{V}{A} \approx 4 \). This value is in a good agreement with Eq. (1) at \( z = 4 \) and \( \alpha = 0.1 \). The value of \( \langle q \rangle \) in the transition point is \( \approx 0.05 \) (i.e. \( -1/2 + 1/2 + \alpha)/2 \). The hysteresis loop collapses with increasing temperature.

![FIG. 1: Average public opinion as a function of \( \frac{V}{A} \) at different temperatures (circles: 0.4, squares: 0.5, triangles: 0.6, plus: 0.7, crosses: 0.8, stars: 0.9, diamonds: 1.0) as given by the simulation. The results obtained for increasing conformity are connected by a solid line, those corresponding to the backward path are connected by dotted lines. For temperatures greater than 0.8 both paths are superimposed (within numerical fluctuations).](image-url)
DISCUSSION OF COMPUTATIONAL RESULTS

As far as we know, the Hamiltonian given by Eq. (1) has never been studied before. At $A = \alpha = 0$ it coincides with the ferromagnetic Ising Hamiltonian and it has a second-order phase transition in the two-dimensional case. We claim that our Hamiltonian has a first order transition at a point given by Eq. (7). It is obvious at $T = 0$. Convincing argument is a singularity in the behavior of energy as a function of $V_A$ shown in Fig. 4 that we interpret as a discontinuity of the first derivative of the energy in the transition point. Our understanding of the hysteresis is that our modeling in real time goes very slowly and therefore we may see a continuation of the phase from which we start above the transition point. That is typical for the first order transition and reminds super-cooled liquid.

On the other hand, we study a very low value of $\alpha$ and transition occurs at small value of $A$. Therefore one should expect that it should be close to the second order phase transition in the Ising model. Therefore we may observe clusters of zealots near the phase transition. As an example the distribution of the zealots at the threshold of the transition is shown in Fig. 5. One can see that the system has a large correlation radius which is typical for the second order phase transition. Thus, we think that we have a weak first order phase transition.

CONCLUSIONS

We propose a simple Hamiltonian that models the drastic opinion changes that can be experienced by human groups submitted to strong mutual influence, even changes that contradict a cultural achievements of the past. This model shows how, due to the conformity, the group may express an opinion opposite to the opinion which each of its isolated members would have. The model contains the following parameters: $T, A, V, \alpha, z$ that can be determined by sociological methods. The level of conformity may be checked by experiments similar to those described in the Introduction. The “temperature” of the society could be found by studying time fluctuations of the public opinion.

Our model is oversimplified, and the Eq. (7) for the critical value at the first order transition point may be not accurate. Nevertheless, if the mechanism of collective phenomenon in the human society can basically be
FIG. 5: Distribution of the zealots at $T = 0.62$, $\frac{V}{\Lambda} = \langle \frac{V}{\Lambda} \rangle_t$, $N = 10^4$, $\alpha = 0.1$. The final distribution of the zealots are indicated by black squares (the $100 \times 100$ lattice is periodical).

described our model, the ratio $\langle \frac{V}{\Lambda} \rangle_t$ as given by Eq. (7) might be an important characteristic of group phenomena.

* Electronic address: efros@physics.utah.edu

[10] We are grateful to D. Stauffer for this remark.