Multifractal Fingerprints in the Visual Arts

J. R. Mureika  
Email: jmureika@jsd.claremont.edu  
W. M. Keck Science Center, The Claremont Colleges  
925 N. Mills Avenue, Claremont, California 91711-5916

G. C. Cupchik  
Email: cupchik@scar.utoronto.ca  
Division of Life Sciences, University of Toronto at Scarborough  
1265 Military Trail, Scarborough, ON Canada M2C 1A4

C. C. Dyer  
Email: dyer@astro.utoronto.ca  
Department of Astronomy and Astrophysics, University of Toronto  
60 St. George Street, Toronto, ON Canada M5S 3H8

Abstract
The similarity in fractal dimensions of paint “blobs” in samples of gestural expressionist art implies that these pigment structures are statistically indistinguishable from one another. This result suggests that such dimensions cannot be used as a “fingerprint” for identifying the work of a single artist. To overcome this limitation, the multifractal spectrum is adopted as an alternative tool for artwork analysis. For the pigment blobs, it is demonstrated that this spectrum can be used to isolate a construction paradigm or art style. Additionally, the fractal dimensions of edge structures created by luminance gradients on the canvas are analyzed, yielding a potential method for visual discrimination of fractally-similar paintings.
The notion that Nature can be described by fractal geometry was first suggested by Benoit Mandelbrot [1]. A fractal is a recursive, self-similar structure whose constituent parts in some way resemble the whole. Mathematically, these are defined as

\[ N(d) \propto d^{D_F}, \]

Here, \( N(d) \) is a measure of the number of “objects” which comprise the set at a viewing scale \( d \), and \( D_F \) is the fractal dimension, which can be interpreted as a measure of the irregularity of the structure. Simple, smooth shapes such as dots, circles, or spheres have fractal dimensions which coincide with their Euclidean dimension. Conversely, a fractal is defined by a non-integer dimension which acts as a measure of the object’s roughness. For example, a line has dimension 1, but if it becomes very jagged at many scale levels, its dimension rises fractionally above this value - it has become a fractal. If the line becomes so jagged and rough that it effectively fills an area, then it has a fractal dimension approaching 2.

A recent surge of interest in fractal geometric “fingerprinting” of natural phenomena has included the study of artistic style. The style in question is gestural expressionism, a mid-twentieth century technique in which the artist’s hand movements are guided by a philosophy of psychic automatism and the resulting images are seemingly disordered and chaotic. Researchers have proposed that a characteristic fractal dimension may be associated with the work of Jackson Pollock, identifying the physical distribution of pigment patterns as the associated fractal [2, 3, 4, 5]. If a distinctive fractal dimension could be uncovered for every artist, this would pave the way for a novel form of artwork authentication. But is this technique sufficient to distinguish artists within the same gestural expressionist group, in this case between Jackson Pollock and the Quebec Automatistes, including artists such as Marcel Barbeau and Jean-Paul Riopelle?

A comparison was made between two groups of 8 paintings by Pollock and Les Automatistes. The images were digitized as 24-bit color files of sides ranging between 1000-2500 pixels, and pigment patterns were filtered out according to a specified target color in RGB space (see Figures 1 and 2). A variance in the values of the R, G, and B channels (each between 0-255 for 24-bit color) up to a specified distance from the target was allowed to account for any small fluctuations in the pigment shade. The fractal dimensions of the resulting patterns were calculated by the standard box counting method.
covering roughly 3 orders of magnitude of scale (1000 pixels to 4 pixels per side), roughly several meters to a few millimeters in terms of the actual canvas dimensions. As the patterns are a result of random monochromatic pigment deposits we hereafter refer to them as “blobs”, an etymology based on the “elongated blobs” of Julesz as distinguishable perceptual objects [6].

A one-way analysis of variance comparing the 8 Pollock (mean $D_F = 1.79$) with the 8 Automatistes ($D_F = 1.73$) paintings indicated that the $D_F$ indices were not significantly different, $F(1, 14) = 1.18, p < .30$. This suggests that the fractal dimension of drip paintings is not unique to any one artist and cannot be used for any such type of authentication scheme. It should be noted that a more recent study [4] has found that a fractal box counting analysis can differentiate between five Pollock and five non-Pollock images. These results can be considered to be consistent with those reported in the paper, since the non-Pollock images could be painted in such a way as to be “non-gestural”. Future analysis can shed more light on this finding.

Since a single fractal dimension rarely represents the true structure of natural objects, the multifractal spectrum of an image may provide a more rigorous way to classify the style or construction paradigm of paintings associated with a particular group such as gestural expressionists. A multifractal is a set whose form is a weave of overlapping self-similar configurations. These geometric formulations have been shown to describe the physical organization of a myriad of natural phenomena, ranging from tree root growth to large-scale galaxy clustering [7, 8]. Unlike simple fractals, multifractals are characterized by an infinite set of dimensions $\{D_q\} = \{D_0, D_1, D_2, \ldots\}$, calculated in a similar manner to $D_F$, which determine the scaling structure as a function of the local pattern density. The subscript $q$ is generally an integer, where $q = 0$ represents the classic fractal dimension ($D_F = D_0$). The regions of densest clustering, represented by extremely large values of $q$ (or $q \rightarrow \infty$) scale according to the dimension $D_\infty \leq D_0$. These two statistics, and all those in between, give a much deeper insight into the physical organization of the object in question, and in fact can be used as a method of identifying the associated formation mechanism (as was discussed in [9]). In the case of a regular fractal, all multifractal dimensions $\{D_q\}$ are equal to $D_F$.

The multifractal spectra $D_q$ were determined for the 8 Pollock (mean $D_0 = 1.60$) and 8 Automatistes (mean $D_0 = 1.58$) paintings, and a one-way analysis of variance reveals that these do not present a clearer means of differentiation than the base dimension, $F(1, 14) = 0.06, p < .80$. The gestural expressionist paintings by Pollock and the Automatistes were then
compared with 6 paintings chosen from a different style, Artonomy or Systematic Art, created by Tsion Avital [10]. This alternate technique involves creating paintings which are grouped in series according to strict rules of transformation and are “meaningless” when individually taken out of context. However, the individual paintings simply serve as a control in this study. A sample of Avital’s Artonomy is shown in Figure 3.

For 6 of these systematic art images, the analysis derived a mean $D_F = 1.60$ and $D_\infty = 1.58$, suggesting instead a monofractal structure ($D_F = D_\infty$). This should be compared with the multifractal “depth” of the Pollock and Automatistes works which show mean differences $D_F - D_\infty = 0.19$ and 0.15, respectively. This indicates that the set of dimensions $\{D_q\}$ of paint blobs can be used only to differentiate between “classes” of painting but not conclusively between different artists within the same class. The multifractal spectrum is interpreted here as the signature of an artistic style [9].

How then can one distinguish between artists within particular stylistic groups? It was noted that humans have a preference for fractals dimensions of about 1.8 [2], suggesting that the gestural expressionists catered their craft to this special dimension. However, according to Berlyne [11] test subjects were found to have a visual propensity for images which are less complex, or contain more symmetric and heterogeneous information. This fact was more recently confirmed independently by Taylor [5], who report that human visual preference is tuned to $D_F \sim 1.3$. The images deemed “pleasing” in reference [11] consist of regularly overlapping Euclidean shapes, which would suggest a fractal dimension closer to (but greater than) 1. This poses the very interesting question of why these artists gear their paintings to such high values if they are not deemed “perceptually favorable”?

Moreover, if there is no appreciable difference between the base fractal statistics for the pigment distributions, as in the case of the images in Figures 1 and 2, what is it about the paintings that can impart different visual sensations? In their seminal work, Hubel and Weisel (see e.g. [12]) have established the principle that the brain is naturally disposed to analyze visual structures in terms of edges. A study of these edges on the canvases should thus reveal new information about the perceptual nature of the artworks.

The standard RGB primary color decomposition can be seen as a reflection of the eye’s sensitivity to specific wavelengths of light via the L, M, and S cone cells. Following the notion of edge detectors in the brain, it makes sense to approach the problem in a different color space representation, namely YIQ. This offers an alternative method to decompose chromaticity.
information in terms of luminance (Y), hue (I), and saturation (Q) instead of red, green and blue primaries (see e.g. [13] for further details on color spaces).

The paintings by Jackson Pollock, Les Automatistes, and Tsion Avital were compared in terms of luminosity gradients, which were obtained by applying a Sobel filter to the luminance channel (Y) whose values again range between 0 (black, no gradient) to 255 (white, high gradient). The edge structure is defined as the regions of strongest color contrast and the associated fractal dimensions $D_F$ for each painting were obtained. In this case, the $D_F$ of these patterns showed decided grouping, unlike those of the physical paint blobs. A highly significant difference was found for Pollock’s edges ($D_F = 1.84$) as compared to those of Les Automatistes ($D_F = 1.48$), $F(1,14) = 14.52$, $p < .002$. The works of Jackson Pollock thus show highly irregular edge structures (characterized by $D_F$ close to 2), compared with those of Les Automatistes, while Artonomy’s edges possess very simple Euclidean organization ($D_F$ roughly 1). It is therefore the irregularity of edges that makes Jackson Pollock’s style unique and is representative of the degree of “expressionism” in the painting.

The suggestion that patterns of similar fractal dimension are perceptually indistinguishable can be related to the work of Julesz [6] who argues that texture discrimination in “effortless” or “immediate” perception can only occur for configurations whose autocorrelation power spectra are different. Such correlation statistics can be implicitly linked to the multifractal spectrum (e.g. $D_2$ is equivalent to the two-point correlation exponent, a structural measure of “pair-clustering” between points on the image) and thus these conclusions provide a natural extension of earlier findings.

The juxtaposition of blobs versus edges on the canvas provides two distinct structures in one painting. One facet of the image results from the deposits of raw pigment on the canvas, while another facet has as its origins the boundary between two adjacent colors. The similarity between these definitions and one of Julesz’s fundamental classes of topological perception units (coined textons in [6,14], analogous to what we term blobs) lends further support to the idea that these are visually discriminable patterns. In fact, as the density of edges on the canvas increases, the edge structures themselves become blobs. Mathematically, this is evident in the sense that $D_F$ (or $D_0$) are very close to 2.

A two-way mixed model ANOVA was conducted treating Artist (Pollock/Les Automatistes) as a between-subjects variable and Structure (blob/edge)
as a within-subjects variable. A significant two-way interaction of Artist and Structure, $F(1, 14) = 5.51, p < .03$, shows that the blobs and the edges that they create are not significantly different for Pollock (see Figure 4). This equivalence of mean fractal dimensions implies a “symmetry” between indistinguishable components which form a cohesive whole. However, for the Automatistes, the density of edges is significantly less than the density for blobs. There is thus a breakdown in the structural symmetry in this case, yielding a perceptual “conflict” of two nested but distinguishable characteristics of the painting. Thus, for the Pollock paintings, the viewer transitions effortlessly between blobs and edges, but not so for Les Automatistes.

The contrast between edges and blobs has figured prominently in art historical analysis as the “linear versus painterly dimension” discussed in [15]. While the linear is characteristic of classical art styles which favor clear edges and structured space, the sketchy baroque and impressionist styles are more painterly, encouraging viewers to complete an image. The linear versus painterly dimension has also consistently emerged as the primary one underlyng perceptual discriminations between pairs of paintings. This applies to paintings selected across a broad spectrum of traditions [16, 17] as well as those produced by Avital’s Systematic Art approach [18].

In sum, this new study has shown that Jackson Pollock is unique within gestural expressionism because of the irregularity or degree of roughness of edges underlying the structure of his paintings. It is precisely the disposition of the brain to discriminate edges [8] that makes it so sensitive to this fractal property in Jackson Pollock. The fact that fractality is a property of the whole implies that order is discerned in seeming chaos and this may provide a foundation for the pleasure experienced by some when viewing his paintings.

Acknowledgments
This work is supported by grants from the Natural Sciences and Engineering Research Council of Canada. The image Reflections of the Big Dipper (1947) by Jackson Pollock (Figure 1(a)) was provided by Art Resource, NY, and has been reproduced with the permission of the Artists Rights Society. We graciously thank Tsion Avital for the permission to reproduce his works in Figure 3.
References


Figure 1: *Reflections of the Big Dipper (1947)*, by Jackson Pollock. Image progression shows (A) original painting, (B) blob structure (black pigment) and (C) luminance edge structure (white regions).
Figure 2: *Tumulte (1973)*, Les Automatistes. (A) blob and (B) edge structure.
Figure 3: Example of Systematic Art, by Tsion Avital (A) Raw image (B) Edge structure.
Figure 4: Interaction of artist and structure for average fractal dimension DF of blob and edge structure.