We derive a low-energy effective theory for gravity with anti-D branes, which are essential to get de Sitter solutions in the type IIB string warped compactification, by taking account of gravitational backreactions of anti-D branes. In order to see the effects of the self-gravity of anti-D branes, a simplified model is studied where a 5-dimensional anti-de Sitter (AdS) spacetime is realized by the bulk cosmological constant and the 5-form flux, and anti-D branes are coupled to the 5-form field by Chern-Simon terms. The AdS spacetime is truncated by introducing UV and IR cut-off branes like the Randall-Sundrum model. We derive an effective theory for gravity on the UV brane and reproduce the familiar result that the tensions of the anti-D branes give potentials suppressed by the forth-power of the warp factor at the location of the anti-D branes. However, in this simplified model, the potential energy never inflates the UV brane, although the anti-D-branes are inflating. The UV brane is dominated by dark radiation coming from the projection of the 5-dimensional Weyl tensor, unless the moduli fields for the anti-D branes are stabilized. We comment on the possibility of avoiding this problem in a realistic string theory compactification.

PACS numbers: 04.50.+h

I. 1.INTRODUCTION

It has been a difficult problem to find de Sitter solutions in the context of string theory. Recently, significant progress has been made in the context of the type IIB string theory. Ref. considered the warped compactification where all moduli fields are stabilized by the flux of the form fields except for the volume modulus for Calabi-Yau space. The ten-dimensional metric is given by

\[ ds_{10} = a(r)^2 g_{\mu\nu} dx^\mu dx^\nu + a(r)^{-2} g_{mn} dr^m dr^n, \]  

where \( a(r) \) is the warp factor. In the intermediate region, the warp factor is approximated as (see figure 1)

\[ a(r)^2 = \left( \frac{r}{\ell} \right)^2. \]  

Then the spacetime looks like \( AdS_5 \times X^5 \) where \( AdS_5 \) is a 5-dimensional Anti-de Sitter spacetime and \( X^5 \) is a compactified manifold. For large \( r \), the spacetime deviates from \( AdS_5 \) spacetime and the geometry is smoothly joined to a Calabi-Yau compactification. This gluing region plays the role of the ultraviolet cut-off of the \( AdS_5 \) spacetime. The geometry also deviates from \( AdS_5 \) and terminates in the far infrared although the spacetime remains smooth. Then the warp factor has a minimum value at small \( r = r_0 \).

In this compactification, the vacuum solution for 4-dimensional spacetime is given by \( AdS \) spacetime. In order to lift the spacetime to de Sitter spacetime, Ref. introduces anti-D branes, which break the supersymmetry and give the potential energy

\[ V_D = \frac{2a_0^4 T_3}{g_s^2} \frac{1}{\sigma^3}, \]  

where \( a_0 \) is the warp factor at the location of the anti-D brane, \( T_3 \) is the tension of the brane, \( g_s \) is the string coupling and \( \sigma \) is the volume modulus. This potential energy can create a de Sitter spacetime. Because the potential energy is minimized at small warp factor, the anti-D branes move towards \( r = r_0 \) and sit at the tip of the warped throat. In order to stabilize the volume modulus \( \sigma \), they introduce non-perturbative effects. Then they get metastable de Sitter spacetime where all moduli-fields are stabilized.

After stabilizing the moduli fields, this model is quite similar to a Randall-Sundrum model, where the UV brane and the IR brane are introduced in \( AdS_5 \) spacetime by hand. In this model, four-dimensional gravity is reproduced.
FIG. 1: Schematic picture of the string warped compactification. The $AdS$ spacetime is truncated at large $r$ (UV) and small $r$ (IR). Anti-D branes are sitting at the tip of the warped throat.

on the UV brane if the IR brane is located at the far infrared. Thus we naively expect that we get 4-dimensional Einstein gravity near the gluing region which plays the role of the UV cut-off brane.

An effective theory for the anti-D brane coupled to gravity is discussed in Ref. [8]. They also introduced a mobile D-brane to have slow-roll inflation. However, their derivation is basically based on a probe brane approximation. A probe brane approximation is adequate for the calculation of the motion of a light brane in a fixed background spacetime. However, if we want to discuss the effect of the brane on the gravity of the UV cut-off brane, we do need to take into account the gravitational back-reaction of the brane on the geometry. In other words, we have to take into account the self-gravity of the brane. Thus it would be desirable to derive the effective gravitational theory by taking into account the self-gravity of the brane.

A difficulty is that we do not have the technology to deal with the self-gravity of infinitely thin branes with codimensions higher than one. If the co-dimension is one, we can use the junction condition to take into account the self-gravity [9]. Thus in this paper, we use a 5-dimensional toy model proposed in Ref. [10, 11]. This is a model motivated by the 5-dimensional supergravity of type IIB theory compactified on $AdS_S \times S^5$ [12]. In order to have $AdS_5$ spacetime as a solution, we assume all moduli fields, including the volume moduli for $S^5$, are stabilized. We also keep only the 5-form field which is relevant for our discussions. Then this model is basically an extension of the Randall-Sundrum model with the 5-form flux. We introduce the UV and IR cut-off branes in $AdS_5$ spacetime and assume that standard model particles are living on the UV cut-off brane. This model is described by the 5D action [10]

$$S = S_5 + \sum_i (S_{brane}^i + S_{CS}^i),$$

$$S_5 = \frac{1}{2\kappa^2} \int d^5x \sqrt{-(5)g} \left( (5)R - 2\Lambda - \frac{1}{2} |G|^2 \right),$$

$$S_{brane}^i = -T_i \int d^4x \sqrt{-g},$$

$$S_{CS}^i = Q_i \int d^4x \sqrt{-g} \frac{1}{4!} \varepsilon^{\mu\nu\rho\sigma} D_{\mu\nu\rho\sigma},$$

where $g_{\mu\nu}$ is the induced metric on the brane, $\varepsilon^{\mu\nu\rho\sigma}$ is a four dimensional epsilon tensor for $g_{\mu\nu}$ which satisfies $\varepsilon^{\mu\nu\rho\sigma} \varepsilon_{\mu\nu\rho\sigma} = -4!$. The 5-form field $G_{ABCDE} = \frac{1}{4!} \partial_{[A} D_{BCDE]}$ has nonzero component $G_{y\mu\nu\rho\sigma} = \partial_y D_{\mu\nu\rho\sigma}$, so

$$|G|^2 = \frac{1}{4!} G_{ABCDE} G^{ABCDE} = \frac{1}{4!} \partial_y D_{\mu\nu\rho\sigma} \partial^y D^{\mu\nu\rho\sigma}.$$

We impose the BPS condition on the tension and charge

$$|T_i| = |Q_i|,$$

for each brane. We also assume $Z_2$ symmetry across the brane for the UV and IR cut-off branes. In order to derive the 4-dimensional effective theory on the UV brane, we need to solve the 5-dimensional Einstein equation. We use a gradient expansion method to solve the 5-dimensional Einstein equations [13].
The structure of the paper is as follows. In section II, we introduce a D-brane in between the cut-off branes and study the back-reaction of the D-brane. We show that there appears no potential if we tune the cosmological constant $\Lambda$ in the bulk appropriately. In section III, we introduce an anti-D brane. In this case, the opposite sign of tension $T_0$ and charge $Q_0$ for the anti-D brane makes it impossible to have a static solution. Thus we must introduce the anti-D-brane perturbatively by assuming $T_0 = -Q_0$ is much smaller than the tension of the UV brane, $T_+$, and the tension of the IR brane, $T_-$. We show that the tension of the anti-D brane gives a potential energy coupled to a moduli that represents the location of the anti-D brane. However, we see that, unless we stabilize this moduli field, the UV brane never inflates by the potential energy. Rather, the UV brane is always a radiation dominated universe. We provide a simple explanation for this fact from a geometrical point of view. In section IV, we generalize our analysis to include many D and anti-D branes. Section V is devoted to conclusions.

II. D BRANE

A. set-up

In this section, we introduce a D-brane in the truncated $AdS_5$ spacetime. The metric is taken as

$$ds^2 = g_{yy}(x) \, dy^2 + g_{\mu\nu}(x,y) dx^\mu dx^\nu = e^{2\phi(x)} dy^2 + e^{-2u(x,y)} h_{\mu\nu}(x,y) dx^\mu dx^\nu,$$

where $\phi(x)$ is an arbitrary function of $x$ and $g_{\mu\nu}$ is the induced metric of $y=\text{constant}$ hypersurfaces. The UV brane, D3 brane and IR brane are located at $y = y_+, y_0$ and $y_-$ respectively. The warp factor $u(x,y)$ is an arbitrary function of $x$ and $y$ at this stage. The physical distances between two branes are given by

$$d_+(x) = \int_{y_0}^{y_+} dy \, e^\phi, \quad d_-(x) = \int_{y_0}^{y_-} dy \, e^\phi,$$

which are called radions. We use the index $i = (+, 0, -)$ to denote the variables related to the branes at $y_+, y_0$ and $y_-$ respectively. We assume all three branes satisfy

$$T_i = Q_i.$$

We consider the bulk with curvature radius and cosmological constant given by $l_+, \Lambda_+$ respectively for $y = (y_+, y_0)$ and $l_-, \Lambda_-$ for $y = (y_0, y_-)$ (see figure 2).

![FIG. 2: The case of a D-brane in the middle of the cut-off branes. The tension of the D-brane is positive for $l_+ > l_-$.](image)
B. Field equations and perturbative solutions

Firstly, the field equation for the form field is given by

\[ \partial_y (\sqrt{-g} \partial^\mu D_i^{\mu\nu\rho\sigma}) = -2\kappa^2 \sqrt{-g} \varepsilon^{\mu\nu\rho\sigma}(g) \sum_i Q_i \delta(y - y_i), \]

where \( \varepsilon^{\mu\nu\rho\sigma}(g) \) is an epsilon tensor defined with respect to \( g_{\mu\nu}(y, x) \). The junction conditions at \( y_+ \) and \( y_- \) are given by

\[ \partial_y D_i^{\mu\nu\rho\sigma} = \begin{cases} -\kappa^2 \varepsilon^{\mu\nu\rho\sigma}(g(y_+, x))Q_+, & y_+ < y < y_0, \\ +\kappa^2 \varepsilon^{\mu\nu\rho\sigma}(g(y_-, x))Q_-, & y_0 < y < y_-, \end{cases} \]

We can easily find the solutions that satisfy these boundary conditions as

\[ -\partial_y D_i^{\mu\nu\rho\sigma} \big|_{y_0+\delta} + \partial_y D_i^{\mu\nu\rho\sigma} \big|_{y_0-\delta} = 2\kappa^2 \varepsilon^{\mu\nu\rho\sigma}(g(y_0, x))Q_0, \]

where \( \varepsilon^{\mu\nu\rho\sigma}(h) \) is an epsilon tensor defined with respect to \( h_{\mu\nu} \). The junction condition at \( y_0 \) is given by

\[ Q_+ + Q_- + 2Q_0 = 0. \]

Next, we consider the Einstein equations. We define the extrinsic curvature on the \( y=\text{constant} \) slicing as

\[ K_{\mu\nu} = \frac{1}{2} e^{-\phi} \partial_y g_{\mu\nu} = \begin{cases} K^+_{\mu\nu}, & y_+ < y < y_0, \\ K^-_{\mu\nu}, & y_0 < y < y_-, \end{cases} \]

and a traceless part \( \bar{K}_{\mu\nu} = K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} K \). Then the Einstein equations can be written as

\[ \partial_y \bar{K}^\mu_{\nu} = -K \bar{K}^\mu_{\nu} + (4) R^\mu_{\nu} - \kappa^2 \left( (5) T^\mu_{\nu} - \frac{1}{4} \delta^\mu_{\nu} T^\rho_{\rho} \right) + \Phi^\mu_{\nu}, \]

\[ \partial_y K = -K^2 + (4) R + \frac{1}{3} \kappa^2 (5) T^\mu_{\nu} + (5) T^\nu_{\rho} - \nabla^2 \phi - (\nabla \phi)^2, \]

\[ \bar{K}_{\mu\nu} \bar{K}^\mu_{\nu} - \frac{3}{4} K^2 + (4) R = -2\kappa^2 (5) T_{yy}, \]

\[ \nabla_{\mu} K^\mu_{\nu} + \nabla_{\nu} K = 0, \]

where

\[ \Phi^\mu_{\nu} = -\nabla^\mu \nabla_\nu \phi + \frac{1}{4} \delta^\mu_{\nu} \nabla^2 \phi - (\nabla^\mu \phi)(\nabla_\nu \phi) + \frac{1}{4} \delta^\mu_{\nu} (\nabla \phi)^2. \]

\( \nabla_{\mu} \) denotes the covariant derivative with respect to the metric \( h_{\mu\nu} \), \( (4) R^\mu_{\nu} \) is the corresponding 4-dimensional Ricci curvature and \( (5) T^\mu_{\nu} \) is the 5-dimensional energy-momentum tensor. We also decompose the 4-dimensional curvature into a trace part \( R = R^\mu_{\nu} \) and a traceless part \( R^\mu_{\nu} = R^\mu_{\nu} - \frac{1}{4} g_{\mu\nu} R \). The junction conditions are given by

\[ (K_{\mu\nu} - K g_{\mu\nu}) \big|_{y_\pm} = \pm \frac{\kappa^2}{2} T_{\pm\mu\nu}, \]

\[ -(K_{\mu\nu} - K g_{\mu\nu})\big|_{y_0+\delta} + (K_{\mu\nu} - K g_{\mu\nu})\big|_{y_0-\delta} = -\kappa^2 T_{0\mu\nu}, \]

where \( \delta \to 0 \).
We derive a low energy theory for this three-brane system by the gradient expansion method \(^{13}\). We assume that the 4-dimensional curvature and the derivative of \(\phi(x)\) with respect to brane coordinates are suppressed as
\[
(4)^4R^\mu_\nu(l) \sim (\nabla_\mu \phi^\nu) l \sim \epsilon \ll 1.
\] (21)
Then the extrinsic curvature can be expanded in terms of \(\epsilon \ll 1\)
\[
K^\mu_\nu = (0)K^\mu_\nu + (1)K^\mu_\nu + \cdots
\] (22)
We can solve the Einstein equations perturbatively in \(\epsilon\).

(1) Zeroth order

At 0-th order, we can put the warp factor in Eq. (7) in the following form
\[
u(x,y) = \begin{cases} 
\frac{e^\phi y}{l_+}, & y_+ < y < y_0, \\
\frac{e^\phi y}{l_-} - e^\phi y_0 \left( \frac{1}{l_-} - \frac{1}{l_+} \right), & y_0 < y < y_-.
\end{cases}
\] (23)
The 0-th order solutions for \(K^\mu_\nu\) are obtained as
\[
(0)K^\pm = -\frac{4}{l_\pm},
\] (24)
\[
(0)K^\pm_\mu = -\frac{1}{l_\pm} \delta^\mu_\nu.
\] (25)
Substituting these solutions into the "Hamiltonian constraint" (17), we get
\[
-\frac{6}{l_\pm^2} = \Lambda_\pm + \frac{1}{4} \kappa^4 Q^2_\pm.
\] (26)
We can think of this as conditions on \(\Lambda_\pm\). On the other hand, the junction conditions give
\[
\left. \begin{aligned}
\frac{6}{l_\pm} &= \pm \kappa^2 T_\pm, \\
T_+ + T_- + 2 T_0 &= 0.
\end{aligned} \right\}
\] (27)
We should note that the constraints for the tensions are consistent with the conditions for the charges if all three branes are D-branes, \(Q_i = T_i\).

(2) First order

At first order, we obtain the solutions as
\[
(1)K = -\frac{l_\pm}{6} R,
\] (29)
\[
(1)\tilde{K}^\mu_\nu = -\frac{l_\pm}{2} \tilde{R}^\mu_\nu - e^{4\phi y/l_\pm} \chi^\mu_\nu(l_\pm, x),
\] (30)
where \(R\) and \(\tilde{R}^\mu_\nu\) are defined with respect to the metric \(g_{\mu\nu}(y, x)\). The integration constants
\[
\chi^\mu_\nu(x) = \begin{cases} 
\chi^\mu_\nu(l_+, x), & y_+ < y < y_0, \\
\chi^\mu_\nu(l_-, x), & y_0 < y < y_-.
\end{cases}
\] (31)
do not depend on \(y\) and satisfy
\[
\chi^\mu_\nu(l_\pm, x) = 0, \quad \nabla_\mu \chi^\nu_\mu(l_\pm, x) = 0.
\] (32)
The junction conditions at \(y_\pm\) and \(y_0\) give the equations
\[
\left. \begin{aligned}
\left[ \frac{l_+}{2} G^\mu_\nu(l_+, y_\pm) + e^{4\phi y_\pm/l_\pm} \chi^\mu_\nu(l_+, x) \right] &= 0, \\
\left[ \frac{l_-}{2} G^\mu_\nu(l_-, y_\pm) + e^{4\phi y_\pm/l_\pm} \chi^\mu_\nu(l_-, x) \right] - \left[ \frac{l_+}{2} G^\mu_\nu(l_-, y_0) + e^{4\phi y_0/l_-} \chi^\mu_\nu(l_-, x) \right] &= 0.
\end{aligned} \right\}
\] (33)
C. Effective equations on the UV brane

Now we are ready to derive the effective equations on the UV brane at $y = y_+$. One can eliminate the integration constants from Eqs. (32, 33) to find the following equation

$$\frac{l_+}{2} \left[ G^\mu_\nu(l_+, y_0) - e^{4d_+/l_+} G^\mu_\nu(l_+, y_+) \right] - \frac{l_-}{2} \left[ G^\nu_\mu(l_-, y_0) - e^{-4d_-/l_-} G^\nu_\mu(l_-, y_-) \right] = 0, \quad (34)$$

where $G^\nu_\mu(l, y)$ denotes the 4-dimensional Einstein tensor on a $y = constant$ hypersurface of the bulk with curvature length $l$. We can rewrite this equation in terms of the induced metric of the UV brane at $y_+$. The Einstein tensor satisfies the following relations:

$$G^\mu_\nu(l_+, y_+) = (4) G^\nu_\mu, \quad (35)$$
$$G^\mu_\nu(l_+, y_0) = G^\mu_\nu(l_-, y_0) = e^{2d_+/l_+} \left[ (4) G^\mu_\nu + \mathcal{F}_\mu_\nu(l_+, d_+) \right], \quad (36)$$
$$G^\nu_\mu(l_-, y_-) = e^{-2d_/l_-} G^\nu_\mu(l_+, y_-) = e^{-2d_/l_-} \left[ (4) G^\nu_\mu + \mathcal{F}_\nu_\mu(l_+, d_+) + \mathcal{F}_\nu_\mu(l_-, d_-) + f^\mu_\nu(l_+, d_+; l_-, d_-) \right], \quad (37)$$

where

$$\mathcal{F}_\mu_\nu(l, d) = \frac{2}{l} \left( \nabla^\mu \nabla_\nu d - \delta^\mu_\nu \nabla^2 d \right) + \frac{2}{l^2} \left( \nabla^\mu \nabla_\nu d + \frac{1}{2} \delta^\mu_\nu \nabla^2 d \right)^2, \quad (38)$$
$$f^\mu_\nu(l_+, d_+; l_-, d_-) = \frac{2}{l_+l_-} \left( \nabla^\mu d_+ \nabla_\nu d_- + \nabla^\mu d_- \nabla_\nu d_+ + \delta^\mu_\nu \nabla d_+ \nabla d_. \right). \quad (39)$$

Then the effective equations on the UV brane are obtained as

$$\left[ 1 - C_0 e^{-2d_+/l_+} - C_- e^{-2d_+/l_+ - d_-/l_-} \right] (4) G^\mu_\nu = C_0 e^{-2d_+/l_+} \mathcal{F}_\mu_\nu(l_+, d_+) + C_- e^{-2d_+/l_+ - d_-/l_-} \left[ \mathcal{F}_\nu_\mu(l_+, d_+) + \mathcal{F}_\nu_\mu(l_-, d_-) + f^\mu_\nu(l_+, d_+; l_-, d_-) \right], \quad (40)$$

where

$$C_0 = \frac{l_+ - l_-}{l_+}, \quad C_- = \frac{l_-}{l_+}. \quad (41)$$

The field equations for the radions are obtained from the traceless conditions for $\chi^\mu_\nu(l_\pm, x)$ as

$$\mathcal{F}_\mu_\nu(l_+, d_+) = 0, \quad (42)$$
$$\mathcal{F}_\mu_\nu(l_-, d_-) + f^\mu_\nu(l_+, d_+; l_-, d_-) = 0. \quad (43)$$

These effective equations can be derived from the following effective action

$$S_{eff} = \frac{1}{2\kappa_4^2} \int d^4 x \sqrt{-h} \left[ (1 - C_0 \Phi^2_0 - C_- \Phi^2_-) (4) R - 6C_0 \partial_\mu \Phi_0 \partial^\mu \Phi_0 - 6C_- \partial_\mu \Phi_- \partial^\mu \Phi_- \right], \quad (44)$$

where $\kappa_4^2 = \kappa^2/l_+$ and we defined two scalar fields as

$$\Phi_0 \equiv e^{-d_+/l_+}, \quad \Phi_- \equiv e^{-d_+/l_+ - d_-/l_-}. \quad (45)$$

The over-all normalization was determined by substituting the solutions into the 5-dimensional action. This effective action is the same as the one for a three-brane system in the Randall-Sundrum model [15]. If we take the limit $l_+ - l_- \to 0$, $C_0 = 0$ and the middle D-brane disappears. Then we reproduce the familiar two brane result [13].

We should mention that the tension of the middle brane is given by

$$T_0 = -\frac{1}{2} (T_+ + T_-) = -\frac{3}{\kappa_4^2} \left( \frac{1}{l_+} - \frac{1}{l_-} \right). \quad (46)$$

Thus if the middle brane has a positive tension we should have $l_+ > l_-$. In this case $C_0 > 0$ and kinetic terms for $\Phi_0$ and $\Phi_-$ are normal. If the middle brane has a negative tension, $C_0 < 0$. In order to see the kinetic term for scalar
fields, it is better to move to the Einstein frame. By defining new fields and metric [13],

\[
\chi = -\ln \left| \frac{1 - \sqrt{1 - \varphi}}{1 + \sqrt{1 - \varphi}} \right|, \quad \varphi = 1 - C_0^2 \Phi_0^2 - C^2 \Phi_0^2, \quad (47)
\]

\[
\psi = \begin{cases} 
\ln \left| \frac{z - 1}{z + 1} \right|, & z = \frac{C_0 \Phi_0}{C_- \Phi_-}, \quad C_0 < 0, \\
2 \arctan z, & z = \frac{C_0 \Phi_0}{C_- \Phi_-}, \quad C_0 > 0,
\end{cases} \quad (48)
\]

where \( g_{E \mu \nu} = \varphi h_{\mu \nu} \),

the effective action can be rewritten as

\[
S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g_E} \left[ R_E - \frac{3}{2} \partial_\mu \chi \partial^\mu \chi + \frac{3}{2} \left( \sinh^2 \frac{\chi}{2} \right) \partial_\mu \psi \partial^\mu \psi \right], \quad (50)
\]

where + for \( C_0 < 0 \) and – for \( C_0 > 0 \). If we consider fluctuations from static solutions \( R = R_E = 0, \psi = \chi = \text{const.} \), the fluctuation for the middle D-brane becomes a ghost if \( C_0 < 0 \), i.e. if the middle brane has a negative tension [14]. We assume the middle D-brane has a positive tension.

Because there are two independent conformal coupled scalar fields, if we consider a homogeneous and isotropic universe, the Friedmann equation on the UV brane is given by

\[
H^2 = C a_4^4, \quad (51)
\]

where \( C \) is an integration constant and \( a_4 \) is the scale factor of the universe on the UV brane. Thus the universe is dominated by the radiation, which is called dark radiation. The integration constant \( C \) is related to the Black Hole mass in the bulk. This is consistent with the effective equations given by

\[
\frac{l_+}{2} G^\mu_\nu = -\chi^\mu_\nu(l_+, x), \quad (52)
\]

where \( \chi^\mu_\nu \) is a transverse-traceless tensor. We can relate \( \chi^\mu_\nu \) to the electric part of the 5-dimensional Weyl tensor projected on the brane, \( E^\mu_\nu \), as [16]

\[
\chi^\mu_\nu(l_+, x) = \frac{l_+}{2} E^\mu_\nu, \quad E_{\mu \nu} = (5) C^{(5)} \gamma_{\mu \nu}. \quad (53)
\]

### III. ANTI-D BRANE

Now, we introduce an anti-D brane at \( y = y_0 \) instead of a D brane. The relation between the tension and the charge is given by

\[
T_0 = -Q_0, \quad (54)
\]

for the anti-D brane. We can immediately see that there is no static solution. The constraint on the charges is unchanged and this can be rewritten in terms of the tensions as

\[
Q_+ + Q_- + 2Q_0 = T_+ + T_- - 2T_0 = 0, \quad (55)
\]

where we assumed \( T_\pm = Q_\pm \). This contradicts the condition to have a static solution, given by \( T_+ + T_- + 2T_0 = 0 \). Thus we assume \( T_0 \ll T_\pm \) and treat \( T_+ + T_- + 2T_0 \) as a first order quantity in the gradient expansion.

We arrive at the same equations as the D brane case but the junction condition at the anti-D brane is modified as

\[
\left[ \frac{l_+}{2} G^\mu_\nu(l_+, y_0) + e^{4e^\gamma y_0/l_+} \chi^\mu_\nu(l_+, x) \right] - \left[ \frac{l_+}{2} G^\mu_\nu(l_-, y_0) + e^{4e^\gamma y_0/l_-} \chi^\mu_\nu(l_-, x) \right] = \frac{1}{2} \kappa_5^2 \Gamma_0 \delta^\mu_\nu, \quad (56)
\]

where

\[
\Gamma_0 = T_+ + T_- + 2T_0 = 4T_0. \quad (57)
\]
Here we used Eq. (55). Because the tension of the anti-D brane is assumed to be small compared with $T_\pm$, we need to assume $|l_+ - l_-| \ll l_\pm$. Then we can get the effective equations on the positive tension branes as
\[
\left[ 1 - C_0 e^{-2d_+ / l_+} - C_- e^{-2(d_+ + d_- / l_-)} \right] (4) G^\mu_{\nu} = C_0 e^{-2d_+ / l_+} F^\mu_{\nu}(l_+, d_+) + C_- e^{-2d_+ / l_+ + d_- / l_-} \left[ F^\mu_{\nu}(l_+, d_+) + F^\mu_{\nu}(l_-, d_-) + f^\mu_{\nu}(l_+, d_+; l_-, d_-) \right]
\]
\[-4T_0 \kappa_4^2 e^{-4d_+ / l_+} \delta^\mu_{\nu}. \] (58)

The equations for the radions are given by
\[
(l_+ - l_-) F^\mu_{\nu}(l_+, d_+) = 16 \kappa_4^2 T_0 e^{-2d_+ / l_+}, \] (59)
\[
F^\mu_{\nu}(l_+, d_+) + F^\mu_{\nu}(l_-, d_-) + f^\mu_{\nu}(l_+, d_+; l_-, d_-) = 0. \] (60)

The effective action is given by
\[
S_{\text{eff}} = \frac{1}{2 \kappa_4^2} \int d^4 x \sqrt{-h} \left[ (1 - C_0 \Phi_0^2 - C_- \Phi_-^2) (4) R - 6 C_0 \partial_{\mu} \Phi_0 \partial^\mu \Phi_0 - 6 C_- \partial_{\mu} \Phi_- \partial^\mu \Phi_- - 8 \kappa_4^2 T_0 \Phi_0^4 \right]. \] (61)

Then we can identify the effective cosmological constant on the UV brane as
\[
\Lambda_4 = 4 \kappa_4^2 T_0 \Phi_0^4 = 4 \kappa_4^2 T_0 e^{-4d_+ / l_+}. \] (62)

This is the desired result. The effective cosmological constant on the UV brane is proportional to the tension of the anti-D brane and it is suppressed by the fourth power of the warp factor at the location of the anti-D brane.

The tension of the anti-D brane is given by
\[
T_0 = \frac{1}{2} (T_+ + T_-) = \frac{3}{\kappa^2} \left( \frac{1}{l_+} - \frac{1}{l_-} \right). \] (63)

Thus, for the anti-D brane, we need $l_+ < l_-$. To have a positive tension brane and a positive cosmological constant (see Figure 3). Then we have $C_0 < 0$ and, in the Einstein frame, the kinetic term for $\psi$ is always positive. Thus there would be a ghost-like excitation, although the situation is more complicated than the D-brane case because the tension of the anti-D brane gives non-trivial potentials for $\chi$ and $\psi$ given by
\[
V(\chi, \psi) = -8 \kappa_4^2 T_0 C_0^{-2} \left( \sinh^4 \frac{\chi}{2} \right) \left( \sinh^4 \frac{\psi}{2} \right). \] (64)

Then there is no static solution. At least classically, the positivity of the kinetic term does not lead to instability of the system.

![FIG. 3: The case of an anti-D-brane in the middle of the cut-off branes. The tension of the anti-D-brane is positive for $l_+ < l_-$.](image)

On the anti-D brane, a de Sitter spacetime is realized because the trace of Eq. (65) gives
\[
R(y_0, l_+) = 16 \frac{\kappa^2}{l_- - l_+} T_0, \] (65)
where we assume $T_0 > 0$ and $l_+ < l_-$ . However, unfortunately, this effective cosmological constant never inflates the UV brane. The Ricci scalar of the UV brane is given by

$$(4) \quad R = e^{-2d_+/l_+} R(l_+, y_0) + \mathcal{F}_\mu(l_+, d_+).$$

Using Eq. (59), we can show $(4) R = 0$. This fact can be easily understood by noting that the effective theory on the UV brane is still given by

$$\frac{l_+}{2} (4) G^\mu_\nu = -\chi^\mu_\nu(l_+, x),$$

where $\chi^\mu_\nu$ is a transverse-traceless tensor. Thus we cannot have a non-zero Ricci scalar. The UV brane is a radiation dominated universe if the brane is homogeneous and isotropic. We can also understand this result by the fact that the moduli describing the location of the anti-D brane is a conformal coupled scalar field and the potential preserves the conformal symmetry. This is related to the difficulty to get slow-roll inflation from an interaction between D-brane and anti-D branes.

The geometrical approach here gives a simple explanation for this. In $AdS$ spacetime with an empty bulk, the radion can induce only dark radiation on the brane if we consider a homogeneous and isotropic brane. In order to get a de Sitter spacetime, we must introduce some matter in the bulk to give a potential for $d_+$. Once $d_+$ is stabilized by bulk matter fields, we can have inflation on the UV brane supported by the tension of the anti-D brane. In a realistic string theory model, the spacetime deviates from $AdS$ in the far infrared and the $AdS$ region terminates smoothly. Then the anti-D brane will sit at the tip of the warped throat. This could effectively stabilize the $d_+$ moduli.

### IV. MANY BRANES

Our result can be easily extended to a many-brane system. The constraint on charges is given by

$$Q_+ + 2 \sum_{i=0}^{n-1} Q_i + Q_- = 0,$$

where we have $n$ branes in between the cut-off branes. This can be rewritten as a condition on the tensions,

$$T_+ + 2 \sum_{i=D} T_i - 2 \sum_{j=\bar{D}} T_j + T_- = 0,$$

where $i = D$ denotes that the branes are D branes $T_i = Q_i$ and $j = \bar{D}$ denotes that the branes are anti-D branes, $T_i = -Q_i$. We defined the tensions in terms of the curvature scales $l_i$ as

$$T_i = \begin{cases} -\frac{3}{\kappa^2} \left( \frac{1}{l_{i-1}} - \frac{1}{l_i} \right) & \text{for } D, \\ \frac{3}{\kappa^2} \left( \frac{1}{l_{i-1}} - \frac{1}{l_i} \right) & \text{for } \bar{D}, \end{cases}$$

where $l_0 = l_+, l_n = l_- \text{ and assume that all branes have positive tensions. These definitions satisfy the constraint Eq. (69).}$ We also define the cosmological constant $\Lambda_i$ in the bulk by

$$\frac{-6}{l_i^2} = \Lambda_i + \frac{1}{4\kappa^4} Q_i^2.$$  

Then we can easily derive the effective action on the UV branes as

$$S = \frac{1}{2\kappa_4^2} \int dx^4 \sqrt{-\left(4g\right)} \left( 1 - \sum_{i=0}^{n} C_i \Phi_i^2 \right) R - 6 \sum_{i=0}^{n} C_i \partial_\mu \Phi_i \partial^\mu \Phi_i - 8\kappa_4^2 \sum_{j=D} T_j \Phi_j^4,$$

where

$$\Phi_i = \exp \left( -\sum_{j=1}^{i} d_{j-1}/l_{j-1} \right), \quad C_i = \frac{l_i - l_{i+1}}{l_+},$$

$l_{n+1} = 0$. $d_i$ is the physical distance between the $(i-1)$-th and $i$-th branes for $0 \leq i \leq n$, where we define the UV brane and IR brane as the $-1$-th brane and $n$-th brane respectively. The contribution for the potential is a linear summation of contributions only from anti-D branes.
V. CONCLUSION

In this paper, we studied a low energy effective theory for gravity in the warped compactification model with anti-D branes. We constructed a simplified model where the UV (positive tension) and the IR (negative tension) cut-off branes are introduced as in the Randall-Sundrum model and the standard model particles are assumed to live on the UV brane. Then we introduce anti-D brane(s) into the truncated $AdS$ spacetime. Due to the opposite sign of the tension and charge of the anti-D brane, the tension of the anti-D brane provides a potential energy suppressed by $a_0^4$, where $a_0$ is the warp factor at the location of the anti-D brane. This could act as a cosmological constant. However, because the moduli field describing the distance between the UV brane and the anti-D brane is a conformal coupled scalar field and the potential preserves the conformal symmetry, this potential energy does not inflate the UV brane. Rather, the UV brane is a radiation dominated universe. From a geometric point of view, the UV brane is dominated by the energy density of the projected 5-dimensional Weyl tensor, known as dark radiation.

We should note that the anti-D brane itself is inflating. Thus, the extra-dimension is quite inhomogeneous. Precisely speaking, the inhomogeneity originates from the inhomogeneous choice of the slicings of the $AdS$ spacetime. The 4-dimensional slicings of $AdS_5$ can have different geometries on them. Without branes, different choices of the slicing are merely different choices of the coordinates. However, once we introduce self-gravitating branes, the differences of the slicing become physical in the sense that the geometries of the branes are determined by the slicings. This is a crucial difference from the conventional Kaluza-Klein compactification where the 4-dimensional spacetime has the same geometry regardless of position in the extra-dimension.

In order to have a de Sitter solution for the UV brane, we must stabilize the moduli fields for anti-D branes. In a realistic string theory compactification, the $AdS$ spacetime terminates smoothly in the far infrared. This can act as a stabilization mechanism for anti-D branes because the anti-D branes will sit at the tip of the throat where the warp factor is minimized. In order to address this issue more precisely, we need to take into account the geometry of the tip instead of introducing the IR cut-off brane by hand. This deserves further investigation.

Acknowledgments

We would like to thank D. Wands for informing us of Ref. [15], R. Maartens for a careful reading of this manuscript and T. Shiromizu for pointing out several typos. KK is supported by PPARC.