Reply To “Comment on ‘Quantum Convolutional Error-Correcting Codes’ ”

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In their comment, de Almedia and Palazzo [1] discovered an error in my earlier paper concerning the construction of quantum convolutional codes [2]. This error can be repaired by modifying the method of code construction.

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de Almedia and Palazzo [1] found a counter-example showing the invalidity of Theorems 2 and 3 in my earlier paper in Ref. [2]. Their counter-example is correct; and the source of error lies with the proof of Lemma 2 in Ref. [2]. In fact, Lemma 2 is not correct and Theorem 3 should be modified as follows. (It is straightforward to extend the modified theorem to cover the case of qudits.)

**Theorem 3.** Let $C_1$ be a classical (block or convolutional) code of rate $r_1$ and distance $d_1$ and let $C_2$ be the $[n_2 = d_2, 1, d_2]$ majority vote classical code of rate $r_2$, namely, the one that maps $|t\rangle$ to $\bigotimes_{j=1}^{n_2} |t\rangle$. We construct a quantum code $C$ by first encoding a quantum state by $C_1$, then by applying a Hadamard transform to every resultant qubit, and finally by encoding each of the Hadamard transformed qubit by $C_2$. The rate and minimum distance of code $C$ equal $r_1 r_2$ and $\min(d_1^2, d_2)$ respectively, where $d_1^2$ is the minimum distance of the (classical) dual code of $C_1$.

**Proof.** Clearly, the rate of code $C$ equals $r_1 r_2$. So, we only need to show that its minimum distance is $\min(d_1^2, d_2)$. Let us examine the classical code $C_1$ and the quantum code $C$ in the stabilizer formalism. We denote the operation of applying $\sigma_x$ ($\sigma_z$) to the $i$th qubit by $X_i$ ($Z_i$). The encoded operation that flips the spin of the $i$th unencoded qubit for the classical code $C_1$ can be expressed in the form $X_i^{f_{1}(1)} \circ X_i^{f_{1}(2)} \circ \cdot \cdot \cdot \circ X_i^{f_{1}(j)}$, where $f_i$ is a binary-valued function. The dual code of $C_1$ is a linear space spanned by vectors in the form $\prod_{j=1}^{n_2} X_j^{g_{j}(1)}$, where $g_j$'s are some binary-valued functions. Since $C_2$ is the majority vote code, from our construction of $C$, the encoded operation that flips the spin (shifts the phase) of the $i$th unencoded qubit for the quantum code $C$ is given by $\prod_{j=1}^{n_2} \prod_{k=1}^{d_2} Z_{n_2(m-1)+1+k} \prod_{j=1}^{n_2} \prod_{k=1}^{d_2} X_{n_2(j-1)+k}$. Furthermore, the stabilizer of $C$ equals the span of $\{Z_{n_2(m-1)+1} \circ Z_{n_2(m-1)+\ell} \prod_{j=1}^{n_2} \prod_{k=1}^{d_2} X_{n_2(j-1)+k} : \ell = 2, 3, \ldots, n_2 \}$ and $m, s \geq 1 \}$. So just like CCS codes, the spin flip and phase shift errors in the quantum code $C$ can be corrected separately. After explicitly writing down the encoded operations and the generators of the stabilizer for the (degenerate) quantum code $C$, it is straightforward to check that $C$ detects all spin errors happened to less than $d_2$ qubits. Moreover, all phase shift errors involving with less than $\min(d_1^2, d_2)$ qubits are in the stabilizer. Thus, the minimum distance of code $C$ is $\min(d_1^2, d_2)$. $\square$

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