CP-violating asymmetries in $B^0$ decays to $K^+K^-K^0_{S(L)}$ and $K^0_SK^0_SK^0_{S(L)}$

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Abstract

Decay rates and time-dependent and direct $CP$ asymmetries in the decays $B^0 \to K^+K^-K^0_{S(L)}$ and $K^0_SK^0_SK^0_{S(L)}$ are studied. Resonant and nonresonant contributions to the three-body decays are carefully investigated. Nonresonant effects on 2-body and 3-body matrix elements are constrained by QCD counting rules. The predicted effects on branching ratios are consistent with the data within the theoretical and experimental errors, though the theoretical central values are somewhat smaller than the experimental ones. Owing to the presence of color-allowed tree amplitudes in $B^0 \to K^+K^-K^0_{S(L)}$, this penguin-dominated mode may be subject to a potentially significant tree pollution and the deviation of the mixing-induced $CP$ asymmetry from that measured in $B \to J/\psi K^0_S$, namely, $\Delta \sin 2\beta_{K^+K^-K^0_{S(L)}} \equiv \sin 2\beta_{K^+K^-K^0_{S(L)}} - \sin 2\beta_{J/\psi K^0_S}$, can be as large as $O(0.10)$. In contrast, the $K^0_SK^0_SK^0_{S(L)}$ modes appear theoretically very clean in our picture with negligible theoretical errors in $\Delta \sin 2\beta_{K^0_SK^0_SK^0_{S(L)}}$. Direct $CP$ asymmetries in $K^+K^-K^0_{S(L)}$ and $K^0_SK^0_SK^0_{S(L)}$ modes are found to be very small.
I. INTRODUCTION

Considerable activity in search of possible New Physics beyond the Standard Model has recently been devoted to the measurements of time-dependent $CP$ asymmetries in neutral $B$ meson decays into final $CP$ eigenstates defined by

$$\frac{\Gamma(B(t) \rightarrow f) - \Gamma(B(t) \rightarrow \bar{f})}{\Gamma(B(t) \rightarrow f) + \Gamma(B(t) \rightarrow \bar{f})} = S_f \sin(\Delta mt) + A_f \cos(\Delta mt),$$

where $\Delta m$ is the mass difference of the two neutral $B$ eigenstates, $S_f$ monitors mixing-induced $CP$ asymmetry and $A_f$ measures direct $CP$ violation (in the BaBar notation, $C_f = -A_f$). The time-dependent $CP$ asymmetries in the $b \rightarrow sq\bar{q}$ penguin-induced two-body decays such as $B^0 \rightarrow (\phi, \omega, \pi^0, \eta', f_0)K_S$ measured by BaBar \[1, 2\] and Belle \[3, 4, 5\] show some indications of sizable deviations from the expectation of the SM where $CP$ asymmetry in all above-mentioned modes should be equal to $S_{J/\psi K_S} = 0.687 \pm 0.032$ [6] with a small deviation at most $O(0.1)$ \[7, 8\]. Based on the framework of QCD factorization \[9\], the mixing-induced $CP$ violation parameter $S_f$ in the seven 2-body modes $(\phi, \omega, \rho^0, \eta', \pi^0, f_0)K_S$ has recently been quantitatively studied in \[10\] and \[11, 12\]. It is found that the sign of $\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}$ ($\eta_f$ being the $CP$ eigenvalue of the final state $f$) at short distances is positive except for the channel $\rho^0 K_S$. After including final-state rescattering effects, the central values of $\Delta S_f$ become positive for all the modes under consideration, but they tend to be rather small compared to the theoretical uncertainties involved so that it is difficult to make reliable statements on the sign at present \[10\].

Time-dependent $CP$ asymmetries in the $b \rightarrow sq\bar{q}$ induced three-body decays $B^0 \rightarrow K^+K^-K_S$ and $K_SK_SK_S$ have also been measured by $B$ factories \[2, 4, 13, 14, 15, 16\] (see Table \[II\]). Three-body modes such as these were first discussed by Gershon and Hazumi \[17\]. While $K_SK_SK_S$ has fixed $CP$-parity, $K^+K^-K_S$ is an admixture of $CP$-even and $CP$-odd components, rendering its $CP$ analysis more complicated. By excluding the major $CP$-odd contribution from $\phi K_S$, the 3-body $K^+K^-K_S$ final state is primarily $CP$-even. A measurement of the $CP$-even fraction $f_+$ in the $B^0 \rightarrow K^+K^-K_S$ decay yields $f_+ = 0.89 \pm 0.08 \pm 0.06$ by BaBar \[2\] and $0.93 \pm 0.09 \pm 0.05$ by Belle \[3\], while the $CP$-odd fraction in $K^+K^-K_L$ is measured to be $f_- = 0.92 \pm 0.33^{+0.13}_{-0.14} \pm 0.10$ by BaBar \[13\]. Hence, while $\eta_f = 1$ for the $K_SK_SK_S$ mode, $\eta_f = 2f_+ - 1$ for $K^+K^-K_S$ and $\eta_f = -(2f_- - 1)$ for $K^+K^-K_L$. It is convenient to define an effective $\sin 2\beta$ via $S_f \equiv -\eta_f \sin 2\beta_{\text{eff}}$. The results of $\sin 2\beta_{\text{eff}}$ for $K^+K^-K_S$ obtained from the measurements of $S_{K^+K^-K_S}$ and $f_+$ are also shown in Table \[II\].

In order to see if the current measurements of the deviation of $\sin 2\beta_{\text{eff}}$ in $KKK$ modes from $\sin 2\beta_{J/\psi K_S}$ signal New Physics in $b \rightarrow s$ penguin-induced modes, it is of great importance to examine and estimate how much of the deviation of $\sin 2\beta_{\text{eff}}$ is allowed in the SM. One of the major uncertainties in the dynamic calculations lies in the hadronic matrix elements which are nonperturbative in nature. One way to circumvent this difficulty is to impose SU(3) flavor symmetry \[18, 19\] or isospin and U-spin symmetries \[20\] to constrain the relevant hadronic matrix elements. While this approach is model independent in the symmetry limit, deviations from that limit can only be computed in a model dependent fashion. In addition, it may have some weakness as discussed in \[19\].
We shall apply the factorization approach in this work as it seems to work even in the case of three-body $B$ decays [21]. By using factorization and kaon time-like form factors extracted from the $e^+e^- \rightarrow K\bar{K}$ process, the predicted $B^0 \rightarrow D^{(*)+} K^- K^0$ rate agrees well with the data [21]. In general, three-body $B$ decays are more complicated than the two-body case as they receive resonant and nonresonant contributions and involve 3-body matrix elements. Nonresonant charmless three-body $B$ decays have been studied extensively [22, 23, 24, 25, 26, 27] based on heavy meson chiral perturbation theory (HMChPT) [28, 29, 30]. However, the predicted decay rates are in general unexpectedly large. For example, the branching ratio of the nonresonant decay $B^- \rightarrow \pi^+\pi^-\pi^-$ is predicted to be of order $10^{-5}$ in [22] and [23], which is too large compared to the BaBar’s preliminary result $(0.68 \pm 0.41) \times 10^{-6}$ [31]. The issue has to do with the applicability of HMChPT. In order to apply this approach, two of the final-state pseudoscalars have to be soft. The momentum of the soft pseudoscalar should be smaller than the chiral symmetry breaking scale $\Lambda_\chi \sim 830$ MeV. For 3-body charmless $B$ decays, the available phase space where chiral perturbation theory is applicable is only a small fraction of the whole Dalitz plot. Therefore, it is not justified to apply chiral and heavy quark symmetries to a certain kinematic region and then generalize it to the region beyond its validity. In order to have a reliable prediction for the total rate of direct 3-body decays, one should try to utilize chiral symmetry to a minimum. Therefore, we will apply HMChPT only to the strong vertex and use the form factors to describe the weak vertex [32]. Moreover, we shall introduce a form factor to take care of the off-shell effect.

As shown in [10], among the aforementioned seven neutral $PK_S$ modes, only the $\omega K_S$ and

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**TABLE I:** Mixing-induced CP asymmetries $-S_f$ (top), direct CP violation $A_f$ (middle) and branching ratios (in units of $10^{-6}$, bottom) for $B^+ \rightarrow K^+ K^- K_S$ and $K_S K_S$ decays. For effective $\sin 2\beta$ for $K^+ K^- K_S$, the third error is due to the uncertainty in the fraction of CP-even contributions to the decay rate. Experimental results are taken from [2, 11, 13, 14, 16].

<table>
<thead>
<tr>
<th>Final State</th>
<th>BaBar</th>
<th>Belle</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ K^- K_S$</td>
<td>$0.42 \pm 0.17 \pm 0.03$</td>
<td>$0.52 \pm 0.16 \pm 0.03$</td>
<td>$0.47 \pm 0.12$</td>
</tr>
<tr>
<td>$(\sin 2\beta)_{K^+ K^- K_S}$</td>
<td>$0.55 \pm 0.22 \pm 0.04 \pm 0.11$</td>
<td>$0.60 \pm 0.18 \pm 0.04^{+0.19}_{-0.12}$</td>
<td>$0.57^{+0.18}_{-0.17}$</td>
</tr>
<tr>
<td>$K^+ K^- K_L$</td>
<td>$0.07 \pm 0.28^{+0.11}_{-0.12}$</td>
<td>$0.07 \pm 0.30$</td>
<td>$0.07 \pm 0.30$</td>
</tr>
<tr>
<td>$(\sin 2\beta)_{K^+ K^- K_L}$</td>
<td>$0.09 \pm 0.33^{+0.13}_{-0.14} \pm 0.10$</td>
<td>$0.09 \pm 0.37$</td>
<td>$0.09 \pm 0.37$</td>
</tr>
<tr>
<td>$K_SK_SK$</td>
<td>$0.63^{+0.28}_{-0.32} \pm 0.04$</td>
<td>$0.58 \pm 0.36 \pm 0.08$</td>
<td>$0.61 \pm 0.23$</td>
</tr>
<tr>
<td>$K^+ K^- K_S$</td>
<td>$-0.10 \pm 0.14 \pm 0.04$</td>
<td>$-0.06 \pm 0.11 \pm 0.07$</td>
<td>$-0.08 \pm 0.10$</td>
</tr>
<tr>
<td>$K^+ K^- K_L$</td>
<td>$-0.54 \pm 0.22^{+0.09}_{-0.08}$</td>
<td>$-0.54 \pm 0.24$</td>
<td>$-0.54 \pm 0.24$</td>
</tr>
<tr>
<td>$K_SK_SK$</td>
<td>$0.10 \pm 0.25 \pm 0.05$</td>
<td>$0.50 \pm 0.23 \pm 0.06$</td>
<td>$0.31 \pm 0.17$</td>
</tr>
</tbody>
</table>

*a*with $\phi(1020)/K_S$ excluded.

*b*with $\phi(1020)/K_L$ excluded.

*c*with the error enlarged by a factor of $S = 1.4$. 

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\( \rho^0 K_S \) modes are expected to have a sizable deviation of the mixing-induced \( CP \) asymmetry \( S_f \) from \( S_{J/\psi K_S} \). More precisely, it is found \( \Delta S_{\omega K_S} = 0.12^{+0.05}_{-0.06} \) and \( \Delta S_{\rho K_S} = -0.09^{+0.03}_{-0.07} \) \(^1\) in the absence of final-state interactions \(^1\). Although the tree contribution in these two modes is color suppressed, the large cancellation between \( a_4 \) and \( a_6 \) penguin terms renders the tree pollution relatively significant. Unlike the above-mentioned case for two-body decays, the tree contribution to the 3-body decay \( B^0 \rightarrow K^+ K^- K_S \) is \textit{color-allowed} and hence it has the potential for producing a large deviation from \( \sin 2\beta \) measured in \( B \rightarrow J/\psi K_S \). We shall see in this work that it is indeed the case. In contrast, the absence of tree pollution in \( K_S K_S K_S \) renders it theoretically very clean in our picture.

The layout of the present paper is as follows. In Sec. II we apply the factorization approach to study \( B^0 \rightarrow K^+ K^- K_S \) and \( K_SK_SK_S \) decays and discuss resonant and nonresonant contributions in Sec. II. Numerical results for decay rates and \( CP \)-violating parameters \( S_f \) and \( A_f \) and discussions are presented in Sec. III. Sec. IV contains our conclusions.

**II. FORMALISM FOR CHARMLESS 3-BODY \( B \) DECAYS**

In the factorization approach, the matrix element of the \( \overline{B} \rightarrow \overline{K} K K \) decay amplitude is given by

\[
\langle \overline{K} K K \rangle |\mathcal{H}_{\text{eff}}| \overline{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \overline{K} K K | T_p | \overline{B} \rangle, \tag{2.1}
\]

where \( \lambda_p \equiv V_{pb} V_{ps}^* \) and \( \mathcal{H}_{\text{eff}} \). \( T_p \) is given by

\[
T_p = a_1 \delta_{pu} (\bar{u}b)_{V-A} \otimes (\bar{s}u)_{V-A} + a_2 \delta_{pu} (\bar{s}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_3 (\bar{s}b)_{V-A} \otimes \sum_q (\bar{q}q)_{V-A}
\]

\[
+ a_4^p \sum_q (\bar{q}b)_{V-A} \otimes (\bar{s}q)_{V-A} + a_5 (\bar{s}b)_{V-A} \otimes \sum_q (\bar{q}q)_{V+} + a_6^p \sum_q (\bar{q}b)_{S-} \otimes (\bar{s}q)_{S+} + a_7 (\bar{s}b)_{V-A} \otimes \sum_q \frac{3}{2} e_q (\bar{q}q)_{V+} + a_8^p \sum_q (\bar{q}b)_{S-} \otimes \frac{3}{2} e_q (\bar{s}q)_{S+} + a_9 (\bar{s}b)_{V-A} \otimes \sum_q \frac{3}{2} e_q (\bar{q}q)_{V-} + a_{10}^p \sum_q (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_q (\bar{s}q)_{V-}, \tag{2.2}
\]

with \( (\bar{q}q')_{V\pm} \equiv \bar{q} \gamma_\mu (1 \pm \gamma_5) q' \) and \( (\bar{q}q')_{S\pm} \equiv \bar{q} (1 \pm \gamma_5) q' \) and a summation over \( q = u, d, s \) being implied. The matrix element \( \langle \overline{K} K K | j \otimes j' | \overline{B} \rangle \) corresponds to \( \langle \overline{K} K | j | \overline{B} \rangle \langle \overline{K} | j' | 0 \rangle \), \( \langle \overline{K} | j | \overline{B} \rangle \langle \overline{K} K | j' | 0 \rangle \) or \( \langle 0 | j | \overline{B} \rangle \langle \overline{K} K K | j' | 0 \rangle \), as appropriate, and \( a_i \) are the NLO effective Wilson coefficients. In this work, we take

\[
a_1 \approx 0.99 \pm 0.37i, \quad a_2 \approx 0.19 - 0.11i, \quad a_3 \approx -0.002 + 0.004i, \quad a_5 \approx 0.0054 - 0.005i, \]

\(^1\) Note that since \( K^+ K^- K_S \) is not a pure \( CP \) eigenstate, we define \( \Delta \sin 2\beta_{\text{eff}} \equiv \sin 2\beta_{\text{eff}} - \sin 2\beta_{J/\psi K} \) with \( \sin 2\beta_{\text{eff}} = -S_f/\eta_f \). In general, the relation \( \Delta S_f = \Delta \sin 2\beta_{\text{eff}} \) holds for the final state with fixed \( CP \) parity.
$$a^4_1 \approx -0.03 - 0.02i, \quad a^4_2 \approx -0.04 - 0.008i, \quad a^5_1 \approx -0.06 - 0.02i, \quad a^5_2 \approx -0.06 - 0.006i,$$

$$a^6_7 \approx 0.54 \times 10^{-4}i, \quad a^6_8 \approx (4.5 - 0.5i) \times 10^{-4}, \quad a^6_9 \approx (4.4 - 0.3i) \times 10^{-4},$$

$$a^6_9 \approx -0.010 - 0.0002i, \quad a^6_{10} \approx (-58.3 + 86.1i) \times 10^{-5}, \quad a^6_{10} \approx (-60.3 + 88.8i) \times 10^{-5},$$

for typical $a_i$ at the renormalization scale $\mu = m_s/2 = 2.1$ GeV which we are working on.

Applying Eqs. 2.1, 2.2 and the equation of motion, we obtain the $\bar{B}^0 \rightarrow K^+ K^- \bar{K}^0$ decay amplitude as

$$\langle \bar{K}^0 \rangle^{K^+ K^- (T_P \bar{B})} = \langle K^+ \bar{K}^0 \rangle ((\bar{u}b)v_{A}|\bar{B}^0) \langle K^- (\bar{s}u)v_{A}|0 \rangle \left[ a_1 \beta_{pm} + a^4_1 + a^5_1 - (a^4_1 + a^5_1)\gamma_{} \right] + \langle K^0 (\bar{s}b)v_{A}|\bar{B}^0 \rangle \langle K^+ K^- (\bar{u}u)v_{A}|0 \rangle \left[ a_2 \beta_{pm} + a_3 + a_5 + a_7 + a_9 \right] + \langle K^0 (\bar{s}b)v_{A}|\bar{B}^0 \rangle \langle K^+ K^- (\bar{u}u)v_{A}|0 \rangle \left[ a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right]$$

$$+ \langle K^0 (\bar{s}b)v_{A}|\bar{B}^0 \rangle \langle K^+ K^- (\bar{s}s)v_{A}|0 \rangle \left[ -2a^6_1 + a^5_1 \right] + \langle K^+ K^- (\bar{s}s)v_{A}|0 \rangle \langle (\bar{d}d)v_{A}|\bar{B}^0 \rangle \left( a^5_1 - \frac{1}{2}a^5_1 \right) + (K^+ K^- (\bar{s}g)v_{A}|0 \rangle \langle (\bar{d}g)v_{A}|\bar{B}^0 \rangle (2.4)$$

with $r_\chi = 2m_2^2/(m_s m_\kappa)$. In the factorization terms, the $K\bar{K}$ pair can be produced through a transition from the $\bar{B}$ meson or can be created from vacuum through $V$ and $S$ operators. There exist two weak annihilation contributions, where the $\bar{B}$ meson is annihilated and a final state with three kaons is created. Note that the OZI suppressed matrix element $\langle K^+ K^- | (\bar{d}d)v_{A}|0 \rangle$ is included in the factorization amplitude since it could be enhanced through the long-distance pole contributions via the intermediate vector mesons such as $\rho^0$ and $\omega$.

To evaluate the above amplitude, we need to consider the $\bar{B} \rightarrow K\bar{K}$, $0 \rightarrow K\bar{K}$ and $0 \rightarrow \bar{K}KK$ matrix elements, the so-called two-meson transition, two-meson and tree-meson creation matrix elements in addition to the usual one-meson transition and creation ones. The two-meson transition matrix element $\langle K^0 (p_1) K^+ (p_2) | (\bar{u}b)v_{A}|\bar{B}^0 \rangle$ has the general expression

$$\langle K^0 (p_1) K^+ (p_2) | (\bar{u}b)v_{A}|\bar{B}^0 \rangle = i\gamma (p_B - p_1 - p_2) - i\gamma (p_2 - p_1) \mu + h \epsilon_{\gamma \alpha \beta \gamma} (p_2 + p_1) \alpha (p_2 - p_1) \beta.$$  

This leads to

$$\langle K^- (p_3) | (\bar{s}u)v_{A}|0 \rangle \langle K^0 (p_1) K^+ (p_2) | (\bar{u}b)v_{A}|\bar{B}^0 \rangle$$

$$= \frac{f_{B_s}}{2} \left[ 2m_2^2 r + (m_B^2 - s_{12} - m_3^2) \omega + (s_{23} - s_{13} - m_2^2 + m_1^2) \omega - \right],$$

where $s_{ij} \equiv (p_i + p_j)^2$. A pole model calculation of the $\bar{B}^0 \rightarrow K^{*0} K^+$ transition matrix element amounts to considering the strong interaction $\bar{B}^0 \rightarrow K^{*0} \bar{B}_s$ followed by the weak transition $\bar{B}_s \rightarrow K^+$ and the result is

$$\left[ \langle K^- (p_3) | (\bar{s}u)v_{A}|0 \rangle \langle K^0 (p_1) K^+ (p_2) | (\bar{u}b)v_{A}|\bar{B}^0 \rangle \right]_{\text{pole}}$$

$$= \frac{f_{B_s}}{f_π} \frac{m_B^2 m_B^2}{s_{23} - m_B^2} F(s_{23}, m_B^2) F^{B_{s}, K}(m_B^2) \left( m_B + s_{23} - m_B^2 - s_{23} \right) \left( 1 - \frac{F_{K_s}(m_B^2)}{F_{K_s}(m_B^2)} \right).$$

5
where \( g \) is a heavy-flavor independent strong coupling which can be extracted from the recent CLEO measurement of the \( D^{*+} \) decay width, \( g = 0.59 \pm 0.01 \pm 0.07 \) [34], and \( F_{0,1}^{B_s K} \) are the \( B_s \rightarrow K \) weak transition from factors in the standard convention [35]. Since \( B_s^* \) can be far from the mass shell, it is necessary to introduce a form factor \( F(s_{23},m_{B_s^*}) \) to take into account the off-shell effect of the \( B_s^* \) pole. Following [36], it is parameterized as

\[
F(s_{23},m_{B_s^*}) = (\Lambda^2 - m_{B_s^*}^2)/(\Lambda^2 - s_{23}) \times \left[ m_1^2 + m_3^2 - s_{13} + \frac{(s_{23} - m_2^2 + m_3^2)(m_2^2 - s_{23} - m_1^2)}{2m_{B_s^*}^2} \right],
\]

(2.7)

where \( s \) is a heavy-flavor independent strong coupling which can be extracted from the recent CLEO measurement of the \( D^{*+} \) decay width, \( s = 0.59 \pm 0.01 \pm 0.07 \) [34], and \( F_{0,1}^{B_s K} \) are the \( B_s \rightarrow K \) weak transition from factors in the standard convention [35]. Since \( B_s^* \) can be far from the mass shell, it is necessary to introduce a form factor \( F(s_{23},m_{B_s^*}) \) to take into account the off-shell effect of the \( B_s^* \) pole. Following [36], it is parameterized as

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\]

(2.7)

where \( \Lambda \) is the cut-off parameter chosen to be \( \Lambda = m_{B_s^*} + \Lambda_{QCD} \).

It is worth making a digression for a moment. In principle, one can apply HMChPT twice to evaluate the form factors \( r, \omega_+ \) and \( \omega_- \) [33]. However, this will lead to too large decay rates in disagreement with experiment [32]. This is because the use of HMChPT is reliable only in the kinematic region where \( K^+ \) and \( \bar{K}^0 \) are soft. Therefore, the available phase space where chiral perturbation theory is applicable is very limited. If the soft meson result is assumed to be applicable to the whole Dalitz plot, the decay rate will be greatly overestimated. Therefore, we employ the pole model to evaluate the aforementioned form factors. We shall apply HMChPT only once to the \( \bar{B}^0 \bar{K}^0 B_s^* \) strong vertex and introduce a form factor to take care of the momentum dependence of the strong coupling.

The resonant pole contributions to the form factors \( r, \omega_\pm \) and \( h \) can be worked out from Eq. (2.7). In principle, there are also nonresonant contributions to these form factors. It turns out that the leading nonresonant contribution can be determined as follows. We notice that the same \( \bar{B} \rightarrow K\bar{K} \) two-meson transition matrix element also appears in the decay \( B^- \rightarrow D^0K^0K^- \) under factorization [21]. The data favors a \( 1^- \) configuration in the \( K^0K^- \) pair [37]. The corresponding two-meson transition matrix element is dominated by the \( \omega_- \) term. Following [21] we shall include a nonresonant contribution to \( \omega_- \) parametrized as

\[
\omega_-^{NR} = \kappa \frac{2p_B \cdot p_2}{s_{12}},
\]

(2.8)

and employ the \( B^- \rightarrow D^0K^0K^- \) data and apply isospin symmetry to the \( \bar{B} \rightarrow K\bar{K} \) matrix elements to determine the unknown parameter \( \kappa \). The denominator in the above parametrization is inspired by the QCD counting rule which gives rise to a \( 1/s_{12} \) asymptotic behavior,\(^2\) while the numerator \( p_B \cdot p_2 = m_B E_{K^+} \) is motivated by the observation that \( K^+ \) contains an energetic \( u \) quark coming from the \( b \rightarrow u \) transition.

The matrix elements involving 3-kaon creation are given by [32]

\[
\langle \bar{K}^0(p_1)K^+(p_2)K^-(p_3)|(sd)_{V-A}|0\rangle\langle 0|(db)_{V-A}|\bar{B}^0\rangle \approx 0,
\]

\[
\langle \bar{K}^0(p_1)K^+(p_2)K^-(p_3)|s\gamma_5d|0\rangle\langle 0|d\gamma_5b|\bar{B}^0\rangle = v\frac{f_{Bm_B^2}}{m_B} \left( 1 - \frac{s_{13} - m_1^2 - m_3^2}{m_B^2 - m_K^2} \right) F_{KKK}(m_B^2),
\]

(2.9)

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\(^2\) As explained in [21], at least two hard gluon exchanges are needed: one creating the \( s\bar{s} \) pair in \( \bar{K}^0K^+ \), the other kicking the spectator to catch up with the energetic \( s \) quark to form the \( K \) meson. This gives rise to a \( 1/s_{12}^2 \) asymptotic behavior.
where
\[ v = \frac{m_{K^+}^2}{m_u + m_s} = \frac{m_{K^-}^2}{m_s - m_d}, \]  
(2.10)
characterizes the quark-order parameter \( \langle \bar{q}q \rangle \) which spontaneously breaks the chiral symmetry. Both relations in Eq. (2.9) are originally derived in the chiral limit and hence the quark masses appearing in Eq. (2.10) are referred to the scale \( \sim 1 \text{ GeV} \). The first relation reflects helicity suppression which is expected to be even more effective for energetic kaons. For the second relation, we introduce the form factor \( F^{KKK} \) to extrapolate the chiral result to the physical region. Following \( \chi \text{PT} \) we shall take \( F^{KKK}(q^2) = 1/[1 - (q^2/\Lambda^2)] \) with \( \Lambda = 0.83 \text{ GeV} \) being a chiral symmetry breaking scale.

We now turn to the 2-kaon creation matrix element which can be expressed in terms of time-like kaon current form factors as
\[
\langle K^+(p_{K^+})K^-(p_{K^-})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^+} - p_{K^-})_\mu F_q^{K^+K^-},
\]
\[
\langle K^0(p_{K^0})\bar{K}^0(p_{\bar{K}^0})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^0} - p_{\bar{K}^0})_\mu F_{q\bar{q}}^{K^0\bar{K}^0}.
\]
(2.11)
The weak vector form factors \( F_q^{K^+K^-} \) and \( F_{q\bar{q}}^{K^0\bar{K}^0} \) can be related to the kaon electromagnetic (e.m.) form factors \( F_{em}^{K^+K^-} \) and \( F_{em}^{K^0\bar{K}^0} \) for the charged and neutral kaons, respectively. Phenomenologically, the e.m. form factors receive resonant and nonresonant contributions and can be expressed by
\[
F_{em}^{K^+K^-} = F_\rho + F_\omega + F_\phi + F_{NR}, \quad F_{em}^{K^0\bar{K}^0} = -F_\rho + F_\omega + F_\phi + F_{NR}'.
\]
(2.12)
It follows from Eqs. (2.11) and (2.12) that
\[
F_{u}^{K^+K^-} = F_{d}^{K^0\bar{K}^0} = F_\rho + 3F_\omega + \frac{1}{3}(3F_{NR} - F_{NR}'),
\]
\[
F_{s}^{K^+K^-} = F_{u}^{K^0\bar{K}^0} = -F_\rho + 3F_\omega,
\]
\[
F_{s}^{K^+K^-} = F_{s}^{K^0\bar{K}^0} = -3F_\phi - \frac{1}{3}(3F_{NR} + 2F_{NR}'),
\]
(2.13)
where use of isospin symmetry has been made.

The resonant and nonresonant terms in Eq. (2.12) can be parametrized as
\[
F_h(s_{23}) = \frac{c_h}{m_h^2 - s_{23} - im_h}, \quad F_{NR}^{(t)}(s_{23}) = \left( \frac{x_1^{(t)}}{s_{23}} + \frac{x_2^{(t)}}{s_{23}} \right) \left[ \ln \left( \frac{s_{23}}{\Lambda^2} \right) \right]^{-1},
\]
(2.14)
with \( \bar{\Lambda} \approx 0.3 \text{ GeV} \). The expression for the nonresonant form factor is motivated by the asymptotic constraint from pQCD, namely, \( F(t) \to (1/t)[\ln(t/\bar{\Lambda}^2)]^{-1} \) in the large \( t \) limit. The unknown parameters \( c_h, x_1, x_2 \) are fitted from the kaon e.m. data, giving the best fit values (in units of GeV\(^2\) for \( c_h \))
\[
c_\rho = 3c_\omega = c_\phi = 0.363, \quad c_{\rho(1450)} = 7.98 \times 10^{-3}, \quad c_{\rho(1700)} = 1.71 \times 10^{-3},
\]
\[
c_{\omega(1420)} = -7.64 \times 10^{-2}, \quad c_{\omega(1650)} = -0.116, \quad c_{\phi(1680)} = -2.0 \times 10^{-2},
\]
(2.15)
and
\[
x_1 = -3.26 \text{ GeV}^2, \quad x_2 = 5.02 \text{ GeV}^4, \quad x_1' = 0.47 \text{ GeV}^2, \quad x_2' = 0.
\]
(2.16)
Note that the form factors $F_{\rho,\omega,\phi}$ in Eqs. (2.12) and (2.13) include the contributions from the vector mesons $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\omega(782)$, $\omega(1420)$, $\omega(1650)$, $\phi(1020)$ and $\phi(1860)$. It is interesting to note that (i) the fitted values of $c_V$ are very close to the vector meson dominance expression $g_{V\gamma}g_{VKK}$ for $V = \rho, \omega, \phi$, where $g_{V\gamma}$ is the e.m. coupling of the vector meson defined by $\langle V|\bar{q}q|0\rangle = g_{V\gamma}\epsilon_V^\gamma$ and $g_{VKK}$ is the $V \rightarrow KK$ strong coupling with, $-g_{\rho KK}K^- \simeq g_{\rho KK}/\sqrt{2} = g_{\omega KK}/\sqrt{2} \simeq 3.03$, and (ii) the vector-meson pole contributions alone yield $F_s^{K^+K^-}(0) \approx 1, -1$ and $F_d^{K^+K^-}(0) \approx 0$ as the charged kaon does not contain the valence $d$ quark. The matrix element in the decay amplitude relevant for our purpose then has the expression

$$
\langle K^0(p_1)|(\bar{s}b)_{V=A}\bar{B}^0\rangle \langle K^+(p_2)K^-(p_3)|(\bar{q}q)_{V=A}|0\rangle = (s_{12} - s_{13})F_1^{BK}(s_3)F_q^{K^+K^-}(s_3). \quad (2.17)
$$

We also need to specify the 2-body matrix element $\langle K^+K^-|\bar{s}s|0\rangle$ induced from the scalar density. It receives nonresonant and non-resonant contributions:

$$
\langle K^+(p_2)K^-(p_3)|\bar{s}s|0\rangle \equiv f_{s}^{K^+K^-}(s_{23}) = \sum_{i} \frac{m_i\tilde{f}_i g_i^{i-KK}}{m_i^2 - s_{23} - im_i\Gamma_i} + f_{s}^{NR},
$$

$$
f_{s}^{NR} = \frac{v}{3} (3F_{NR} + 2F_{NR}^3) + \frac{\sigma}{s_{23}} \left[ \ln \left( \frac{s_{23}}{\Lambda^2} \right) \right]^{-1}, \quad (2.18)
$$

where the scalar decay constant $\tilde{f}_i$ is defined in $\langle i|\bar{s}s|0\rangle = m_i\tilde{f}_i$, $g_i^{i-KK}$ is the $i \rightarrow KK$ strong coupling, and the nonresonant terms are related to those in $F_s^{K^+K^-}$ through the equation of motion. The main scalar meson pole contributions are those that have dominant $\bar{s}s$ content and large coupling to $KK$. It is found in [41] that among the $f_0$ mesons, only $f_0(980)$ and $f_0(1530)$ have the largest couplings with the $KK$ pair. Note that $f_0(1530)$ is a very broad state with the width of order 1 GeV [41]. To proceed with the numerical calculations, we use $g_{f_0(980)}^{KK} = 1.5$ GeV, $g_{f_0(1530)}^{KK} = 3.18$ GeV, $\Gamma_{f_0(980)} = 80$ MeV, $\Gamma_{f_0(1530)} = 1.160$ GeV [41], $\tilde{f}_{f_0(980)}(\mu = m_0/2) \approx 0.39$ GeV [42] and $\tilde{f}_{f_0(1530)} \approx \tilde{f}_{f_0(980)}$. The sign of the resonant terms is fixed by $f_{s}^{K^+K^-}(0) = v$ from a chiral perturbation theory calculation (see, for example, [43]). It should be stressed that although the nonresonant contributions to $F_s^{KK}$ and $F_s^{KK}$ are related through the equation of motion, the resonant ones are different and not related a priori. To apply the equation of motion, the form factors should be away from the resonant region. In the large $s_{23}$-region, the nonresonant contribution dominated by the $1/s_{23}$ term is far away from the resonant one. In contrast, the $1/s_{23}^2$ term dominates in the low $s_{23}$-region where resonant contributions cannot be ignored. The $1/s_{23}^2$ term in $F_s$ is not necessarily conveyed to $F_S$ through the equation of motion.

3 The vector meson e.m. couplings are given by $g_{\rho\gamma} = e_s m_\rho f_\rho$, $g_{\omega\gamma} = [(e_u - e_d)/\sqrt{2}]m_\rho f_\rho$ and $g_{\omega\gamma} = [(e_u + e_d)/\sqrt{2}]m_\omega f_\omega$ where $e_q$ is the quark’s charge and $f_\gamma$ is the vector decay constant.

4 The sign convention is fixed by using $\langle M(q\bar{q}',p)|\bar{M}(q\bar{q}',p')\bar{q}_{\gamma\rho}q|0\rangle = \langle M(q\bar{q}',p)|\bar{q}_{\gamma\rho}q|\bar{M}(q\bar{q}',p')\rangle = (p - p')_{\mu}F_q^{MM}$. The use of equations of motion also leads to

$$
f_{s}^{K^+K^-} = -v F_s^{K^+K^-}. \quad (2.19)
$$

Note that the pole contribution to $F_s^{K^+K^-}$ should be dropped in the above relation as it applies only to nonresonant contributions.
Hence, the $1/s_{23}^2$ term in Eq. (2.18) is undetermined and a new parameter $\sigma$, which is expected to be of similar size as $x_2$, is assigned and will be determined later by fitting to the data. The corresponding matrix element is now given by

$$
\langle K^0(p_1)|\bar{s}b|\overline{B}^0\rangle\langle K^+(p_2)K^-(p_3)|\bar{s}s|0\rangle = \frac{m_B^2 - m_K^2}{m_b - m_s} F_{B0}^{BK}(s_{23}) f_s^{K^+K^-}(s_{23}).
$$

(2.20)

Collecting all the relevant matrix elements evaluated above, we are ready to compute the amplitude $A(\overline{B}^0 \rightarrow K_{S(L)}K^+K^-) = \pm A(\overline{B}^0 \rightarrow \overline{B}^0 K^+K^-)/\sqrt{2}$. Since under CP-conjugation we have $K_S(\bar{p}_1) \rightarrow K_S(\bar{p}_1)$, $K^+(\bar{p}_2) \rightarrow K^-(\bar{p}_2)$ and $K^-(\bar{p}_3) \rightarrow K^+(\bar{p}_3)$, the $\overline{B}^0 \rightarrow K_SK^+K^-$ amplitude can be decomposed into CP-odd and CP-even components

$$
A[\overline{B}^0 \rightarrow K_S(p_1)K^+(p_2)K^-(p_3)] = A(s_{12},s_{13},s_{23}) = A_{CP^-} + A_{CP^+},
$$

(2.21)

Correspondingly, we have

$$
\Gamma = \Gamma_{CP^+} + \Gamma_{CP^-},
$$

$$
\Gamma_{CP^\pm} = \frac{1}{(2\pi)^3 \frac{1}{32m_B^3}} \int |A_{CP^\pm}|^2 ds_{12} ds_{13} = \frac{1}{(2\pi)^3 \frac{1}{32m_B^3}} \int |A_{CP^\pm}|^2 ds_{12} ds_{23}.
$$

(2.22)

The vanishing cross terms due to the interference between CP-odd and CP-even components can be easily seen from the (anti)symmetric properties of the amplitude and the integration variables under the interchange of $s_{12} \leftrightarrow s_{13}$. Similar relations hold for the conjugated $B^0$ decay rate $\overline{\Gamma}$. The CP-even fraction $f_+$ is defined by

$$
f_+ \equiv \frac{\Gamma_{CP^+} + \Gamma_{CP^+}}{\Gamma + \Gamma_{CP^-}} \phi_{K_S} \text{ excluded.}
$$

(2.23)

Note that results for the $K^+K^-K_L$ mode are identical to the $K^+K^-K_S$ ones with the CP eigenstates interchanged. For example, results for $(K^+K^-K_L)_{CP^+}$ and hence $f_+$ in $K^+K^-K_S$ corresponds to $f_-$ in $K^+K^-K_L$.

We next turn to the $\overline{B}^0 \rightarrow K_SK_SK_S, K_SK_SK_L$ decays. The decay amplitudes are given by

$$
A[\overline{B}^0 \rightarrow K_S(p_1)K_S(p_2)K_{S,L}(p_3)] = \left(\frac{1}{2}\right)^{3/2} \left\{ \pm A[\overline{B}^0 \rightarrow K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)]
$$

$$
+ A[\overline{B}^0 \rightarrow K^0(p_2)\overline{K}^0(p_3)\overline{K}^0(p_1)]
$$

$$
+ A[\overline{B}^0 \rightarrow K^0(p_3)\overline{K}^0(p_1)\overline{K}^0(p_2)] \right\},
$$

(2.24)

with

$$
A[\overline{B}^0 \rightarrow K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)] = \frac{G_F}{\sqrt{2}} \sum_{\bar{u},c} \lambda_p \left[ \langle K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)\rangle \langle \bar{s}d \rangle_{V-A} \langle \overline{B}^0 \rangle \langle \overline{K}^0(p_2)\overline{K}^0(p_3)\rangle \langle \bar{s}d \rangle_{V-A} \langle 0 \rangle 
$$

$$
+ \langle K^0(p_1)\overline{K}^0(p_3)\overline{K}^0(p_2)\rangle \langle \bar{s}d \rangle_{V-A} \langle \overline{B}^0 \rangle \langle \overline{K}^0(p_1)\overline{K}^0(p_2)\rangle \langle \bar{s}d \rangle_{V-A} \langle 0 \rangle \right]
$$

$$
\times \left( a_{G}^{p} + \frac{1}{2} a_{T}^{p} - \left( a_{G}^{p} - \frac{1}{2} a_{T}^{p} \right) r_{\chi} \right)
$$

$$
+ \left[ \langle \overline{K}^0(p_2)\overline{s}b \rangle \langle \overline{B}^0 \rangle \langle K^0(p_1)\overline{K}^0(p_3)\rangle \bar{s}s \langle 0 \rangle \right]
$$

(2.25)
where the last term will not contribute to the purely CP-even decay $B^0 \to K_SK_SK_S$. Decay rates for the $K_SK_SK_S$ and $K_SK_SK_L$ modes can be obtained from Eq. (2.22) with an additional factor of $1/3!$ and $1/2!$, respectively, for identical particles in the final state.

We now consider the $CP$ asymmetries for $B^0 \to K^+K^-K_{S(L)}$, $K_SK_SK_{S(L)}$ decays. The direct $CP$ asymmetry and the mixing induced $CP$ violation are defined by

$$ A_{KKK} = \frac{\Gamma - \Gamma} {\Gamma + \Gamma}, $$

$$ S_{KKK,CP_\pm} = \frac{2 \int |e^{-2i\beta} A_{CP_\mp} \bar{A}_{CP_\pm}| ds_{12} ds_{23}} {\int |A_{CP_\mp}|^2 ds_{12} ds_{23} + \int |A_{CP_\pm}|^2 ds_{12} ds_{23}}, $$

$$ S_{KKK} = \frac{2 \int |e^{-2i\beta} \bar{A}^*| ds_{12} ds_{23}} {\int |A|^2 ds_{12} ds_{23} + \int |\bar{A}|^2 ds_{12} ds_{23}}, $$

where $\bar{A}$ is the decay amplitude of $B^0 \to K^+K^-K_{S(L)}$ or $K_SK_SK_{S(L)}$. For the $K^+K^-K_S$ mode, it is understood that the contribution from $\phi K_S$ is excluded. It is expected in the SM that $S_{KKK,CP_+} \equiv \sin 2\beta_{\text{eff}} \approx \sin 2\beta$, $S_{KKK,CP_-} \approx -\sin 2\beta$ and hence $S_{KKK} \approx -(2f_+ - 1) \sin 2\beta$.\(^6\)

### III. NUMERICAL RESULTS AND DISCUSSIONS

To proceed with the numerical calculations, we need to specify the input parameters. For the CKM matrix elements, we use the Wolfenstein parameters $A = 0.825$, $\lambda = 0.22622$, $\bar{\rho} = 0.207$ and $\bar{\eta} = 0.340$, corresponding to $(\sin 2\beta)_{\text{CKM}} = 0.724$.\(^{14}\) For $B \to K$ form factors we shall use those derived in the covariant light-front quark model \(^{45}\) with the assigned error to be 0.03, namely, $F_{0,1}^{DKK}(0) = 0.35 \pm 0.03$. The parameter $\kappa$ in Eq. (2.28) is determined from the $B^- \to D^0 K^0 K^-$ data. From the measured branching ratio $B(B^- \to D^0 K^0 K^-) = (5.5 \pm 1.4 \pm 0.8) \times 10^{-4}$\(^{37}\), we obtain $\kappa = 3.1^{+5.1}_{-1.8} \text{ GeV}$ where use of $a_1^{DKK} = 0.935$ and $a_2^{DKK}(\approx a_2^{D\rho}) = 0.4 \pm 0.2$ has been made\(^{21}\). For the quark masses and the unitarity angle $\gamma$, we shall use $m_b(m_b) = 4.2 \text{ GeV}$, $m_s(m_b/2) = 80 \pm 20 \text{ MeV}$ and $\gamma = (58.6 \pm 7)\(^{14}\). The $K_SK_SK_S$ rate sensitive to the parameter $\sigma$

\(^{6}\) Writing the $CP$-conjugated decay amplitude as $\bar{A} = \bar{A}_{CP_+} + \bar{A}_{CP_-}$, we have $\bar{A}_{CP_\pm} = \mp A_{CP_\pm}$ with $\lambda_p \to \lambda_p^*$. This leads to $S_{KKK,CP_-} \approx -S_{KKK,CP_+}$.\(^{44}\)
TABLE II: Branching ratios for $B^0 \rightarrow K^+K^-K_S, K_SK_SK, K_SK_SK_L$ decays and the fraction of $CP$-even contribution to $B^0 \rightarrow K^+K^-K_S$, $f_+$ [see Eq. (2.23)]. The branching ratio of $CP$-odd $K^+K^-K_S$ with $\phi K_S$ excluded is shown in parentheses. Results for $(K^+K^-K_L)_{CP\mp}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. Theoretical errors correspond to the uncertainties in (i) $\kappa$, (ii) $m_s$, $F_0^{BK}$ and $\sigma$ (constrained by the $K_SK_SK_S$ rate), and (iii) $\gamma$.

<table>
<thead>
<tr>
<th>Final State</th>
<th>$B(10^{-6})_{\text{theory}}$</th>
<th>$B(10^{-6})_{\text{expt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+K^-K_S$</td>
<td>$7.33^{+8.38+2.31+0.70}_{-1.08-1.59-0.10}$</td>
<td>$12.4 \pm 1.2$</td>
</tr>
<tr>
<td>$(K^+K^-K_S)_{CP+}$</td>
<td>$5.45^{+5.29+1.48+0.05}_{-0.65-1.13-0.06}$</td>
<td></td>
</tr>
<tr>
<td>$(K^+K^-K_S)_{CP-}$</td>
<td>$1.88^{+3.08+0.83+0.04}_{-0.33-0.46-0.04}$</td>
<td>$(0.48^{+2.98+0.54+0.03}_{-0.40-0.22-0.03})$</td>
</tr>
<tr>
<td>$K_SK_SK_S$</td>
<td>input</td>
<td>$6.2 \pm 1.2$</td>
</tr>
<tr>
<td>$K_SK_SK_L$</td>
<td>$5.74^{+6.02+2.24+0.02}_{-0.88-1.40-0.03}$</td>
<td></td>
</tr>
<tr>
<td>$f_+^{\text{theory}}$</td>
<td>$f_+^{\text{expt}}$</td>
<td></td>
</tr>
<tr>
<td>$K^+K^-K_S$</td>
<td>$0.92^{+0.06+0.04+0.00}_{-0.16-0.08-0.00}$</td>
<td>$0.91 \pm 0.07$</td>
</tr>
<tr>
<td>$K^+K^-K_L$</td>
<td>$0.92^{+0.06+0.04+0.00}_{-0.16-0.08-0.00}$</td>
<td>$0.92 \pm 0.37$</td>
</tr>
</tbody>
</table>

TABLE III: Mixing-induced and direct $CP$ asymmetries $\sin 2\beta_{\text{eff}}$ (top) and $A_f$ (in %, bottom), respectively, in $B^0 \rightarrow K^+K^-K_S$ and $K_SK_SK_S$ decays. Results for $(K^+K^-K_L)_{CP\mp}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. Experimental results are taken from Table I.

<table>
<thead>
<tr>
<th>Final State</th>
<th>$\sin 2\beta_{\text{eff}}$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K^+K^-K_S)_{\phi K_S}$ excluded</td>
<td>$0.749^{+0.080+0.024+0.004}_{-0.013-0.011-0.015}$</td>
<td>$0.57^{+0.18}_{-0.17}$</td>
</tr>
<tr>
<td>$(K^+K^-K_S)_{CP+}$</td>
<td>$0.770^{+0.113+0.040+0.002}_{-0.031-0.023-0.013}$</td>
<td></td>
</tr>
<tr>
<td>$(K^+K^-K_L)_{\phi K_L}$ excluded</td>
<td>$0.749^{+0.080+0.024+0.004}_{-0.013-0.011-0.015}$</td>
<td>$0.09 \pm 0.34$</td>
</tr>
<tr>
<td>$K_SK_SK_S$</td>
<td>$0.748^{+0.000+0.000+0.007}_{-0.000-0.000-0.018}$</td>
<td>$0.65 \pm 0.25$</td>
</tr>
<tr>
<td>$K_SK_SK_L$</td>
<td>$0.748^{+0.000+0.000+0.007}_{-0.000-0.000-0.018}$</td>
<td></td>
</tr>
<tr>
<td>$A_f(%)$</td>
<td>Expt.</td>
<td></td>
</tr>
<tr>
<td>$(K^+K^-K_S)_{\phi K_S}$ excluded</td>
<td>$0.16^{+0.95+0.29+0.01}_{-0.11-0.32-0.02}$</td>
<td>$-8 \pm 10$</td>
</tr>
<tr>
<td>$(K^+K^-K_S)_{CP+}$</td>
<td>$-0.09^{+0.73+0.16+0.01}_{-0.00-0.27-0.01}$</td>
<td></td>
</tr>
<tr>
<td>$(K^+K^-K_L)_{\phi K_L}$ excluded</td>
<td>$0.16^{+0.95+0.29+0.01}_{-0.11-0.32-0.02}$</td>
<td>$-54 \pm 24$</td>
</tr>
<tr>
<td>$K_SK_SK_S$</td>
<td>$0.74^{+0.02+0.00+0.005}_{-0.06-0.01-0.06}$</td>
<td>$31 \pm 17$</td>
</tr>
<tr>
<td>$K_SK_SK_L$</td>
<td>$0.77^{+0.12+0.08+0.06}_{-0.28-0.11-0.07}$</td>
<td></td>
</tr>
</tbody>
</table>

in Eq. (2.18) is used to determine $\sigma = (-10.4^{+5.4}_{-4.8})$ GeV$^4$, where the errors include the uncertainties in the $K_SK_SK_S$ decay rate, the strange quark mass and the $F_0^{BK}$ form factor.

Results for the decay rates and $CP$ asymmetries in $B^0 \rightarrow K^+K^-K_{S(L)}, K_SK_SK_{S(L)}$ are exhibited in Table III and Table III respectively. The theoretical errors shown are from the uncertainties in (i) the parameter $\kappa$ which governs the nonresonant contribution to the form factor
Since the central value of our $f_{\text{ap}}$ term is reduced significantly. For the first error, we note that the larger the value of $\text{CP}$ the larger the uncertainty in $\beta$ fraction can see from Table II, the predicted rates for $K$ are dominated by the sizable error in $\kappa$ on rates are somewhat smaller than the experimental ones. Theoretical errors on the branching ratios are in accordance with the data within errors, though the theoretical central values on rates are somewhat smaller than the experimental ones. Theoretical errors on the branching ratios are dominated by the sizable error in $\kappa$ and the uncertainty in the strange quark mass as the penguin term $a_6 r_\chi$ and the parameter $v$ are very sensitive to $m_s$. Note that the second error in rates (including the contribution from the uncertainty in $\sigma$) are constrained from the $K_S K_S K_S$ rate and hence are reduced significantly. For the first error, we note that the larger the value of $|\kappa|$ we have, the larger rate on $\text{CP}$-odd $K^+ K^- K_S$ is obtained, leading to a smaller value of $f_+(K^+ K^- K_S)$. Since the central value of our $f_+(K^+ K^- K_S)$ agrees well with data, $\kappa$ is preferred to be around its central value.

The $K^+ K^-$ mass spectra of the $B^0 \to K^+ K^- K_S$ decay from $\text{CP}$-even and $\text{CP}$-odd contributions are shown in Fig. 1. In the spectra, there are peaks at the threshold and a milder one in the large $m_{K^+ K^-}$ region. For the $\text{CP}$-even part, the threshold enhancement arises from the $f_0(980)K_S$ and the nonresonant $f_S^{K^+ K^-}$ contributions [see Eq. (2.18)], while the peak at large $m_{K^+ K^-}$ comes from the nonresonant two-meson transition $B^0 \to K^+ K_S$ followed by a current produced $K^-$. Since the nonresonant term [Eq. (2.8)] favors a small $m_{K^+ K_S}$ region, the spectrum should peak at the large $m_{K^+ K^-}$ end. For the $\text{CP}$-odd spectrum the bump at the large $m_{K^+ K^-}$ end originates from the same two-meson transition term, while the peak on the lower end corresponds to the $\phi K_S$ contribution, which is also shown in the insert. The full $K^+ K^- K_S$ spectrum is basically the sum of the $\text{CP}$-even and the $\text{CP}$-odd parts. Note that although we include $f_0(1530)K_S$ contribution, its effect is not as prominent as one may expect from the $K^- K^+ K^-$ spectrum where a large $f_X(1500)K^-$ contribution is found [40].
For the mixing-induced $CP$ asymmetry in the $K^+K^-K_S$ mode, we compute the effective $\sin 2\beta$ in two different ways: In one way, we calculate $S$ with $\phi K_S$ excluded in $K^+K^-K_S$ and then apply the relation $S = -(2f_+ - 1)\sin 2\beta_{\text{eff}}$ and the theoretical value of $f_+$ to obtain $\sin 2\beta_{\text{eff}}$. This procedure follows closely the BaBar and Belle method of measuring the effective $\sin 2\beta$. In the other way, we calculate $S$ directly for the $CP$-even $K^+K^-K_S$ and identify $S_{KKK,CP+}$ with $\sin 2\beta_{\text{eff}}$. As for the $K_SK_SK_S$ mode, there is no such ambiguity as it is a purely $CP$-even state. As shown in Table I and Fig. 2, the resulting $\sin 2\beta_{\text{eff}}$ is slightly different in these two different approaches.

The deviation of the mixing-induced $CP$ asymmetry in $B^0 \rightarrow K^+K^-K_S$ and $K_SK_SK_S$ from that measured in $B \rightarrow J/\psi K_S$ (or the fitted CKM’s $\sin 2\beta$ [44]), namely, $\Delta \sin 2\beta_{\text{eff}} \equiv \sin 2\beta_{\text{eff}} - \sin 2\beta_{J/\psi K_S (CKM)}$, is calculated from Table I to be

$$\Delta \sin 2\beta_{K^+K^-K_S} = 0.06^{+0.08}_{-0.02} \times (0.02^{+0.08}_{-0.02})$$

$$\Delta \sin 2\beta_{K_SK_SK_S} = 0.06^{+0.00}_{-0.00} \times (0.02^{+0.00}_{-0.00}).$$

(3.1)

Note that part of the deviation comes from that between the measured $\sin 2\beta_{J/\psi K_S}$ and the fitted CKM’s $\sin 2\beta$. The $K^+K^-K_S$ has a potentially sizable $\Delta \sin 2\beta$, as this penguin-dominated mode is subject to a tree pollution due to the presence of color-allowed tree contributions. For the $K_SK_SK_S$ mode, the central value and the error on $\Delta \sin 2\beta$ are small.

It is instructive to see the dependence of $\sin 2\beta_{\text{eff}}$ on the $K^+K^-$ invariant mass, $m_{K^+K^-} \equiv m_{23} = \sqrt{s_{23}}$. For the phase space integration in Eq. (2.20), for a given $s_{23}$, the upper and lower bounds of $s_{12}$ are fixed. The invariant mass $m_{23}$ is integrated from $m_{23} = m_2 + m_3$ to $m_{23}^{\text{max}} = m_B - m_1$. When the variable $s_{23}$ or $m_{23}$ is integrated from $m_{23}^{\text{max}}$ to a fixed $m_{23}^{\text{max}}$ (of course, $m_{23} < m_{23}^{\text{max}} \leq m_{23}^{\text{max}}$), the effective $\sin 2\beta$ thus obtained is designated as $\sin 2\beta_{\text{eff}}(m_{23}^{\text{max}})$. Fig. 2 shows the plot of $\sin 2\beta_{\text{eff}}(m_{K^+K^-}^{\text{max}})$ versus $m_{K^+K^-}^{\text{max}}$ for $K^+K^-K_S$. Since there are two different methods for the determination of $\sin 2\beta_{\text{eff}}$, the results are depicted in two different curves. It is interesting that $\sin 2\beta(m_{23}^{\text{max}})$ is slightly below $\sin 2\beta_{\text{CKM}}$ at the bulk of the $m_{K^+K^-}$ region and gradually increases and becomes slightly larger than $\sin 2\beta_{\text{CKM}}$ when the phase space is getting saturated. The deviation $\Delta \sin 2\beta_{K^+K^-K_S}$ arises mainly from the large $m_{K^+K^-}$ region.

Direct $CP$ violation is found to be very small in both $K^+K^-K_S$ and $K_SK_SK_S$ modes. It is interesting to notice that direct $CP$ asymmetry in the $CP$-even $K^+K^-K_S$ mode is only of order $10^{-3}$, but it becomes $0.2 \times 10^{-2}$ in $K^+K^-K_S$ with $\phi K_S$ excluded. Since these direct $CP$ asymmetries are so small they can be used as approximate null tests of the SM.

Since direct $CP$ violation in charmless $B$ decays can be significantly affected by final-state rescattering [30], we have studied to what extent indications of possibly large deviations of the mixing-induced $CP$ violation seen in the penguin-induced two-body decay modes from $\sin 2\beta$ determined from $B \rightarrow J/\psi K_S$ can be accounted for by final-state interactions [10]. It is natural to extend the study of final-state rescattering effect on time-dependent $CP$ asymmetries to $B \rightarrow KKK$ decays. Final-state interactions in three-body decays are expected to be much more complicated than the two-body case. For example, the color allowed tree decay $T^0 \rightarrow D_s^{(*)+}D_s^{(*)-}$ can rescatter into a $K^+K^-K_S$ final state, where we have $D_s^{(*)+} \rightarrow K^+D_s^{(*)0}$, $D_s^{(*)-} \rightarrow K_SD_s^{(*)-}$ followed by a $D_s^{(*)0}D_s^{*} \rightarrow K^-$ fusion. These diagrams are too complicated and will not be included in this study.  

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7 In passing we note that these diagrams could have the effect of increasing somewhat our predictions for
FIG. 2: Mixing-induced CP asymmetry $\sin 2\beta_{\text{eff}}(m_{K^+K^-}^{\text{max}})$ (see the text for the definition) versus the invariant mass $m_{K^+K^-}^{\text{max}}$ for $K^+K^-K_S$ with $\phi K_S$ excluded (solid line) and for CP-even $K^+K^-K_S$ (dashed line). When $m_{K^+K^-}^{\text{max}}$ approaches the upper limit $m_B - m_{K_S}$, the whole phase space is saturated and $\sin 2\beta_{\text{eff}}(m_{K^+K^-}^{\text{max}})$ is reduced to the usual $\sin 2\beta_{\text{eff}}$. This result also applies to the $K^+K^-K_L$ mode.

Nevertheless, we attempt to incorporate final state rescattering effects in a simple way by including resonance contributions to the corresponding kaon pairs in the final state \[47\]. We note that another attempt in this direction has recently been made by Furman et al. \[48\]. They considered rescattering of $\pi\pi$ and $K\bar{K}$ pairs in the $\pi\pi$ effective mass range from threshold to 1.1 GeV. While their predicted direct CP asymmetry is very small, the parameter $S$ is found to be $-0.64$ or $-0.77$, depending on the set of penguin amplitudes. However, due to the limitation on phase space, the calculated branching ratios of order $1 \times 10^{-6}$ for $K^+K^-K_S$ and $K_SK_SK_S$ are only small portions of the total experimental rates (see Table II) and, consequently, the predictions of $S$ may be affected when the whole phase space is taken into consideration.

IV. CONCLUSIONS

In the present work we have studied the decay rates and time-dependent CP asymmetries in the decays $B^0 \to K^+K^-K_{S(L)}$ and $K_SK_SK_S(L)$ within the framework of factorization. Our main results are as follows:

1. Resonant and nonresonant contributions to the hadronic matrix elements are carefully investigated. We incorporate final state rescattering effects in a simple way by including resonance contributions to the corresponding kaon pairs in the final state. Instead of applying heavy meson chiral perturbation theory to the matrix element for $B \to KK$, which is valid only the rates of 3K final states. Although these contributions carry negligible CP-odd (weak) phases, they also contribute to the strong phases and hence will tend to dilute our prediction on $\Delta S$ but not necessarily on direct CP asymmetries.

14
for a small kinematic region, we consider the resonant contribution from the $B_s^*$ pole and nonresonant contributions constrained by QCD counting rules.

2. Using the $K_S K_S K_S$ decay rate as an input, the predicted branching ratio of $K^+ K^- K_{S(L)}$ modes and the $CP$-even (-odd) fraction of $B^0 \rightarrow K^+ K^- K_{S(L)}$ are consistent with the data within the theoretical and experimental errors, though the theoretical central values on rates are somewhat smaller than the experimental ones.

3. Owing to the presence of color-allowed tree contributions in $B^0 \rightarrow K^+ K^- K_{S(L)}$, this penguin-dominated mode is subject to a potentially significant tree pollution and the deviation of the mixing-induced $CP$ asymmetry from that measured in $B \rightarrow J/\psi K_S$, namely, $\Delta \sin 2\beta_{K^+ K^- K_{S(L)}} \equiv \sin 2\beta_{K^+ K^- K_{S(L)}} - \sin 2\beta_{J/\psi K_S}$, can be as large as $O(0.10)$. The deviation $\Delta \sin 2\beta_{K^+ K^- K_{S(L)}}$ arises mainly from the large $m_{K^+ K^-}$ region.

4. The $K_S K_S K_{S(L)}$ mode appears theoretically very clean in our picture: The uncertainties in $\Delta \sin 2\beta_{\text{eff}}$ are negligible.

5. Direct $CP$ asymmetries are very small in both $K^+ K^- K_{S(L)}$ and $K_S K_S K_{S(L)}$ modes.

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Note added: After the paper was submitted for publication, BaBar has presented a Dalitz plot study of $B^0 \rightarrow K^+ K^- K_S^0$ decays [49]. The BaBar results constrain the tree contribution (incorporated via Eq. (2.18) in the present work) in rates and, as a result, a small $\Delta \sin 2\beta_{K^+ K^- K_S}$ is preferable.
[31] BaBar Collaboration, B. Aubert et al., hep-ex/0408032.
[49] BaBar Collaboration, B. Aubert et al., [hep-ex/0507094]