Secular Evolution in Mira Variable Pulsations

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ABSTRACT

Stellar evolution theory predicts that asymptotic giant branch stars undergo a series of short thermal pulses that significantly change their luminosity and mass on timescales of hundreds to thousands of years. These pulses are confirmed observationally by the existence of the short-lived radioisotope technetium in the spectra of some of these stars, but other observational consequences of thermal pulses are subtle, and may only be detected over many years of observations. Secular changes in these stars resulting from thermal pulses can be detected as measurable changes in period if the star is undergoing Mira pulsations. It is known that a small fraction of Mira variables exhibit large secular period changes, and the detection of these changes among a larger sample of stars could therefore be useful in evolutionary studies of these stars. The American Association of Variable Star Observers (AAVSO) International Database currently contains visual data for over 1500 Mira variables. Light curves for these stars span nearly a century in some cases, making it possible to study the secular evolution of the pulsation behavior on these timescales. In this paper, we present the results of our study of period change in 547 Mira variables using data from the AAVSO. We use wavelet analysis to measure the period changes in individual Mira stars over the span of available data. By making linear fits to the period versus time measurements, we determined the average rates of period change, $d\ln P/dt$, for each of these stars. We find non-zero $d\ln P/dt$ at the 2-$\sigma$ significance level in 57 of the 547 stars, at the 3-$\sigma$ level in 21 stars, and at the level of 6-$\sigma$ or greater in eight of the 547. The latter eight stars have been previously noted in the literature, and our derived rates of period changes largely agree with published values. The largest and most statistically significant $d\ln P/dt$ are consistent with the rates of period change expected during thermal pulses on the AGB. A number of other stars exhibit non-monotonic period change on decades-long timescales,
the cause of which is not yet known. In the majority of stars, the period variations are smaller than our detection threshold, meaning the available data are not sufficient to unambiguously measure slow evolutionary changes in the pulsation period. It is unlikely that more stars with large period changes will be found among heretofore well-observed Mira stars in the short term, but continued monitoring of these and other Mira stars may reveal new and serendipitous candidates in the future.

Subject headings: stars: AGB and post-AGB — stars: evolution — stars: oscillations — stars: mass loss — stars: variables: Miras — astronomical data bases: miscellaneous

1. Introduction

The asymptotic giant branch (AGB) is an important evolutionary stage for stars of low to intermediate mass. It represents the most luminous stage of these stars’ lives, and the stage during which they begin to return a significant fraction of their mass to the interstellar medium (Willson 2000). The AGB is also the location of the Mira variables, which are high-amplitude, pulsating variable stars; as with several other types of pulsating variables, the Miras are also useful as distance indicators (Feast et al. 2002). Thus observational studies of stars on the AGB, particularly the pulsating stars, provide information useful for several other important astrophysical problems.

The Mira stars are also reasonably easy targets of study. They make up a significant percentage of the catalogued variable stars in the nearby universe (Kholopov et al. 1985). Their large luminosities and large amplitudes make them easy to detect and monitor, and their long periods allow for easy study since observations can be widely spaced in time. Many hundreds of Mira variables have been regularly observed by amateur and professional astronomers for over a century, and the long spans of data collected on these stars now enable the study of secular changes in pulsation behavior. These secular changes can potentially provide useful information on several aspects of Mira variable and AGB evolution, including thermal pulses and pulsation-driven mass-loss.

Mira variables have long been known to have varying periods and other aperiodic behavior. Plakidis and Eddington (Eddington & Plakidis (1929); Plakidis (1932)) investigated the effects of both measurement error and intrinsic cycle-to-cycle variations on the $O-C$ diagrams of Mira stars and LPVs, and Plakidis (1932) states that “...the periods of [R Aql and R Hya] are well-known to decrease.” R Aql and R Hya were again identified as stars with
statistically significant period changes by Sterne & Campbell (1937). The ability to detect period changes increased along with the span of available observations, and by the 1970’s, very large data sets could be used to detect and study such behavior. Since the 1980’s, work in this field has blossomed; for example, Wood & Zarro (1981) noted that W Dra was a candidate for period changes, Alksne & Alksnis (1985) noted a significant period increase in LX Cyg, and Bateson et al. (1988) found the same in BH Cru. The most spectacular case of period change in a Mira star is that of T UMi, described independently by Gál & Szatmáry (1995) and Mattei & Foster (1995). Since then, several surveys and discussions of period change have appeared in the literature, and it is now generally (but not universally) accepted that large period changes are caused by thermal pulses.

The existence of thermal pulses follows from standard stellar evolution models (Wood & Zarro 1981; Iben & Renzini 1983; Boothroyd & Sackmann 1988; Vassiliadis & Wood 1993), and also from the observation of technetium in the spectra of some AGB stars (Lebzelter & Hron 1999). However, the consequences of thermal pulses should also be observable in the pulsation behavior of variable stars on the AGB. The expected rate of period change of the fundamental mode is given by

\[ \frac{d \ln P}{dt} = -0.32 \frac{d \ln M}{dt} + 0.55 \frac{d \ln L}{dt} \]  

(1)

where \( d \ln P, d \ln M, \) and \( d \ln L \) are the period, mass, and luminosity normalized by the average value over the interval \( dt \). This equation is obtained by combining the fundamental mode period-mass-radius relation of Ostlie & Cox (1986) with the relation \( d \log R / d \log L = 0.68 \) from Iben (1984). A similar equation results for overtone pulsations or for constant \( Q = P \sqrt{\rho / \rho_\odot} \), but with somewhat smaller coefficients. The Wood & Zarro (1981) interpretation of R Aql and R Hya as shell flashing giants assumed overtone pulsation, but recent observations make it clear that Miras are fundamental mode pulsators (Perrin et al. 2004; Weiner 2004). For the Mira variables, \( d \ln L / dt \sim -d \ln M / dt \), so both mass loss and luminosity changes will significantly affect pulsation periods on the AGB. During thermal pulses, the rates of mass and luminosity change may increase on secular timescales, causing detectably large changes in the period.

First, thermal pulses will change the stellar luminosity and temperature, and readjust the interior structure of the star on the thermal timescale. Evolution models predict that for a short period of time after the pulse onset (less than 1000 years), large and observable changes will occur to the star. Such changes would manifest themselves as changes in pulsation behavior, because of changes in both the interior sound speed and stellar radius. Mass loss also occurs on the AGB, and may affect pulsations in Mira variables on observable
timescales. First it would cause a decrease in envelope mass, which affects the pulsation period(s) as shown in equation (1). Second, it may also cause an increase in dust and molecular opacity, which would affect the absolute magnitude of the star and change the spectral energy distribution. Third, Iben (1984) evolutionary models (for example) predict that as the mass decreases, the radius increases (as per equation 1 above) during late states of evolution for stars more massive than about 1.16 M⊙. The mass loss rates of these stars can be as high as a few times 10⁻⁵ M⊙/yr, affecting the pulsation period over the course of a century even though the amount of mass lost would be relatively small (a few thousandths of a solar mass).

According to most evolutionary models (Wood & Zarro 1981; Boothroyd & Sackmann 1988; Vassiliadis & Wood 1993), the largest period changes caused by thermal pulses should occur over relatively short periods of time, perhaps a few thousand years at most. The interpulse phases of AGB evolution may last for a few hundred thousand years. If we assume that Mira pulsations can occur during most of the pulse and interpulse phases, then we might expect that a few percent of all observed Mira variables exhibit significant changes to their pulsation behavior over the span of available observations. These changes may manifest themselves as changes to (i) pulsation period, (ii) mean magnitude, and (iii) amplitude and/or light curve shape.

A third possible reason for secular changes in Mira variable pulsations unrelated to evolutionary processes is nonlinear behavior. In most other types of pulsating variable, the pulsations have small enough amplitudes that they do not significantly deviate from strictly periodic behavior, and do not impact the equilibrium structure of the star. This is why linear perturbation theory has been so effective in modeling stellar pulsations in general. However, for the Mira stars this is not necessarily the case. It is well-known that the Miras have cycle-to-cycle variations in period and amplitude, suggesting nonlinear behavior. Nonlinearity can also affect the propagation of atmospheric shocks into regions forbidden by linear theory, where $P > P_{\text{acoustic}}$ (Bowen 1990), leaving open the possibility of additional nonlinear effects. Some work has also been done to determine whether chaos plays a role in Mira pulsations (Cannizzo et al. 1990; Bedding et al. 1998; Kiss & Szatmáry 2002), although not all of these studies confirmed chaotic behavior. The semi-regular and long-period irregular variables are in similar evolutionary stages as the Miras, and nonlinear dynamics and chaos may be at work in these stars as well. Any studies of Mira variable evolution based on pulsation behavior must keep this in mind. More modeling work along these lines is clearly warranted.

In this paper, we present a time-series analysis of 547 Mira variable star light curves from the American Association of Variable Star Observers (AAVSO) International Database (ID), to search for evolutionary secular changes among Mira stars. We use the technique of wavelet
analysis to measure changes in the pulsation periods of these stars over the span of available observations. We then investigate the stars which exhibit statistically significant changes in period to assess whether these changes may have arisen from evolutionary events. In section 2 of this paper, we describe the AAVSO data and our sample selection criteria. In section 3, we describe our wavelet analysis algorithm and procedure for finding statistically significant period variations. In section 4, we describe the results of our time-series analysis and compare our results with those previously published. In section 5, we discuss the implications of our results on our understanding of Mira stars and AGB evolution.

2. Data

In this work, we make use of visual magnitude estimates collected and archived by the AAVSO. These visual magnitude estimates were made primarily by amateur astronomers using small telescopes or binoculars. Because individual stars may have hundreds of different observers contributing observations, the data are necessarily heterogeneous due to variations in the response of the eyes of different individuals and in the type of observing aid used. However, the amount of data available for many of these stars is so large that the resulting averaged light curves are of very high quality. Furthermore, all observers used the same comparison star sequence, which improves the consistency of data taken by different observers. The statistical properties of the data within a given light curve are largely constant over time, so the data may be treated as statistically homogeneous. The data have also undergone a validation process by which clearly discrepant points are flagged, checked by AAVSO staff, and removed from the light curves (if need be) prior to analysis (Waagen & Mattei 2003). Finally, the visual data are averaged into 10-day bins to produce the final light curve which we then analyze.

For this paper, we selected stars for analysis based upon two criteria: the AAVSO ID must contain at least 500 valid data points for the star, and the most recent version of the General Catalogue of Variable Stars (Kholopov et al. 1985) must classify the object as Mira (M) or suspected Mira (M:). Using these criteria, the data for 547 stars were selected from the AAVSO ID.

As an example, Figure 1 shows the 10-day mean light curve of visual observations of S Arietis, an ordinary Mira variable with moderate light curve coverage typical of our sample. The light curve shows both the strengths and weaknesses of our data. The light curves track the behavior of this variable well during most of the past 90 years, particularly in the coverage of maxima and minima which show amplitude variations. However, increasingly poor coverage and larger data gaps earlier in the data set make it more difficult to track
the earliest behavior of this object. Furthermore, even the most recent data are not so well-covered that the variations in times and magnitudes of maxima are exactly defined, and there is still scatter of a few tenths of a magnitude in the brightnesses of minima. Figure 2 shows a portion of the visual light curve overlaid with CCD $V$-band measurements of this star, also taken by AAVSO observers. Although the two data sets agree for a substantial portion of the light curves, there are significant differences in places, notably near JD 2451850 (minimum) and JD 2452250 (maximum). Such differences were noted by Zissell (2003), who compared AAVSO visual observations to calibrated $B$-, $V$-, and $R$-band data taken specifically for transformation purposes, and noted that several factors including color differences between comparison stars and physiological differences between observers can contribute. He also noted that such effects can make transformation between visual magnitude estimates and standard photometric systems very difficult. However, we are most concerned that the visual observations represent the general behavior of the star properly, regardless of whether their absolute calibration is correct, and the visual and CCD data are in clear qualitative agreement in the behavior of the star; the same conclusion was reached by Lebzelter & Kiss (2001). Therefore the visual observations form a valid data set for performing this kind of study.

The intrinsic 1-$\sigma$ errors of visual brightness estimates are approximately 0.1 to 0.3 magnitudes, with larger scatter commonly observed for redder stars. Because the data are not evenly sampled, binning of data may result in varying RMS errors per point along the light curve, but the average RMS error per point is between 0.1 and 0.3 magnitudes. Figure 3 shows the raw light curves for two Mira variables having approximately the same period, but very different spectral types. The carbon Mira WX Cygni shows significantly more scatter in its light curve than does the oxygen Mira U Aurigae. This increased scatter arises from a combination of factors, including differences in ocular red-sensitivity between observers, observations with equipment of different aperture sizes, different observing techniques, and ultimately, the Purkinje effect. For a longer discussion of this topic, we refer the reader to Hallett (1998) and Stanton (1999). Because of the increased scatter observed in redder stars, we expect higher uncertainties in the data for S- and C-type Miras, particularly those with lower amplitudes.

3. Analysis method

To search for period changes, we use a version of the weighted wavelet Z-transform (WWZ) developed by Foster (1996) specifically for analyzing AAVSO data. The WWZ algorithm projects the data onto a set of sine and cosine trial functions (the wave form); a
test sine wave having fixed frequency is fit to the data using the Gaussian wavelet window function

\[ W(\omega, \tau) = \exp(-c\omega^2(t - \tau)^2) \] (2)

as the weighting function of the data, where \( \omega \) is the test frequency, \( \tau \) is the center of the test window, and \( c \) is a tuning constant used to adjust the width of the window. The algorithm was designed with irregularly sampled data in mind, and is analogous to date-compensated Fourier methods such as the Ferraz-Mello (1981) date-compensated DFT, and the Lomb-Scargle periodogram (Scargle 1982). Hawkins et al. (2001) studied the secular evolution of R Cen using the WWZ algorithm, and they showed that WWZ produces similar results to dividing the data into finite segments and performing a date-compensated DFT on each segment. WWZ is preferable to manually dividing the data, as the former automatically selects how much data to include in each iteration based upon the test frequency and the window width, rather than having a fixed span of data at a given time step.

For our tests, we step the wavelet window over the data at time intervals, \( \Delta \tau \), of 500 days, and scan a frequency range of 0.0005 to 0.02 cycles per day (corresponding to periods of 2000 to 50 days respectively). The Gaussian window width is frequency-dependent, such that the number of cycles of the test frequencies appearing within the window is constant. We used a tuning constant, \( c \), of 0.001, to smooth out the cycle-to-cycle variations and emphasize the longer-term trends in pulsation behavior. A value of 0.001 sets the e-folding full-width of the weighting window at approximately 20 cycles; at a distance of ten cycles on either side of the wavelet window center, the data weights have fallen from 1 to 1/e. We tested several values of \( c \) between 0.012 and 0.0001, and found 0.001 gave the best results for our purposes. Values smaller than 0.001 tended to wash out all of the variations including the long-term ones because smaller values of \( c \) result in larger windows. Increasing the value of \( c \) decreased the frequency resolution and increased the scatter in peak frequency, making it difficult to track subtle changes.

As Foster (1996) noted, WWZ is well-suited to detecting period variations but not amplitude or mean magnitude changes. This problem is most acute where the data contain gaps due to solar interference, seasonal weather variations, or lack of observers. For example, the amplitude and mean magnitude are often underestimated due to missed minima; many of the Mira variables in our sample have minima below \( m_{\text{vis}} = 15 \), beyond the capabilities of observers with small- to moderate-sized telescopes. WWZ also underestimates total amplitude even for well-sampled, regular light curves because our particular implementation of the transform assumes purely sinusoidal variations. WWZ detects higher-order Fourier harmonics, but our implementation does not compute the phase information and so the Fourier
harmonics were not combined to determine the total light amplitude.

Following the automated wavelet analysis of the 547 light curves, we inspected the results of each by eye. For our analysis, we require only the peak period as a function of lag time, $\tau$, rather than the full spectrum. We found that 89 of the 547 stars showed either noise spikes or sharp discontinuities in plots of peak period versus time. In most cases, these points are spurious, due to gaps in the data. In a few cases (including DH Cyg, DU Cyg, R Nor, and R Cen), the light curves are or appear to be double-peaked, and the discontinuities result from WWZ selecting the shorter period of the smaller peaks rather than the longer period of the main variation. We removed these points before we analyzed the WWZ output for period changes.

Finally, we estimated the average rate of period change for all the stars in the sample by fitting a straight line through the values of $P(\tau)$, and obtaining $d\ln P/d\tau$ from the slope of the line. In some cases, assuming a linear period change is clearly incorrect. In the unique case of T UMi (discussed below), we fit a line through the epoch of declining period only. For those stars that do not exhibit monotonic period change (such as RU Tau and S Ori) we also compute the largest deviations from the mean period during the span of the observations. For both calculations, we assume that the error in period is the half-width at half-maximum of the $Z$-statistic, $Z(\omega, \tau)$ (Foster 1996). We measured this for several test stars, and found that the half-width at half-maximum was typically on the order of 1-2% of the period. Therefore, we conservatively assumed that the 1-$\sigma$ error bars, $\sigma_P$, of individual measurements of $P(\tau)$ at each timestep are fixed at 0.02$P(\tau)$ for all stars at all times. In reality, $\sigma_P$ varies as a function of the number of data points within a given window because the coverage of the individual light curves is not uniform. However, our use of an average error for simplicity is reasonable; we exclude regions of the light curves where the data are very poorly-covered, and the value of 0.02$P(\tau)$ is likely overly conservative for well-covered light curves. Therefore, we are confident we are not drastically underestimating the errors in period, but may be overestimating them for well-observed stars.

Once the curves of $P(\tau)$ versus $\tau$ were obtained, we made linear fits to each star’s curve and obtained a slope, $dP/d\tau$, standard deviation of the slope, $\sigma_{fit}$, and average period, $\bar{P}$. The rate of period change, $d\ln P/d\tau$, is then defined as the slope divided by the average period, and the significance level was obtained by dividing the slope by $\sigma_{fit}$. Stars with significance levels greater than two were flagged, and those greater than three were considered significant.
4. Results

Our main goal is to search for period changes in Mira stars and determine whether they are caused by evolutionary changes within the star. First, a preliminary discussion of the global properties of the sample is in order. Figure 4 shows the number distribution of periods within our sample. Overall, the sample has a wide distribution in period, with an average period of 307 days. Figure 5 shows the same histogram divided into the three subtypes (M-, S-, and C-type Miras) and those that are unclassified. The majority of the Miras in our sample are M-type (458), with the S-type (35), C-type (32), and unclassified Miras (21) making up a much smaller fraction. The S- and C-type Miras also have longer average periods than the M-types; the S- and C-types have average periods of 373 and 400 days, respectively, while the M-types have an average period of 298 days. The period distributions of the M- and S-type Miras in our sample agree with those of all Miras in the GCVS, both in terms of mean period and of the distribution of short-, medium-, and long-period Miras of each sub-type. However, our sample of C-type Miras is underabundant in short-period stars relative to the GCVS distribution.

Figure 6 shows the measured values of $d \ln P / dt$ versus period for the 547 stars in the sample. The first result is that the majority of stars do not have measurable $d \ln P / dt$ significantly different from zero. Stars on the AGB should spend most of their time in the interpulse phase, where $d \ln P / dt$ is of order $10^{-5}$. In fact, we find the average error ($\sigma_{\text{fit}}$) is ten times larger than this, meaning that the slow increase in period is not generally detectable with our analysis technique. We found 57 stars with $d \ln P / dt$ more than $2\sigma$ different from zero, which we list in Table 1. (The complete Table 1, listing the average periods and rates of period change for all 547 stars is available online.) Twenty-one of these stars had $d \ln P / dt$ significant at the $3\sigma$ level or greater, and eight stars exhibited period changes at the $6\sigma$ level or greater. Plots of period versus time for these stars are shown in Figures 7a and 7b.

The eight stars with the most-significant period changes have all been noted previously in the literature. They are: T UMi, LX Cyg, BH Cru, R Aql, Z Tau, W Dra, R Cen, and R Hya. Because the historical literature for these eight bears summarizing, we discuss and compare our results for each of these stars with those previously published in the following subsections. We then discuss the general results, followed by a short comparison of analysis methods.
4.1. Rapid period changes

4.1.1. T UMi

T Ursae Minoris is the most dramatic case of a Mira star with a period change. Our analysis of the AAVSO visual data indicates a rate of period change of $-8.4 \times 10^{-3} \, \text{yr}^{-1}$, the largest known of any Mira variable. The period change began after the star had been monitored and maintained a constant period for many decades, and size of the period change has been very large (nearly 25 percent). The first note of a modest period change (a decline of 10-15 days) appeared in the *General Catalogue of Variable Stars* (Kholopov et al. 1985), which gave a period of 301 days for the epoch JD2445761. In 1993, a mention of the behavior of T UMi was made on a variable star mailing list, and though no archive of that discussion survives, it did trigger further investigations. Mattei (1994) used a period of 280 days in the calculation of predicted maxima for 1994, indicating that a larger period discrepancy was known and measured by January 1994. The nature of the period change was subsequently investigated independently by Gál & Szatmáry (1995) and Mattei & Foster (1995), who obtained similar results. The most recent published analysis of T UMi was by Szatmáry et al. (2003) who interpret the period variations as arising from a thermal pulse. They used a variety of statistical methods to measure the period change, but their linear fitting method yielded $-3.4 \pm 0.5 \, \text{d} \, \text{yr}^{-1}$, or $-1.19 \pm 0.18 \times 10^{-2} \, \text{yr}^{-1}$ in units of $d \ln P / dt$. The slight disagreement between their value and ours is likely due to their use of a shorter span of data, beginning at JD 2444000 rather than our JD 2440000. Our data range includes the early, shallower portion of the period change onset, and a slightly smaller average rate is therefore expected. Otherwise, the values derived are very similar despite being done with wholly independent data sets, a good indication that our visual data are consistent with other data sets, and that our analysis method is sound. We show our WWZ results for the full span of available AAVSO data for T UMi in Figure 8.

As Szatmáry et al. (2003) state, the onset and rate of period change are entirely consistent with what has been predicted by theoretical evolution models of period change in thermal pulses. However, the modeling of the early stages of thermal pulses is difficult. Most evolution models assume that the local and global thermal time scales are much shorter than the nuclear time scale, which is not the case during thermal pulses. Thus it is difficult to model the early stages of thermal pulses without resorting to thermonuclear hydrodynamic models instead. Wood & Zarro (1981) and Vassiliadis & Wood (1993) do not explicitly state the time steps used in their calculations, but we expect their model accuracy to be lowest during the earliest phases of the thermal pulse in which we believe T UMi to be. The rates of period change for the Vassiliadis & Wood (1993) models are on the order of $10^{-2} \, \text{yr}^{-1}$, consistent with our result. Because of the numerical difficulties in modeling this evolution phase
with standard models, it is difficult to assess whether the observed rate of period change is in quantitative agreement with physical expectation. However, the qualitative behavior of the star is very similar to what is expected during this phase, and changes in theoretical models will likely not produce differences larger than a factor of two. Therefore, we concur with the general consensus that the observed period change in T UMi indicates that it is a star in the earliest phases of a thermal pulse.

4.1.2. LX Cyg

Period change in LX Cyg was first noted by Alksne & Alksnis (1985), and Mattei used periods of 500 days (Mattei 1995), 550 days (Mattei 1997), and 600 days (Mattei 1999) to predict dates of maxima for those years. Broens et al. (2000) derived a period of 568d for the epoch of JD2450377, and the nature of the period variation was investigated in some detail by Templeton et al. (2003), who found a smooth variation in period over the entire span of available observations. Zijlstra et al. (2004) re-analyzed the available AAVSO data (also using WWZ) and confirmed the results of Templeton et al. (2003). They also noted the similarity of LX Cyg to BH Cru (discussed below), and that LX Cyg has a transitional spectrum of type SC, indicating carbon enrichment has occurred or is ongoing.

Our reanalysis of a slightly longer span of AAVSO visual observations again clearly shows the period variation, and a linear fit of the period variation yields $d \ln P / dt$ of $+6.5 \times 10^{-3}$ y$^{-1}$. Published evolution models suggest that a period increase occurs later on in the thermal pulse, if indeed LX Cyg is undergoing one. However, we note that the rate of period change in LX Cyg appears to be decelerating. As Figure 7a shows, its period change has clearly not been linear, and has slowed in recent years. Further monitoring of LX Cyg in the coming decades is required, and may help to show whether this object is in the later stages of a thermal pulse, or whether the period variations are caused by some other phenomenon.

4.1.3. BH Cru

Period change in BH Cru was first noted by Bateson et al. (1988). Whitelock (1999) later discussed the spectral behavior and tentatively made the connection between the changing period found by Bateson et al. (1988), and the spectral changes observed over the past 30 years; though BH Cru was originally assigned a spectral type of SC (Catchpole & Feast 1971), it now shows carbon bands indicative of a CS spectral type. Whitelock (1999) attributed the period evolution as being due to a thermal pulse, and argues that the changing spectral
type may be caused by a dredge-up of processed nuclear material in real-time. However, Zijlstra et al. (2004) analyzed the visual observations of the Royal Astronomical Society of New Zealand (RASNZ) along with the spectrum of this object, and suggested that both the period changes and concurrent spectral changes could be due to a global change in effective temperature, rather than a change in structure caused by a thermal pulse. They suggest that feedback between opacities and stellar structure could result in structural changes to the star, causing a shift in mode period without requiring a thermal pulse. Thus they raise the possibility that these stars may undergo large period changes without thermal pulses.

We analyzed the AAVSO visual observations of BH Cru, and determined the rate of period change to be $+3.7 \times 10^{-3} \, \text{y}^{-1}$, consistent with the Zijlstra et al. (2004) analysis of the RASNZ visual data. As with LX Cyg, the trend in period change is not linear, but has exhibited an almost sinusoidal variation over the span of available observations. Such behavior is observed in some other Mira stars with long periods, and they have been dubbed “meandering Miras” by Zijlstra & Bedding (2002). However, most of the long-period Miras with unstable periods only vary by a few percent about a mean period, while BH Cru (and LX Cyg) have changed their periods by over 10 percent over the available span of observations. While we agree that both BH Cru and LX Cyg may simply be manifesting unstable periods, we are not yet prepared to discard them as thermally-pulsing candidates. We discuss possible physical mechanisms for their period variations in Section 5.

4.1.4. R Aql and R Hya

R Aql and R Hya have long been known as stars exhibiting large period changes. (See Müller & Hartwig (1918) for historical reviews.) Period changes in these stars were detected and discussed by both Plakidis (1932) and Sterne & Campbell (1937), though neither quantified the rate of change or offered a physical explanation for them. Both stars were cited as possible examples of thermally-pulsing objects by Wood & Zarro (1981), who obtained reasonable fits between the observed period variations and the theoretical period variations from evolution model calculations (although they assumed first-overtone pulsations). The rate of period change in R Aql has remained essentially unchanged over the available span of observations (see Greaves & Howarth (2000) for a review). Zijlstra et al. (2002) studied both modern and historical data for R Hya dating back to 1662. They found that the historical data indicate the onset of a near-constant period change beginning around 1770, when the period was approximately 495 days. Their derived rate of period decline is $-1.48 \times 10^{-3} \, \text{y}^{-1}$, a factor of two larger than the rate derived from the linear fit to the AAVSO data spanning JD 2419389 to 2452894. This decline continued until 1950 when the period levelled out at a
nearly constant value, and has remained near 395 days since that time. Our smaller derived value for the period change is a reflection of our shorter span of data during which the rate of period change rapidly decelerated. For this star, the longer span of observational data used by Zijlstra et al. (2002) is clearly a better reflection of the long-term evolution of R Hya. In fact, the behavior of R Hya for the latter half of the 20th century resembles the behavior of the meandering-period stars.

As Woold & Zarro (1981) clearly show, the behavior of the periods of both R Aql and R Hya are reasonably interpreted as the result of radius variations after the peak luminosity in a thermal pulse. Woold & Zarro (1981) determined the “time-scale” for these stars, equivalent to the inverse of $d \ln P/ dt$, obtaining values of 950 and 650 years for R Hya and R Aql, respectively. Our values of $d \ln P/ dt$ yield timescales of 1440 and 641 years, respectively. The value for R Aql is in excellent agreement. The much longer value for R Hya is a reflection of the much slower rate of period change in the more recent times covered by our data set.

4.1.5. W Dra

W Dra has also long been known to have period variations (Müller & Hartwig 1918). The third edition of the GCVS (Kukarkin et al. 1969) noted that the period of W Dra is variable, and subsequent studies showed that it varied in a monotonic fashion. Woold & Zarro (1981) first suggested W Dra as another candidate for a thermal pulse after analyzing the $O-C$ diagrams of 48 well-observed Mira stars. Unlike R Hya and R Aql, the period of W Dra was increasing, suggesting that it was in a different stage of the thermal pulse. Woold & Zarro (1981) suggested it is near the first peak in period following the onset of a thermal pulse, and that the period change should cease and/or change sign over the next century. Koen & Lombard (1995) and Percy & Colivas (1999) confirmed that the period change in W Dra was real by analyzing times of maximum, and Greaves (2000) used the AMPSCAN (Howarth 1991) procedure to measure the period change in this star. He found that the period increased from about 255 days in the early part of the 20th century, to about 280 days in the late 1970’s, where it has remained since. We obtained similar results, and derived a $d \ln P/ dt$ of $+1.0 \times 10^{-3}$ y$^{-1}$, though we note we included the most recent constant-period data in the fit, which underestimates the rate of period change earlier in the data. It is not yet clear whether the most recent few decades of nearly constant period represent a fundamental change in the behavior of W Dra, or whether it is a short-term excursion within a longer, more constant trend. Observations in coming decades will clarify this, and monitoring of W Dra should continue.
4.1.6. R Cen

R Cen has exhibited substantial variations in its light curve shape as well as its period during the course of its observed history. R Cen and R Nor are the prototypes of the “double-peaked” Miras, whose light curves exhibit two distinct maxima per cycle. However, unlike R Nor, the light curve of R Cen has undergone significant changes in morphology in recent decades, where the double peaks have become far less pronounced at times. The amplitude has also decreased with time, and is currently about one third of the amplitude in the early part of the 20th century. The true period of R Cen is also debatable; Feast (1963) suggested that the difference between the absorption and emission line velocities in the spectrum of R Cen were consistent with it being a star with half the GCVS period of 550 days, and this assertion has been adopted by others since (Keenan et al. 1974; Jura 1994). However, the characteristics of R Cen are very similar to other stars with known or suspected double maxima, and all of these stars have long periods (Lebzelter et al. 2005; Templeton & Willson 2005). Walker & Greaves (2001) noted that the two maxima have different (B − V) colors, indicating that the physical causes of each maximum are different, and that the time between two maxima having the same color is the true period.

The general behavior of the period variation was discussed by Walker & Greaves (2001), and the AAVSO visual data for R Cen between 1950 and 2000 were analyzed in detail by Hawkins et al. (2001). The latter derived a rate of period change of about -1 day per year, yielding $d\ln P/dt = -1.9 \times 10^{-3}$ y$^{-1}$. Because they only analyzed the most recent data with the strongest period decline, their rate is more than twice our value derived using all data from 1917 to 2002. Despite the fact that the period was reasonably constant from 1917 to 1950 and then seems to have begun an accelerated decline like T UMi, the behavior of R Cen resembles that of the other long-period stars such as LX Cyg and BH Cru. In fact, both of the latter are considered members of the “double-maxima” class (Kholopov et al. 1985) and so the comparison to these stars is appropriate.

4.1.7. Z Tau

Coherent period change in Z Tau was first noted by Nagel (1986), who analyzed AAVSO $O-C$ data covering the period of 1915 to 1983. Percy & Au (1999) did not give a specific rate of period change but note that the rate of decline was larger than $10^{-3}$ days day$^{-1}$; the corresponding value of $d\ln P/dt$ would be $7.7 \times 10^{-4}$ y$^{-1}$. Using AAVSO visual data from 1917 to 2003, we derive an average value of $-1.15 \times 10^{-3}$ y$^{-1}$, fully consistent with the lower limit of Percy & Au (1999). This rate of period change appears to be relatively constant through the span of available data. It is also consistent with theoretical rates derived from
models of the later stages of thermal pulses.

4.2. Slower period changes

Forty-nine more of 547 stars showed period variations whose linear fits were statistically different from zero at the 2-$\sigma$ level, with 13 of these being significant at the 3- to 6- $\sigma$ level. For many of these stars, the period variations are not linear in nature, and the use of a linear fit is clearly not representative of their behavior; its utility is in assessing the general trend of period change throughout the data set. (A few stars, notably DU Cyg and S Tri, exhibited modest period changes throughout the span of observations, but their net period change was small enough that they did not meet our selection criteria.) Among the stars that do not exhibit clear linear trends in period are several whose periods appear to vary in a seemingly sinusoidal fashion. The best examples of this behavior are RU Tau and S Ori, with S Sex, TY Cyg, and U Dra being slightly weaker examples. Zijslstra & Bedding (2002) described such objects as “meandering Miras,” suggesting that as many as 10-15% of all stars of long period may exhibit such behavior. Such behavior, on timescales of decades rather than centuries, is not well-modeled by evolutionary processes, and may be a manifestation of some other type of instability. We discuss this further in section 5.

At least one star in our sample, Y Per, underwent a drastic change in pulsation behavior, switching from Mira-like variations to semi-regular variations in less than two pulsation cycles. This change was also coincident with a modest drop in period. This behavior was first noted by Kiss et al. (2000), who suggested that vigorous convection may be at work. Cadmus et al. (1991) noted that the semi-regular stars RV And, S Aql, and U Boo exhibited significant variations in pulsation period and amplitude, and suggested that mode-switching could explain the behavior. Other Miras have changed from regular pulsators to semi-regular or irregular ones in the past, notably W Tau and RT Hya (Mattei et al. 1990), both of which have similar periods to Y Per. We note these two objects appear to be re-establishing more regular pulsations, but more observations are required to confirm this. The transition from Mira to semi-regular may be transient in nature, as may changes in period. Objects like Y Per, W Tau, and RT Hya should be continually monitored in the future.

A large fraction of the stars showing “variations” are marginal detections, between 2- and 3-$\sigma$. While such objects meet the statistical requirement for mention in our results, the period variations appear to be erratic at best, and do not represent a coherent change in period; CQ And and RZ Sco are particularly notable examples of this. We suggest that for many of these stars, the measured “period changes” are simply manifestations of the random period changes known to occur in Mira stars. Miras can have substantial jitter in period.
from cycle to cycle, and it is reasonable to assume that in any large sample of stars, some objects may exhibit period variations large enough to be flagged by our criteria.

4.3. The utility of wavelet analysis for deriving rates of period change

Our work has highlighted the many different methods for searching for period changes in data. The classical method for performing such studies was the use of maxima and minima data to create $O - C$ diagrams. Linear trends in period appear as parabolic curves in $O - C$, making the detection of period changes reasonably straightforward. This method was employed in the earliest work of Eddington and Plakidis in the 1920’s and 1930’s, and continues to be employed today. See the series of papers by Koen & Lombard (1993) and Percy et al. (Percy & Au 1999; Percy & Colivas 1999) for more details. The major limitation of such studies is that they require the measurement of times of maxima and minima – in effect a “pre-processing” of the data. More computationally-intensive methods such as wavelet, or other “time-frequency” analyses allow us to use the data directly. Such methods are not perfect; variations in light curve shape, long known to affect the Mira variables, can have a substantial effect on the Fourier spectra of these objects, and the additional computational time required for such methods has only become “trivial” relatively recently. As the extensive work by both Koen & Lombard and Percy et al. shows, our analysis method and others similar to it should be considered complimentary to $O - C$ and time-of-maximum analyses, rather than be viewed as replacements.

One clear example of this is in the detection of period changes with very long data sets composed only of times of maximum, when actual observations are not available. Sterken et al. (1999) used $O - C$ analysis to detect a small but significant period change in $\chi$ Cyg dating to its discovery 1686. The data from the 20th century has period changes fully consistent with the random variations seen in some Mira variables, and since $\chi$ Cyg is a long-period star, such period variations are not surprising. Our data and the Sterken et al. (1999) data set produce similar results in this regard. However, when the time-of-maximum data were analyzed, longer term trends began to emerge. The full 320-year $O - C$ diagram indeed revealed a parabolic curve, and Sterken et al. (1999) determined that the rate of period change is consistent with the rate expected in the late stages of a thermal pulse, after the hydrogen-burning shell has fully re-established itself. Percy & Au (1999) also suggest that even recent time-of-maximum data can be used to detect the long-term period evolution in Mira stars, though as they state, the statistical significance of their result is less than 2-$\sigma$. Unfortunately, historical data are unlikely to exist for a significant number of Mira variables, since only a handful were discovered prior to the late 19th century.
5. Discussion

The incidence of statistically significant, long-term period variations in our sample is on the order of ten percent, and we see large changes in about one percent of our sample. The latter fraction is reasonable given the expected durations of the rapid shell-flashing phase and the slower inter-pulse phase. Because our sample is a heterogeneous mixture of stars with different ages and masses, it is not possible to make an exact prediction of the numbers of stars we expect to undergo such changes. The physical mechanism for period changes in Mira variables has not been proven, though we expect thermal pulses to occur on the AGB, and we expect that some stars must be Mira variables when these pulses occur. Therefore, we expect to observe changes in pulsation behavior in at least some Mira variables, given a large enough sample size. Below, we discuss potential evolutionary and non-evolutionary mechanisms for the observed period changes in our sample.

5.1. Evolutionary period changes

From theoretical models of thermal pulses, we expect to see large and relatively monotonic variation in period on timescales of centuries. This is indeed what we see for eight stars of the 547, although even for some of those, there are secondary period variations apparent on the period vs time curves. Thus, the interpretation of these secular changes as resulting from shell flashing appears both natural and appropriate. However, the fact that secondary variations do appear, and that other stars only show these secondary, meandering variations leaves room for either a different or modified explanation for large, secular period variations.

Even if the thermal pulse explanation is correct, it is then difficult to estimate the number of Miras we expect to exhibit strong period changes in a given population. The Miras in our sample are a mixture of spectral types and must also have a range of masses based upon the range of periods observed. If these stars only spend part of their AGB lifetimes within the Mira instability strip, then we may see period changes in other types of AGB variables, such as semi-regulars and obscured carbon and OH-IR stars, all of which have similar luminosities as the Miras. We are currently analyzing data for the semi-regular variables found in the AAVSO ID to search for period changes, but note the existence of at least one well-known semi-regular variable with a period change, namely RU Vul, which we show in Figure 9. The large period change was first noted by Zijlstra & Bedding (2002), and though the star was noted as having a double period by Kiss et al. (1999), both its period change and its declining pulsation amplitude are remarkably similar to those of T UMi. Continued monitoring of this star in the coming years is strongly encouraged.
According to theoretical models of Vassiliadis & Wood (1993), stars with solar metallicity and masses larger than 1 M$_\odot$ spend between 70,000 and 100,000 years between thermal pulses. The pulses themselves cause large changes in period for perhaps 2,000 years, and thus the ratio of pulse to interpulse lifetimes is between 2 and 3 percent. If the Miras are evenly distributed throughout the thermal-pulsing AGB, then we might expect large, secular period changes in the same percentage of Miras. Eight of 547 stars represents 1.6% of the sample, in reasonable agreement with this expectation.

We note that when AGB stars undergo a thermal pulse, they may become or cease to be Mira stars for a period of time. The models only lie within the Mira instability region for part of the thermally-pulsing and interpulse phases of AGB evolution. During the pulse, transient oscillations may develop in the atmosphere, turning the stars into semi-regular variables, or quenching the pulsations altogether. The thermal pulse may also be accompanied by mass-loss events, turning the stars into extreme carbon or OH-IR stars, and rendering them optically faint or invisible. In the former case, period changes may indeed be visible among the semi-regular variables, and we are currently analyzing the semi-regular variables found in the AAVSO ID for a forthcoming paper.

5.2. Non-evolutionary period changes

Several stars in our sample exhibited non-monotonic period variations with time-scales of a few decades, either about the mean period, or superimposed upon a larger trend. The most prominent examples of this are RU Tau and S Ori, whose period variations appear to be nearly sinusoidal. Although we cannot definitively claim the period variations are truly oscillatory in nature (because the timescales are of the same order as the span of the data), the “periods” of these variations are on the order of 10 to 80 years. We do not have an immediate explanation for why such variations exist, but we speculate that they may be related to thermal pulses. While the decades-long timescales of these variations are far shorter than the those predicted for pulse-induced global changes, they may represent thermal oscillations in the envelope with characteristic Kelvin-Helmholtz timescales, $\tau_{KH}$, on the order of a few decades. The Kelvin-Helmholtz timescale can be derived if one can reliably estimate the radial order of the pulsation mode, the luminosity, and the mass of the Mira star. The precise value of $\tau_{KH}$ for individual stars must be derived using known values of stellar parameters, and for most objects in our sample, these values are not known. Studies of mass loss and evolution (see Bowen & Willson (1991) for example) allow us to estimate the mass based upon the period, and for nearly all reasonable values of these quantities, $\tau_{KH}$ is on the order of a few decades.
We can estimate the Kelvin-Helmholtz timescale, $\tau_{KH}$, as

$$\tau_{KH} \sim \frac{GM^2}{RL} \sim 3.1 \times 10^7 \frac{(M/M_\odot)^2}{(R/R_\odot)(L/L_\odot)} \text{y}. \quad (3)$$

Ostlie & Cox (1986) calculated many AGB models in the range of 0.8 to 2.0 $M_\odot$, and obtained Kelvin-Helmholtz timescales between 6 and 200 years. A Mira star with $M = 1.0M_\odot$, $L = 5000L_\odot$, and $R = 250R_\odot$ would have $\tau_{KH}$ of $\sim 25$ years, in good agreement with the observed timescales. Thus we suggest that the shorter-timescale period variations may be thermal relaxation oscillations in the stellar envelope, as it responds to the global changes caused by a thermal pulse. A more comprehensive study of these variations will appear in a future paper.

We also note two additional non-evolutionary explanations for such behavior. One is that Mira variable pulsations are intrinsically non-linear in nature, and that they exhibit low-dimensional chaotic behavior. Kiss & Szatmáry (2002) argue that amplitude (not period) variations in the light curve of R Cygni are caused by low-dimensional chaos, perhaps through the non-linear interaction of two or more pulsation modes. Such a mechanism was suggested by Buchler et al. (1996) to describe the irregular behavior of RV Tauri-type stars, and a similar (albeit less irregular) mechanism may be at work in the Mira stars.

A second explanation for large period changes was put forth by Ya'ari & Tuchman (1996), who suggested that Mira pulsations may be strong enough to modify the interior structure as the star pulsates. They suggest that the pulsations modify the entropy structure of the star over time, causing both mode-switching and a readjustment of the equilibrium structure of the star. Their theory was tested with non-linear, hydrodynamic models in which a fundamental-mode Mira model changed to first-overtone pulsation. The “new” model period was significantly different from the initial equilibrium first-overtone period as a result of structural changes wrought by the strong pulsations. More modeling and investigations along both of these lines would be useful, and may explain several other aspects of Mira variability such as the small, cycle-to-cycle variations in period and amplitude.

In closing, we also note that some of the stars with “long-term” period changes may in fact be exhibiting the same behavior as stars showing these shorter-term variations, but on a longer time-scale. In particular, LX Cyg, BH Cru, and R Hya seem to be nearing plateaus or troughs in their periods, and may indeed begin to change again in the future. Such stars bear close watching in the coming decades, both in regards to their periods and to their spectra. It is entirely possible that LX Cyg, BH Cru, and R Hya are undergoing similar behavior to RU Tau, and that we simply do not have a span of observations long enough to detect this. Continued monitoring of these stars is necessary and encouraged, as the coming
decades may clarify their behavior.

6. Conclusions

We have performed a wavelet analysis of 547 well-observed Mira variables from the AAVSO International Database. We are unable to detect long-term variations in period in the majority of the stars having $d\ln P/dt$ below our detection threshold. About 10% of the sample shows period variations on timescales of decades at the $2-\sigma$ level or greater and only eight of the 547 (1.6%) have highly significant ($> 6-\sigma$) monotonic period changes. This latter percentage is consistent with the fraction of stars we expect to be in the early post-thermal pulse phase at any given moment, during which period change is predicted to be greatest. One star, T Ursae Minoris, showed a sudden onset of large period decrease which is consistent with the onset of a thermal pulse during the span of observations. In several stars, we detect significant but non-monotonic “meandering” changes. The physical mechanism for such variations in period is not clear, but the timescale for these variations is similar to the Kelvin-Helmholtz cooling timescale for the envelope.

Continued monitoring of Mira stars and other long-period variables is clearly warranted, and will almost certainly continue given the level of interest in these stars among both the amateur and professional astronomical communities. The behavior of stars such as T Ursae Minoris suggests that even “well-behaved” stars with nearly constant periods are deserving of continued monitoring given the possibility of detecting the sudden onset of period change. However, the lack of thus-far undetected large-scale period changes among well-observed Miras suggests that few additional thermally-pulsing candidates will be found among Mira stars in the solar neighborhood, at least in the short-term.

In addition to continued monitoring of known Mira stars, we encourage the continued long-term monitoring of the large numbers of Mira and Mira-like variables detected in the Galactic center and in other galaxies, since they may produce additional candidates for thermal pulses once sufficient data has been collected to measure period changes. Such monitoring efforts will require decades-long commitments of telescope time, but this may be easily achievable with many of the current and planned all-sky monitoring projects, as well as with the continuing efforts of volunteer and amateur observers.

Dr. Janet Mattei passed away shortly before the completion of this work, and her coauthors respectfully dedicate it to her memory. We are indebted to the many thousands of variable star observers worldwide whose work and dedication over the past century made this research possible. We wish to thank Rebecca Turner and Elizabeth Waagen of the AAVSO
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Fig. 1.— Light curve of 10-day means of S Arietis (P = 292 days). The light curve was obtained from visual observations, and spans nearly 90 years. This light curve highlights some of the features and drawbacks of the AAVSO data. The light curve coverage is sparse up to $\sim JD2435000$, but markedly improves later on, as an increasing number of observers contribute their observations. After $JD2440000$, the maxima and minima are well-covered, showing the modest variability in the magnitudes of maximum and minimum light.
Fig. 2.— 10-day means of S Ari versus $V$-band CCD observations. The agreement between the CCD and visual data is good, indicating that the visual data are reliable estimators of the star’s photometric behavior.
Fig. 3.— Raw visual light curves of U Aur and WX Cyg, M- and C-type Mira variables, respectively. The larger scatter in visual observations of WX Cyg is due to this star being redder (B-V = 3.2; Alksnis et al. (2001)) than U Aur (B-V = 2.1). Light curves of very red stars are more susceptible to observational effects such as the Purkinje effect.
Fig. 4.— Histogram of the 547 Mira variables in our sample, binned by period. The average period of our sample is 307 days.
Fig. 5.— Histograms of the period distribution of the 547 Miras in our sample, subdivided by spectral type. The M- and S-type Miras have broad distributions of period, while the C-type Miras are more densely concentrated at longer periods. When the period distributions are compared to those of all Miras listed in the GCVS, the AAVSO sample of C-type Miras appears to be biased relative to the period distribution of the GCVS C-type sample, but the M- and S-types in our sample are not statistically different from the GCVS sample.
Fig. 6.— Histograms of \((d\ln P/dt)\) for the 547 stars in our sample. The 1-\(\sigma\) error in \((d\ln P/dt)\) is marked by the vertical dashed lines. The majority of stars in our sample have \(|d\ln P/dt|\) less than this, and thus do not have measurable period changes given the current data. Fifty-seven of the 547 stars have period changes significant at the 2-\(\sigma\) level or greater.
Fig. 7a.— Period versus time the 57 stars with non-zero $d \ln P/dt$ significant at the 2-$\sigma$ level or greater. Solid points are the period at which the wavelet $Z$-transform is maximal at each value of $t$, separated by 500 days. Error bars are 1-$\sigma$, defined by 0.02$P$. Solid lines are the best linear fit through the data points. Clearly, some stars are not well-fit by a linear period change, such as RU Tau and S Ori. Others, like T UMi and LX Cyg are very well-fit by linear trends.
Fig. 7b.— Period versus time the 57 stars with non-zero $d\ln P/dt$ significant at the 2-$\sigma$ level or greater. Solid points are the period at which the wavelet $Z$-transform is maximal at each value of $t$, separated by 500 days. Error bars are 1-$\sigma$, defined by $0.02P$. Solid lines are the best linear fit through the data points. Clearly, some stars are not well-fit by a linear period change, such as RU Tau and S Ori. Others, like T UMi and LX Cyg are very well-fit by linear trends.
Fig. 8.— Period versus time for T Ursae Minoris, including the entire span of the data. The rapid period decline clearly begins midway through the span of data. Prior to JD 2440000, the period was essentially constant at 315 days, but has since fallen to 240 days. The amplitude (not shown) has declined along with the period.
Fig. 9.— Period versus time for RU Vulpeculae, a semi-regular variable with an apparent period change. The period has declined from approximately 155 days to about 110 days at present, a decline of nearly 30 percent in 65 years. The amplitude of pulsation has dramatically declined as well, and the star may cease pulsating altogether in the coming decades.
Table 1. Measured periods and rates of period change for 57 Mira stars with $d \ln P/dt$ greater than 2-$\sigma$ above the measurement error.

<table>
<thead>
<tr>
<th>Name</th>
<th>$\bar{P}$ (d)</th>
<th>$d \ln P/dt$ ($10^{-3}$ y$^{-1}$)</th>
<th>N-$\sigma$</th>
<th>$d \ln P$</th>
<th>$(d \ln P/dt)^{-1}$ (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T UMi</td>
<td>285.49 ± 1.10</td>
<td>−8.47 ± 0.35</td>
<td>23.90</td>
<td>0.27</td>
<td>120</td>
</tr>
<tr>
<td>LX Cyg</td>
<td>520.15 ± 2.01</td>
<td>6.47 ± 0.36</td>
<td>17.86</td>
<td>0.19</td>
<td>150</td>
</tr>
<tr>
<td>R Aql</td>
<td>293.00 ± 0.72</td>
<td>−1.56 ± 0.09</td>
<td>17.02</td>
<td>0.14</td>
<td>640</td>
</tr>
<tr>
<td>Z Tau</td>
<td>476.63 ± 1.19</td>
<td>−1.15 ± 0.10</td>
<td>11.66</td>
<td>0.10</td>
<td>870</td>
</tr>
<tr>
<td>W Dra</td>
<td>270.56 ± 0.66</td>
<td>1.03 ± 0.09</td>
<td>11.42</td>
<td>0.09</td>
<td>970</td>
</tr>
<tr>
<td>R Cen</td>
<td>538.14 ± 1.35</td>
<td>−0.84 ± 0.10</td>
<td>8.63</td>
<td>0.08</td>
<td>1190</td>
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<tr>
<td>R Hya</td>
<td>393.89 ± 0.95</td>
<td>−0.71 ± 0.09</td>
<td>8.01</td>
<td>0.07</td>
<td>1410</td>
</tr>
<tr>
<td>BH Cru</td>
<td>511.87 ± 2.35</td>
<td>3.70 ± 0.61</td>
<td>6.09</td>
<td>0.08</td>
<td>270</td>
</tr>
<tr>
<td>V Del</td>
<td>528.85 ± 1.46</td>
<td>−0.43 ± 0.10</td>
<td>4.08</td>
<td>0.05</td>
<td>2350</td>
</tr>
<tr>
<td>S Ori</td>
<td>419.99 ± 1.03</td>
<td>0.35 ± 0.09</td>
<td>3.81</td>
<td>0.09</td>
<td>2870</td>
</tr>
<tr>
<td>TY Cyg</td>
<td>350.60 ± 0.86</td>
<td>0.36 ± 0.09</td>
<td>3.79</td>
<td>0.05</td>
<td>2790</td>
</tr>
<tr>
<td>RU Sco</td>
<td>365.28 ± 0.91</td>
<td>−0.36 ± 0.10</td>
<td>3.70</td>
<td>0.06</td>
<td>2770</td>
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<tr>
<td>DF Her</td>
<td>335.40 ± 1.40</td>
<td>1.69 ± 0.46</td>
<td>3.69</td>
<td>0.04</td>
<td>590</td>
</tr>
<tr>
<td>BK Ori</td>
<td>339.38 ± 1.31</td>
<td>−1.29 ± 0.36</td>
<td>3.56</td>
<td>0.04</td>
<td>780</td>
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<tr>
<td>T Lyn</td>
<td>410.63 ± 1.15</td>
<td>−0.49 ± 0.14</td>
<td>3.53</td>
<td>0.04</td>
<td>2030</td>
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<tr>
<td>RU Tau</td>
<td>568.74 ± 1.39</td>
<td>0.32 ± 0.09</td>
<td>3.52</td>
<td>0.10</td>
<td>3090</td>
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<tr>
<td>DN Her</td>
<td>226.97 ± 0.86</td>
<td>−1.19 ± 0.34</td>
<td>3.49</td>
<td>0.04</td>
<td>840</td>
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<tr>
<td>RS Peg</td>
<td>413.72 ± 0.99</td>
<td>0.29 ± 0.09</td>
<td>3.40</td>
<td>0.03</td>
<td>3400</td>
</tr>
<tr>
<td>W Lac</td>
<td>320.47 ± 1.31</td>
<td>1.41 ± 0.43</td>
<td>3.28</td>
<td>0.04</td>
<td>710</td>
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<tr>
<td>RZ Sco</td>
<td>159.32 ± 0.40</td>
<td>0.31 ± 0.10</td>
<td>3.24</td>
<td>0.08</td>
<td>3190</td>
</tr>
<tr>
<td>AN Peg</td>
<td>272.46 ± 1.14</td>
<td>−1.39 ± 0.46</td>
<td>3.02</td>
<td>0.04</td>
<td>720</td>
</tr>
<tr>
<td>Y Per</td>
<td>250.77 ± 0.60</td>
<td>−0.25 ± 0.08</td>
<td>2.97</td>
<td>0.05</td>
<td>4000</td>
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<tr>
<td>BG Cyg</td>
<td>288.35 ± 0.82</td>
<td>−0.42 ± 0.14</td>
<td>2.94</td>
<td>0.05</td>
<td>2360</td>
</tr>
<tr>
<td>R Nor</td>
<td>498.20 ± 1.25</td>
<td>0.29 ± 0.10</td>
<td>2.89</td>
<td>0.04</td>
<td>3490</td>
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<tr>
<td>RR Cas</td>
<td>299.48 ± 0.75</td>
<td>−0.28 ± 0.10</td>
<td>2.88</td>
<td>0.04</td>
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<tr>
<td>RS Hya</td>
<td>336.45 ± 0.85</td>
<td>−0.29 ± 0.10</td>
<td>2.86</td>
<td>0.04</td>
<td>3460</td>
</tr>
<tr>
<td>BU And</td>
<td>381.86 ± 1.47</td>
<td>−1.03 ± 0.36</td>
<td>2.85</td>
<td>0.04</td>
<td>970</td>
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<tr>
<td>S Sex</td>
<td>259.52 ± 0.73</td>
<td>−0.40 ± 0.14</td>
<td>2.85</td>
<td>0.06</td>
<td>2520</td>
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<tr>
<td>WZ Gem</td>
<td>332.88 ± 1.13</td>
<td>0.69 ± 0.24</td>
<td>2.83</td>
<td>0.05</td>
<td>1450</td>
</tr>
<tr>
<td>EL Lyr</td>
<td>235.69 ± 0.98</td>
<td>1.29 ± 0.46</td>
<td>2.81</td>
<td>0.04</td>
<td>780</td>
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</table>
Table 1—Continued

<table>
<thead>
<tr>
<th>Name</th>
<th>$\bar{P}$ (d)</th>
<th>$d\ln P/dt$ (10$^{-3}$ y$^{-1}$)</th>
<th>N-$\sigma$</th>
<th>$d\ln P$ (d ln $P/dt$)$^{-1}$ (y)</th>
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</thead>
<tbody>
<tr>
<td>AB Cep</td>
<td>323.24 ± 1.24</td>
<td>1.00 ± 0.36</td>
<td>2.77</td>
<td>0.03</td>
</tr>
<tr>
<td>Z Car</td>
<td>384.51 ± 0.97</td>
<td>0.28 ± 0.10</td>
<td>2.73</td>
<td>0.06</td>
</tr>
<tr>
<td>T Scl</td>
<td>203.82 ± 0.62</td>
<td>0.49 ± 0.18</td>
<td>2.72</td>
<td>0.04</td>
</tr>
<tr>
<td>U Lyr</td>
<td>455.88 ± 1.10</td>
<td>-0.24 ± 0.09</td>
<td>2.70</td>
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</tr>
<tr>
<td>CQ And</td>
<td>190.73 ± 0.76</td>
<td>1.09 ± 0.40</td>
<td>2.69</td>
<td>0.05</td>
</tr>
<tr>
<td>VY Aur</td>
<td>395.69 ± 1.58</td>
<td>1.08 ± 0.41</td>
<td>2.66</td>
<td>0.04</td>
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<tr>
<td>T Cas</td>
<td>444.74 ± 1.03</td>
<td>-0.20 ± 0.08</td>
<td>2.61</td>
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</tr>
<tr>
<td>T CVn</td>
<td>290.32 ± 0.69</td>
<td>0.22 ± 0.08</td>
<td>2.61</td>
<td>0.05</td>
</tr>
<tr>
<td>SU Vir</td>
<td>209.26 ± 0.52</td>
<td>-0.24 ± 0.10</td>
<td>2.48</td>
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<tr>
<td>U Dra</td>
<td>318.09 ± 0.77</td>
<td>0.22 ± 0.09</td>
<td>2.46</td>
<td>0.05</td>
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<tr>
<td>RS Aqr</td>
<td>216.59 ± 0.54</td>
<td>0.25 ± 0.10</td>
<td>2.46</td>
<td>0.06</td>
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<tr>
<td>UZ Hya</td>
<td>262.60 ± 1.01</td>
<td>0.88 ± 0.36</td>
<td>2.44</td>
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<tr>
<td>X Aql</td>
<td>346.07 ± 0.86</td>
<td>-0.23 ± 0.09</td>
<td>2.41</td>
<td>0.03</td>
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<tr>
<td>RT Sco</td>
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<td>-0.43 ± 0.18</td>
<td>2.40</td>
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<td>342.86 ± 0.83</td>
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<tr>
<td>BF Cep</td>
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<td>1.15 ± 0.49</td>
<td>2.34</td>
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</tr>
<tr>
<td>S Scl</td>
<td>364.65 ± 0.89</td>
<td>0.21 ± 0.09</td>
<td>2.27</td>
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<tr>
<td>Z Cas</td>
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<td>0.20 ± 0.09</td>
<td>2.22</td>
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<td>U UMi</td>
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<td>2.21</td>
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<td>R Leo</td>
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<td>-0.17 ± 0.08</td>
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<td>0.03</td>
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<td>TT Mon</td>
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<td>0.89 ± 0.41</td>
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<td>CF Her</td>
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<td>-0.82 ± 0.38</td>
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<td>S Pic</td>
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<td>-0.21 ± 0.10</td>
<td>2.08</td>
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<td>R Tel</td>
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<td>0.21 ± 0.10</td>
<td>2.04</td>
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<td>TY Cas</td>
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<td>-0.87 ± 0.43</td>
<td>2.01</td>
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<td>T Col</td>
<td>226.05 ± 0.57</td>
<td>0.20 ± 0.10</td>
<td>2.00</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Note. — $d\ln P/\text{d}t$ is defined as $dP/\text{d}t/\bar{P}$, where $dP/\text{d}t$ and $\bar{P}$ are obtained from the linear fit. $d\ln P$ is defined as $(P_{\text{max}} - P_{\text{min}})/\bar{P}$. The error in $d\ln P$ is approximately constant at $\sim 0.04$. 