Quantum evaporation of a naked singularity

Rituparno Goswami*, Pankaj S. Joshi*, Parampreet Singh†
* Tata Institute for Fundamental Research, Colaba, Mumbai 400005, India and
† Institute for Gravitational Physics and Geometry,
Pennsylvania State University, University Park, PA 16802, USA

We investigate here gravitational collapse of a scalar field model which classically leads to a naked singularity. We show that non-perturbative semi-classical modifications near the singularity, based on loop quantum gravity, give rise to a strong outward flux of energy. This leads to the dissolution of the collapsing cloud before a naked singularity can form. Quantum gravitational effects can thus censor naked singularities by avoiding their formation. Further, quantum gravity induced mass flux has a distinct feature which can lead to a novel observable signature in astrophysical bursts.

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Naked singularities are one of the most exotic objects predicted by classical general relativity. Unlike their black hole siblings, they can be in principle directly observed by an external observer. This has led to extensive debates on their existence, with a popular idea being cosmic censorship conjectures which forbid classical nakedness [1]. However, naked singularities are boon for testing cosmic censorship conjectures which forbid classical nakedness [1]. However, naked singularities are boon for testing cosmic censorship conjectures which forbid classical nakedness [1].

One of the non-perturbative quantizations of gravity is loop quantum gravity [2] whose key predictions include Bekenstein-Hawking entropy formula [3]. Its application to symmetry reduced mini-superspace quantization of homogeneous spacetimes is known as loop quantum cosmology [4] whose success includes resolution of the big bang singularity [5], initial conditions for inflation [6, 7], and possible observable signatures in cosmic microwave background radiation [8]. These techniques have also been applied to resolve black hole singularity in a scalar field collapse scenario [9].

Since the dynamics of a generic collapse is very complicated and tools to address such a problem in quantum gravity are still under development, it is useful to work with a simple collapse scenario as of a scalar field. It serves as a good toy model to gain insights on the role of quantum gravity effects at the end state of gravitational collapse. Existence of naked singularities in these models is well-known [2] and one of the simplest setting is to consider a homogeneous and isotropic scalar field $\Phi = \Phi(t)$ with a potential $V(\Phi)$ [10], where it has been shown that fate of the singularity being naked or covered depends on the rate of gravitational collapse. For an appropriately chosen potential, formation of trapped surfaces can be avoided even as the collapse progresses, resulting in a naked singularity with an outward energy flux, in principle observable. Since the interior of homogeneous scalar field collapse is described by a FRW metric, techniques of loop quantum cosmology can be used to investigate the way quantum gravity modifies the collapse.

Let us first consider the classical collapse of a homogeneous scalar field $\Phi(t)$ for the marginally bound ($k = 0$) case. The interior metric is given by

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 d\Omega^2]$$

(1)

with classical energy density and pressure of the scalar field,

$$\rho(t) = \frac{1}{2} \dot{\Phi}^2 + V(\Phi), \quad p(t) = \frac{1}{2} \ddot{\Phi}^2 - V(\Phi) .$$

(2)

The dynamical evolution of the system is obtained from the Einstein equations which yield

$$\dot{R}^2 R = F(t, r) ,$$

(3)

$$\rho = \frac{F_r}{8\pi GaR^2}, \quad p = \frac{-\dot{F}}{8\pi GR^2} .$$

(4)

Here $F(t, r)$ is an arbitrary function with the interpretation of the mass function of the collapsing cloud, with $F \geq 0$ and $R(t, r) = ra(t)$ is the area radius of a shell labeled by comoving coordinate $r$. In a continual collapse the area radius of a shell at a constant value of comoving radius $r$ decreases monotonically. The spacetime region is trapped or otherwise, depending on the value of mass function. If $F$ is greater (less) than $R$, the the region is trapped (untrapped). The boundary of the trapped region is given by $F = R$.

The collapsing interior can be matched at some suitable boundary $r = r_b$ to a generalized Vaidya exterior geometry, given as [11],

$$ds^2 = - \left( 1 - \frac{2M(r_v, v)}{r_v} \right) dv^2 - 2dvdr_v + r_v^2 d\Omega^2 .$$

(5)

The Israel-Darmois conditions then yield [12, 13],

$$r_b a(t) = r_v (v), \quad F(t, r_b) = 2M(r_v, v),$$

(6)

$$M(r_v, v) = \frac{F}{2r_b a} + r_b^2 a \ddot{a} .$$

(7)
The form of the potential that leads to a naked singularity can be determined as follows. The energy density of scalar field can be written in a generic form as 
\[ \rho = l^{n-4}a^{-n}, \]
where \( n > 0 \) and \( l \) is a proportionality constant. Using energy conservation equation, this leads to the pressure \( p = (n-3)/3 \) \( l^{n-4}a^{-n} \), which on using eq. (2) gives,
\[ \Phi = -\sqrt{\frac{n}{8\pi G}} \ln a, \quad V(\Phi) = (1 - n/6)l^{n-4}e^{\sqrt{8\pi G n} \Phi}. \] (8)
It is then straightforward to check that for \( 0 < n < 2 \), if no trapped surfaces exist initially then no trapped surfaces would form till the epoch \( a(t) = 0 \), which is determined by
\[ a(t) = \left(1 - \frac{n t}{2\sqrt{3}}\right)^{2/n}. \] (9)
Absence of trapped surfaces is accompanied by a negative pressure which implies that for a constant value of the comoving coordinate \( r \), \( \Phi \) is negative and hence the mass contained in the cloud of that radius keeps decreasing. This leads to a classical outward energy flux. As the collapse proceeds, the scale factor vanishes in finite time and physical densities blow up, leading to a naked singularity. Since no trapped surfaces form during collapse, the outward energy flux shall in principle be observable. However, near the singularity when energy density is close to Planckian values, this classical picture has to be modified and we need to investigate the scenario incorporating quantum gravity modifications to the classical dynamics.

Let us hence consider the non-perturbative semi-classical modifications based on loop quantum cosmology for the interior. The underlying geometry for the FRW spacetime in loop quantum cosmology is discrete and dynamics is governed by quantum difference equations. However, for scale factors \( a_0 = \sqrt{\ell_P} \lesssim a \lesssim a_* = \sqrt{j\gamma/3\ell_P} \), dynamics can be described by modifications to Friedmann dynamics on a continuous spacetime. Here \( j \approx 0.2375 \) is the Barbero-Immirzi parameter, \( \ell_P \) is Planck length and \( j \) is a half-integer free parameter which shall be greater than unity for existence of this semi-classical regime. It has been shown that such modifications approximate very well the dynamics governed by quantum difference equations. We note that since semi-classical modifications for inhomogeneous case are still not known, we cannot do a complete quantum analysis of interior and exterior. The exterior is assumed to remain classical. Further, as a continuous spacetime can be approximated till scale factor \( a_0 \), the matching of interior and exterior spacetimes remains valid during the semi-classical evolution.

An important feature of loop quantum modifications is the change in behavior of classical geometrical density \((1/a^3)\) for scales \( a \lesssim a_* \), whose eigenvalue spectrum is approximated by
\[ d_j(a) = D(q) a^{-3}, \quad q := \frac{a^2}{a_*^2}, \quad a_* := \sqrt{\gamma \ell_P} \] (10)
with
\[ D(q) = \frac{8\pi G}{77} q^{3/2} \left(7 \left(q + 1\right)^{11/4} - \left|q - 1\right|^{11/4}\right) - 11q \left(q + 1\right)^{7/4} - \text{sgn}(q - 1) \left|q - 1\right|^{7/4}\right)^{1/4}. \] (11)
The scale factor \( a_* \) serves as the critical scale below which the geometrical density becomes proportional to the positive powers of scale factor and decreases with decrease in scale factor. In the case when \( a \ll a_* \),
\[ d_j(a) \approx \left(\frac{12}{7}\right)^{6/7} \left(\frac{a}{a_*}\right)^{15} a^{-3} \] (12)
and for \( a \gg a_* \) it behaves classically \( d_j \approx a^{-3} \). This change in the behavior of geometrical density is key to modifications to dynamical equations and resultant physical effects.

The eigenvalues of the scalar field Hamiltonian in the semi-classical regime can be approximated by
\[ \mathcal{H}_\Phi = d_j(a) P_\Phi^2/2 + a^3 V(\Phi). \] (13)
Here \( P_\Phi \) is the canonical conjugate momentum to the scalar field. The Hamilton equations of motion for the scalar field
\[ \ddot{\Phi} = d_j(a) P_\Phi, \quad \dot{P}_\Phi = -a^3 V_\Phi(\Phi) \] (14)
yield the Klein-Gordon equation
\[ \ddot{\Phi} + \left(\frac{3\dot{a}}{a} D(q)/D(q)\right) \dot{\Phi} + D(q) V_\Phi(\Phi) = 0. \] (15)
At classical scales \( a \gg a_* \), \( D(q) \approx 1 \) and the modified dynamics reduces to the standard Friedmann dynamics. It can be easily verified that for \( a \ll a_* \), \( D(q)/D(q) \approx 15\dot{a}/a \), \( D \ll 1 \) and the kinetic term in Klein-Gordon equation flips its sign for \( a < a_* \). This results in friction for the collapsing scalar field in contrast to the classical anti-friction, thus radical modifications to the classical collapse are expected.

The modified energy density and pressure of the scalar field in the semi-classical regime can be obtained from the eigenvalues of density operator and using the stress-energy conservation equation. They are
\[ \rho_{\text{eff}} = d_j(a) \mathcal{H}_\Phi = \ddot{\Phi}^2/2 + D(q) V(\Phi) \] (16)
and
\[ p_{\text{eff}} = \left[1 - \frac{2}{3} \left(\frac{1}{\dot{a}/a} D(q)\right) \ddot{\Phi}^2/2 - D(q) V(\Phi) - \frac{\dot{D}(q)}{3(\dot{a}/a)} V(\Phi)\right]. \] (17)
It is then straightforward to check that \( p_{\text{eff}} \) is generically negative for \( a \lesssim a_* \) and for \( a \ll a_* \) it becomes very
is proportional to \( \rho \). The energy density modifications to collapse is following.

The parameters chosen are shows evolution of energy density (in Planck units) with time. The parameters chosen are \( n = 1.9 \) and \( j = 100 \).

strong. For example, at \( a \sim a_0 \), \( p_{\text{eff}} \approx -9\rho_{\text{eff}} \). This is much stronger than its classical counterpart \( p = (n - 3)/3 \rho \) with \( 0 < n < 2 \). Thus we expect a strong burst of outward energy flux in the semi-classical regime. Further, for \( a \ll a_* \), \( D(q) \ll 1 \) and the Klein-Gordon equation yields \( \Phi \propto a^{12} \). Hence from the eq. (16) we easily see that the effective density, instead of blowing up, becomes extremely small and remains finite.

The mass function of the collapsing cloud can be similarly evaluated using eq. (6) and the modified Friedmann dynamics,

\[
F = \frac{8\pi G}{3} \left( \frac{d_j^{-1} \dot{\Phi}^2}{2} + a^3 V(\Phi) \right) r^3 .
\] (18)

In the regime \( a \sim a_0 \), the potential term becomes negligible and \( d_j^{-1} \dot{\Phi}^2 \) becomes proportional to \( a^{12} \). Thus mass function becomes vanishingly small at small scale factors.

The picture which thus emerges from loop quantum modifications to collapse is following.

- Before the area radius of the collapsing shell reaches \( R_\ast = ra_\ast \) at \( t = t_\ast \), collapse proceeds as per classical dynamics and as smaller scale factors are approached \( \Phi \) and the energy density \( \rho \propto a^{-n} \) increase. The mass function is proportional to \( a^{n-3} \) and (as \( 0 < n < 2 \)) it decreases with decreasing scale factor so there is a mass loss to the exterior, which is also understood from existence of negative classical pressure.

- As the collapsing cloud reaches \( R_\ast \), the geometric density classically given by \( a^{-3} \), modifies to \( d_j \) and the dynamics is governed by the modified Friedmann and Klein-Gordon equations. The scalar field which experienced anti-friction in classical regime, now experiences friction leading to decrease of \( \Phi \).

- The slowing down of \( \Phi \) because of quantum friction decreases the rate of collapse and formation of singularity is delayed. Eventually when scale factor becomes smaller than \( a_0 \) this leads to breakdown of continuum spacetime approximation and semi-classical dynamics. Discrete quantum geometry emerges at this scale \( [14, 15] \) and the dynamics can only be described by quantum difference equation. The naked singularity is thus avoided till the scale factor at which a continuous spacetime exists.

We show the evolution of area radius in time as collapse proceeds in Fig.1. The semi-classical evolution (solid curve) closely follows classical trajectory (dashed) till the time \( t_\ast \). Within a finite time after \( t_\ast \), the classical collapse leads to a vanishing \( R \) and naked singularity. However, the area radius never vanishes in the loop modified semi-classical dynamics and the naked singularity does not form as long as the continuum spacetime approximation holds. The inset of Fig.1 shows the evolution of energy density in Planck units. Classical energy density (dashed curve) blows up whereas it remains finite and in fact decreases in the semi-classical regime.

The phenomena of delay and avoidance of the naked singularity in continuous spacetime is accompanied by a burst of matter to the exterior. If the mass function at scales \( a \gg a_\ast \) is \( F_i \) and its difference with mass of the cloud for \( a < a_\ast \) is \( \Delta F = F_i - F \), then the mass loss can be computed as

\[
\frac{\Delta F}{F(a_\ast)} = \left[ 1 - \frac{\rho_{\text{eff}} d_i^{-1}}{\rho_{\text{eff}} a_i^{n-3}} \right] .
\] (19)

For \( a < a_\ast \), as the scale factor decreases, the energy density and mass in the interior decrease and the negative pressure strongly increases. This leads to a strong burst of matter. The absence of trapped surfaces enables the quantum gravity induced burst to propagate via the generalized Vaidya exterior to an observer at infinity. The evolution of mass function is shown in Fig.2. In the semi-classical regime, \( \Delta F/F_i \) approaches unity very rapidly. This feature is independent of the choice of parameter \( j \). The choice of potential causes mass loss to exterior in classical collapse also, but it is much smaller and in any case the classical description cannot be trusted at energy density greater than Planck, when we must consider quantum effects as above.

Interestingly, for a given collapsing configuration, the scale at which the strong outward flux initiates depends on the loop parameter \( j \) which controls \( a_\ast \). If \( j \) is large then burst occurs at an earlier area radius and vice versa.

The inset of Fig.2 shows the mass loss ratio for different values of \( j \). For all choices, \( \Delta F/F_i \rightarrow 1 \), but the profile of outgoing flux varies with change in value of \( j \) (the profile would be similar to classical for \( j \) less than unity). The loop quantum burst has a distinct signature, at \( a \sim a_\ast \) the flux decreases for a short period and then rapidly increases. Since the causal structure of classi-
cal spacetime is such that trapped surface formation is avoided, this quantum gravitational signature can be in principle observed by an external observer as a slight dimming and subsequent brightening of the collapsing star. This peculiar phenomena is directly related to the peak in the function $d_j(a)$, and depends solely on the value of parameter $j$. Thus an observer can estimate the loop quantum parameter $j$ by observing the flux profile of an astrophysical burst based on this collapse scenario and measuring the variation in luminosity of the collapsing cloud. The signal may have additional features due to role of other parameters in a more general scenario, however the important result is the possible existence of such novel quantum gravity signatures astrophysical bursts.

During such a burst most of the mass is ejected and this may dissolve the singularity. Thus non-perturbative semi-classical modifications may not allow formation of naked singularity as the collapsing cloud evaporates away due to super-negative pressures in the late regime. In this sense loop quantum effects imply a quantum gravitational cosmic censorship, alleviating the naked singularity problem. We note that the semi-classical effects do not show that the singularity is absent, it is only avoided till scale factor $a_0$, below which the semi-classical dynamics and matching may break down. If for a given choice of initial data, semi-classical dynamics is unable to completely dissolve the singularity, the final fate of naked singularity must be decided by using full quantum evolution. Even in such cases we have valuable insights from semi-classical loop quantum effects. In the toy model considered, we showed that the classical outcome and evolution of collapse is radically altered by the non-perturbative modifications to the dynamics. The possibility of such observable signatures in astrophysical bursts, as originating from quantum gravity regime near singularity is intriguing, indicating that gravitational collapse scenario can be used as probes to test quantum gravity models.

Our considerations are of course within the mini-superspace setting, and the general case of inhomogeneities and anisotropies remains open. It would also be useful to examine generic scalar fields with arbitrary potentials, and the well explored gravitational collapse scenarios such as dust and other perfect fluids [18], to investigate singularity resolution and possible observable signatures.

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[18] Apart from the modification of inverse scale factor, there might be additional higher order perturbative corrections to Friedmann dynamics due to discrete quantum effects as considered in Ref. [10]. We work with initial configurations such that these corrections are negligible.