Misinterpretations in Lorentz Transformations

Diego Saá

P.A.C.S: 01.55.+b; 03.30.+p; 01.70.+w

1 Introduction

In the present paper some reasons are given to suggest that the interpretation of the Lorentz’ transformations as if they referred to coordinates instead of to intervals could be incorrect. Besides, the usual form of such transformations, by using variables that represent finite values instead of differentials, could be another imprecision.

2 THE INFINITESIMAL CHARACTER OF THE LORENTZ TRANSFORMATIONS

A seemingly overlooked error, preserved since the original works of Lorentz, Minkowski and Einstein, is that the Lorentz transformations are written as if they referred to finite magnitudes, when in fact they should refer to infinitesimals. The present author believes that to this error can be traced most of the so-called “paradoxes” that pervade Special Relativity.

The Lorentz transformations have the purpose of finding the coordinates of an event, from the point of view of one coordinate system, given the coordinates of the same event as seen from a second coordinate system.

Einstein wrote \[2\]: “Any such event is represented with respect to the co-ordinate system K by the abscissa x and the time t, and with respect to the system K’ by the abscissa x’ and the time t’. We require to find x’ and t’ when x and t are given.”

Einstein wrote the Lorentz transformation in finite form, in Appendix I.

---

1Escuela Politécnica Nacional. Quito – Ecuador. email: dsaa@server.epn.edu.ec
of his book [2] and also in his original paper of 1905 [1] as follows:

\[ t' = \gamma(t - \frac{V x}{c^2}) \quad (1) \]

\[ x' = \gamma(x - V t) \quad (2) \]

where \( V \) represents the relative velocity between the two frames of reference, let the two frames be called \( S' \) and \( S \) (Einstein named \( K \) and \( K' \), but that is insignificant). Let us assume that the frame \( S' \) moves to the right in the \( x \) direction (or the frame \( S \) moves to the left).

![Figure 1: Coordinates](image.png)

The inverses of these equations are:

\[ t = \gamma(t' + \frac{V x'}{c^2}) \quad (3) \]

\[ x = \gamma(x' + V t') \quad (4) \]

These equations can be used, in theory, to compute the coordinates \( t' \) and \( x' \) of a certain event, as seen from the frame of reference \( S' \), if we know
the coordinates $t$ and $x$ of the same event as seen from the frame $S$, or vice versa.

Let us try now to reveal that they cannot be used to accomplish this goal.

First, assume that the relative velocity $V$ between the two frames is zero. Then, the Lorentz contraction factor

$$\gamma = (1 - V^2/c^2)^{-\frac{1}{2}}$$

becomes equal to 1 and the transformations are simplified to $t' = t$ and $x' = x$. This means that the coordinates of the event are the same when they are seen from the two frames of reference and when those frames of reference do not have a relative movement between them. It can also be concluded that the two origins are coincident.

Now let us assume that the origin of $S'$ is displaced a certain given distance to the left of the origin of $S$. No matter what the relative velocity between the two frames of reference, but in particular if it is zero, it would be necessary to, somehow, add that distance to the coordinate $x$ in order to find $x'$ (or subtract that distance from $x'$ in order to find $x$).

The coordinate transformations that include the mentioned constant distance are called Poincaré transformations. But, in the same sense, are mistakenly interpreted as that they transform coordinates instead of intervals.

Let us provide some numbers in a given example and then let us do the computations suggested by equations (1) and (2). Assume that the origin of the frame $S$ is at rest in our laboratory; also, the velocity $V$ of the frame $S'$ (moving to the right) is so small that, in practice, can be ignored (assume, for example, one meter per day). Now, the problem is to compute the coordinate $x'$ at the instant in which the origin of the frame $S'$ is at 100 meters to the left of the origin of the frame of reference $S$ and the coordinate $x$ of the event (for example a tennis ball hits the floor), is produced at 1000 meters to the right of the origin of the frame $S$.

A quick “mental” estimation of the value for $x'$, assuming that both frames of reference are static, produces 1100 meters; on the other hand, according to equation (2), the coordinate $x'$ is given by: $\gamma (1000\text{ meters} - 1\text{ meter/day} \cdot t)$, where $\gamma$ is very close to 1 and you should tell me now what is the time $t$. Did you obtain minus one hundred days? Fine.

The computation is something like:

$$1100m = 1000m - 1m/day \cdot (-100\text{day})$$
What is the meaning of this time? It is, evidently, the interval of time since
the event was produced until the origin of the primed frame of reference
reaches the origin of the unprimed frame of reference. From this computa-
tion we can conclude that the magnitude of the time variable is greater when
the speed is smaller, so as to be close to infinite when the relative velocity of
both frames of reference is close to zero. So, one conclusion almost obvious
here, is that it is not an independent time coordinate, because it must de-
pend on the relative positions of the origins of the two frames of reference at
the time when the event occurs and, also, such time depends on the relative
velocity between both frames of reference. In this context we realize that the
phenomena is behaving, and should be explained, as a differential equation.
The supposed time coordinate is not such thing.

It seems that we still have to do another “Gedanken Experiment” to re-
alyze what is happening. Let us assume again that the two frames have a
relative velocity $V$. The argument is simplified a little if we assume a small
velocity, for example one meter per day. For such a small velocity we can
ignore $\gamma$ again. But, strictly speaking, it is not necessary this simplification
because the problem to be revealed here is too gross to be overlooked, what-
ever the value of $\gamma$. Also, assume that the event occurs close to you, at the
origin of the $x$-axis of the $S$ frame. This means that the $x$ coordinate of the
event is $x=0$. With this assumption, the transformation (2) simplifies to: $x' = -Vt$. Finally, in order to define the time coordinate of the event, assume
that the event occurs at the precise instant at which you look at your wrist-
watch. Thus, the time coordinate, $t$, is precisely the hour you have at this
moment in your watch. If, sometime later, one of your neighbors passes near
you in his/her car, whose frame of reference is $S'$, and you inform him/her,
written in a piece of paper, the coordinates you just recorded for the event in
your frame $S$, he/she could, in theory, compute the coordinate $x'$ where the
event happened, with respect to his/her frame of reference, if he/she replaces
those coordinates in the original equations (1) and (2), or in the simplified
transformation $x' = -Vt$.

I am almost sure that he/she will not be able to compute a reasonable
result, in the first place because the origin of the time coordinate is usually
different for different observers. For example, the Gregorian calendar marks
currently a few more than 2000 years, a few more than 1380 years in the
Islamic chronology and a few more than 5760 years in the Jewish calendar.
Those numbers make nonsense if you try to use them as origins of the tem-
poral coordinates in the above equations. So, for example, if I have the time
coordinate of 2005 years, he/she could have 5765 or 5766 years. It does not help much if the beginning of the time coordinate is assumed midnight, because it is highly probable that I do not have the same hour as you do, due to our respective geographical position. And you should not suppose that synchronizing our clocks then you can use the above “coordinate transformations” because, even if we synchronize our time coordinate, in years, days, hours, and seconds, the product of a time coordinate by a velocity, produces nonsense for the problem at hand.

The only reasonable way to compute the interval $x'$ is if I further specify that the clock is not a clock but a chronometer or stop-watch that can be put to zero when the event takes place. In other words, you need to know what is the elapsed time since the occurrence of the event. You can obtain a reasonable result only if you know, for example, that the event took place one hundred days before the origins of the $x$ coordinates of both frames became coincident.

The conclusion of the above argument is that the variables used in equations (1) and (2) are not coordinates but intervals. If you use space and time intervals, which are still finite, those equations work better but not quite.

The Lorentz transformations should be written with differentials, because, in that case, the needed constant could enter as a constant of integration. The same can be sustained for the time transformation (1).

The mathematicians should explain if it is correct, as is usual and accepted in current Physics, to interpret the Lorentz transformations in differential form as if they were equivalent to the finite transformations, or if a new proof is needed.

The Lorentz transformations with the use of differentials would be the following:

$$\frac{dt'}{\gamma dt^2} = \gamma (dt - V dx/c^2)$$  \hspace{1cm} (6)

$$\frac{dx'}{\gamma dx^2} = \gamma (dx - V dt)$$  \hspace{1cm} (7)

Take note that the event is instantaneous and consequently does not have a velocity of displacement associated with it. The variable $V$ was used in this section, and should be interpreted, as the relative velocity between the two frames of reference.

Please verify that these equations are identical to the equations (1) and (2), except that we are now using differentials instead of intervals.
Let us note that if we look at the event from the origin of a third frame of reference, such as $S''$, then the velocity, $v''$, to be used both explicitly in the equations as well as in the Lorentz contraction, should be now the relative velocity between the frames of reference $S''$ and $S'$, whose corresponding time and space intervals we are trying to compute now.

If we require to compute both the time and space differentials corresponding to the frame of reference $S'$, given the coordinate differentials defined for the frame of reference $S''$, we will still have to use equations (6) and (7), with the necessary corrections, which produce the following end result:

\[
dt' = \gamma'' (dt'' - v'' dx''/c^2) \quad (8)
\]

\[
dx' = \gamma'' (dx'' - v'' dt'') \quad (9)
\]

where $v''$ is interpreted currently as the relative velocity of the frame of reference $S''$ with respect to the frame of reference $S'$ (or vice versa).

Equating equations (6), (8) and (7), (9) we come up with:

\[
dt' = \gamma (dt - V dx/c^2) = \gamma'' (dt'' - v'' dx''/c^2) = \ldots \quad (10)
\]

\[
dx' = \gamma (dx - V dt) = \gamma'' (dx'' - v'' dt'') = \ldots \quad (11)
\]

The velocities $V, v'', \ldots$ associated with each frame of reference are equal to the velocities between the frame of reference $S'$ and each one of the frames of reference $S$ and $S''$ of the observers (or vice versa).

The equations for the different $\gamma$'s are functions of the corresponding velocities.

Solving the last two equalities (10) and (11) for $dt''$ and $dx''$ we obtain:

\[
dt'' = \Gamma (dt - v dx/c^2) \quad (12)
\]

\[
dx'' = \Gamma (dx - v dt) \quad (13)
\]

where $v$ is the abbreviation of the expression:

\[
v = \frac{V - v''}{1 - \frac{V \cdot v''}{c^2}} \quad (14)
\]
which represents the velocity between the frames of reference S and S” and is comparable with Einstein’s equation for “composition of velocities”.

Whereas \( \Gamma \) is defined as:

\[
\Gamma = \sqrt{1 - \frac{v^2}{c^2}}
\] (15)

The previous equations reveal the group character of the Lorentz transformations.

Equations (12) and (13) can be rewritten and simplified by replacing \( \frac{dx}{dt} \) by \( c \):

\[
\frac{dt''}{dt} = \frac{dx''}{dx} = \sqrt{\frac{1 - v/c}{1 + v/c}}
\] (16)

The proportion of time and space differentials is the same expression and represents the Doppler redshift of an event as observed from two arbitrary frames of reference.

Some observations, concerning some effects and objective reality of the “Lorentz transformations” are presented by the author in other paper.

## 3 Conclusions

1. I felt that I could be offending the intelligence of the reader by explaining in so much detail elementary examples such as the one analyzed in the section “the infinitesimal character of the Lorentz transformation” that, in principle, can be grasped by any motivated layman, but the case is that, apparently, it has not been recognized, throughout the last one hundred years, that the finite Lorentz transformation should work with intervals and not with coordinates. Or is the case that physicists, and in particular the “relativists” didn’t want to recognize it? My suspicion is that we, as humans, have great difficulty in capturing concepts, doing deductions and considering the cumulus of objections which have been sustained through the years by many investigators. As Feyerabend says: “the lasting success of our categories and the omnipresence of some specific point of view is not signal of excellence or indicative that truth has been found. Rather it is indication of the failure of reason” to find adequate alternatives to overcome
an intermediate stage of our knowledge” (my own translation from the Spanish version of this book). Even though some physicists have been able to identify the many contradictions related with Special Relativity (one should have been enough), they have been incapable of constructing an integrated and coherent theory. Our more advanced reasoning seem to use deductive chains of about five resolution steps. Chains of reasoning with about ten resolution steps are almost impossible for us to find and accomplish without error. An alternative line of investigation, worth of further study, is about the complexity of the knowledge; this should help to pinpoint, and be careful with, arguments where the validation of the truth of the premises and of the chains of deduction are approaching the limits of human comprehension.

2. It is a well-known fact that Special Relativity is based on the finite coordinate transformations, which can be traced back to Voigt, Lorentz and Minkowski but popularized by, essentially, the original works of Einstein. As these transformations have been here proved wrong, because they should use neither finite nor coordinate distances, many of the applications and paradoxes based in such equations collapse and disappear.

3. A theory should always be subject to analysis and improvements. In particular, Special Relativity, as a theory, seems that has not been honest in the estimation of the degree of uncertainty that it was conveying, even though the technical aspects had been better founded. This paper has revealed some misinterpretations, which seem to be better qualified as errors, that cannot be corrected or improved with experiments but by changing the theory.

References

