QCD corrections to the electroproduction of hadrons with high $p_T$

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Abstract. We compute the order $\alpha_s^2$ corrections to the one particle inclusive electroproduction cross section of hadrons with non vanishing transverse momentum. We compare our results with H1 data on forward production of $\pi^0$, and conclude that the data is well described by the DGLAP approach within the theoretical uncertainties.

INTRODUCTION

The precise measurement of final state hadrons in lepton nucleon deep inelastic scattering constitutes an excellent benchmark for different features of perturbative quantum chromodynamics. Among them, the calculation of higher order corrections, which have been explored and validated for most processes up to next to leading order (NLO) accuracy. For the one particle inclusive processes only very recently there has been progress beyond the leading order (LO) [1, 2, 3, 4, 5], and up to now there were no analytic computation of the $O(\alpha_s^2)$ corrections for the electroproduction of hadrons with non vanishing transverse momentum. The analytic computation of the $O(\alpha_s^2)$ corrections allows us to check factorization in a direct way, which means that collinear singularities showing up in the partonic cross section factorize into parton densities (PDFs) and fragmentation functions (FFs). As a consequence of this explicit cancellation, the resulting cross section is finite and can be straightforwardly convoluted with PDFs and FFs in a fast and stable numerical codes. The analytical result is still sufficiently exclusive and keeps the dependence on the rapidity and the transverse momentum of the produced hadron, allowing a detailed comparison with the experimental data. In the following we summarize the results obtained in Ref. [6]

$O(\alpha_s^2)$ QCD CORRECTIONS

We consider the process

$$l(l) + P(P) \rightarrow l'(l') + h(P_h) + X,$$

where a lepton of momentum $l$ scatters off a nucleon of momentum $P$ with a lepton of momentum $l'$ and a hadron $h$ of momentum $P_h$ tagged in the final state. Omitting
target fragmentation at zero transverse momentum, which has been discussed at length in \[1, 2\], the cross section for this process can be written as

\[
\frac{d\sigma^h}{dx_B dQ^2} = \sum_{i,j,n} \int_0^1 d\xi \int_0^1 d\zeta \int dPS^{(n)} \left[ f_i(\xi) D_{h/i}(\zeta) \frac{d\sigma_i^{(n)}}{dx_B dQ^2 dPS^{(n)}} \right] \tag{2}
\]

where \(\sigma_i^{(n)}\) is the partonic level cross section corresponding to the process and is calculated order by order in perturbation theory through the related parton-photon squared matrix elements \(\mathcal{H}_{\mu\nu}^{(n)}(i, j)\) for the \(i + \gamma \rightarrow j + X\) processes.

\[
\frac{d\sigma_i^{(n)}}{dx_B dQ^2 dy dz} = \frac{\alpha_{em}^2}{\epsilon^2} \frac{1}{x_B S_H^2} \left( Y_M (-g^{\mu\nu}) + Y_L \frac{4x_B^2}{Q^2} p^\mu p^\nu \right) \sum_n \mathcal{H}_{\mu\nu}^{(n)}(i, j). \tag{3}
\]

in terms of the standard kinematical variables \[3\].

At order-\(\alpha_s^2\), the partonic cross sections receive contributions from the following reactions:

\[
\begin{align*}
\text{Real contributions} & : & \gamma + q(\bar{q}) & \rightarrow g + g + q(\bar{q}) \\
& & \gamma + q_i(\bar{q}_i) & \rightarrow q_i(\bar{q}_i) + q_j + \bar{q}_j \quad (i \neq j) \\
& & \gamma + q_i(\bar{q}_i) & \rightarrow q_i(\bar{q}_i) + q_i + \bar{q}_i \\
& & \gamma + g & \rightarrow g + g + \bar{q} \\
\text{Virtual contributions} & : & \gamma + q(\bar{q}) & \rightarrow g + q(\bar{q}) \\
& & \gamma + g & \rightarrow q + \bar{q}
\end{align*} \tag{4}
\]

where any of the outgoing partons can fragment into the final state hadron \(h\).

At variance with the \(|p_T| = 0\) case, where the integration over final states leads to overlapping singularities along various curves in the residual phase space, here the only remaining singularities are found at \(z = 0\) and thus they can be dealt with the standard method. After combining real and virtual contributions to a given partonic process, the cross section can be written as

\[
\frac{d\sigma_i^{(2)}}{dx_B dQ^2 dy dz} = \frac{c_q C_v^2}{x_B S_H^2} \left\{ \frac{1}{\epsilon} \mathcal{P}_{1ij}^{(2)}(\rho, y, z) + C_{ij}^{(2)}(\rho, y, z) + \mathcal{O}(\epsilon) \right\}, \tag{5}
\]

where the coefficient of the single poles, \(\mathcal{P}_{1ij}^{(2)}(\rho, y, z)\), as well as the finite contributions \(C_{ij}^{(2)}(\rho, y, z)\), include ‘delta’ and ‘plus’ distributions in \(z\). The IR double poles present in the individual real and virtual contributions cancel out in the sum, providing the first straightforward check on the angular integration of real amplitudes and the loop integrals in the virtual case. In the real terms, the above mentioned double poles come from the product of a pole arising in the integration over the spectators phase space and a single pole coming from the expansion of \(z^{-1+\epsilon}\) factors. Double poles in the virtual contributions always arise from loop integrals.

The remaining singularities, contributing to the single pole, are of UV and collinear origin. The former are removed by means of coupling constant renormalization, whereas the latter have to be factorized in the redefinition of parton densities and fragmentation functions.
PHENOMENOLOGY

In Figure 1 we show the LO and NLO predictions for the electroproduction of neutral pions as a function of $x_B$ and $p_T$, respectively, in the kinematical range of the H1 experiment \[7\], together with the most recent data for the range $p_T \geq 3.5$ GeV. The cross sections are computed as described in the previous sections, applying H1 cuts and using MRST02 parton densities \[8\]. Similar results are found using other sets of modern PDFs. For the input fragmentation functions, we use two different sets, the ones from reference \[9\] denoted as KKP and those from \[10\] referenced as K. We set the renormalization and factorization scales as the average between $Q^2$ and $p_T^2$, and we compute $\alpha_s$ at NLO(LO) fixing $\Lambda_{QCD}$ as in the MRST analysis. The plots clearly show that the NLO cross sections are much larger than the LO ones, even by the required order of magnitude in certain kinematical regions. Another interesting feature is that the uncertainty due to the choice of a fragmentation functions set is also quite noticeable, this fact driven by the different gluon content of the two sets considered here. Low $Q^2$ bins seem to prefer KKP set, which have a larger gluon-fragmentation content, whereas for larger $Q^2$ both sets agree with the data within errors. LO estimates show a much smaller sensitivity on the choice of fragmentation functions, since gluon fragmentation does not contribute significantly to the cross section at this order.

The rather large size of the K-factor can, then, be understood as a consequence of the opening of a new dominant (‘leading-order’) channel, and not to the ‘genuine’ increase in the partonic cross section that might otherwise threaten perturbative stability. The dominance of the new channel is due to the size of the gluon distribution at small $x_B$ and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \rightarrow g + q + \bar{q}$ partonic process.

In Figure 2 we show the different contributions to the cross section discriminated by the underlying partonic process. Notice that at very small $x_B$ the $gg$ term can be by itself several times larger than the LO contribution, remaining larger or comparable even for

![Figure 1](image_url)
higher $x_B$ values. The forward selection is also responsible of the scale sensitivity of the cross section, as it suppresses large components with small scale dependence whereas it stresses components as $gg$ whose scale dependence would be partly canceled only at NNLO.

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**REFERENCES**