A 2-Dimensional Cellular Automaton for Agents Moving from Origins to Destinations

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Abstract

We develop a two-dimensional cellular automaton (CA) as a simple model for agents moving from origins to destinations. Each agent moves towards an empty neighbor site corresponding to the minimal distance to its destination. The stochasticity or noise \((p)\) is introduced in the model dynamics, through the uncertainty in estimating the distance from the destination. The friction parameter \(\mu\) is also introduced to control the probability that the movement of all agents involved to the same site (conflict) is denied at one time step. This model displays two states; namely the freely moving and the jamming state. If \(\mu\) is large and \(p\) is low, the system is in the jamming state even if the density is low. However, if \(\mu\) is large and \(p\) is high, a freely moving state takes place whenever the density is low. The cluster size and the travel time distributions in the two states are studied in detail. We find that only very small clusters are present in the freely moving state while the jamming state displays a bimodal distribution. At low densities, agents can take a very long time to reach their destinations if \(\mu\) is large and \(p\) is low (jamming state); but long travel times are suppressed if \(p\) becomes large (freely moving state).

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1 Introduction

Cellular Automata (CA) micro-simulation has emerged as a tool for simulating traffic flow and modelling transport networks. In CA, time and space are discrete. The space is represented as a uniform lattice of cells with finite number of states, subject to a uniform set of rules, which drives the behavior of the system. These rules compute the state of a particular cell as a function of its previous state and the state of the neighboring cells.

Agents moving from origins to destinations across networks may represent several real entities, as for example: ants, biological organisms, small robots, transport in micro-mechanical systems, crowd flow, packets transport in the Internet, etc.... It was found that the motion of the biological organisms is usually controlled by interactions with other organisms in their neighborhood and randomness also plays an important role. Real ants have been shown to be able to find shortest paths towards destinations using as only information the pheromone trail deposited by other ants. The finding of shortest paths, inspired from ants’ behavior, are successfully applied to several physical problems such as, pedestrians, traffic flow, combinatorial optimization and circuit switched communications network problems. The problems of movement of agents with origins and destinations were studied using a two dimensional cellular automata. In these models an agent tries to reach its destination using simple rules. Transitions from the freely moving to the jamming states were studied. A variant 2-dimensional CA model for simulation of agents moving from origins to destinations will be presented here. Agents moving across the network have sensors to perceive their local neighborhood and their destinations and then affect their environment. This is done especially by estimating the distance metric to the destination site. The concern herein will be with the movement, propagation, and interaction of agents in low and high density situations. This will be done by exploring the patterns and behaviors of the spatio-temporal organization of agents. The objective of this research is to provide insight into modelling complex dynamics using CA microsimulation and capturing general features of agents travelling from origins to destinations. The paper is organized as follows. In Sec. 2, we describe our model for movements of agents with origins and destinations. In Sec. 3, we present our numerical results where we give the phase diagrams of the system. A detailed description of the cluster size and the travel time distributions are also presented. Finally, we conclude with some conclusions in Sec. 4.
2 The cellular automata model

The CA model is a two-dimensional cellular automaton based on a square lattice with periodic boundary conditions. There is a fixed number of agents on the lattice at all times. Only one agent at most can occupy a given site. At any time step, an agent can move at most to one of its 4 neighboring sites. Updating of the CA occurs in parallel where the rules are applied to all agents at the same time. Agents are associated with given origin-destination sites. The origin and destination sites must be different. An agent travels from the origin site towards its destination site, whereupon it disappears. Each disappeared agent is immediately replaced by a new agent, and so the agent number is always constant in the lattice. A new origin-destination pair is then chosen randomly for this new agent. If, however, there is an agent already present at the chosen origin, then another origin site is selected. Agents will move towards their destinations at all times by selecting an unoccupied neighboring site which has the minimal distance from that site to the destination site (see figure 1). An agent examines the unoccupied neighboring sites. For each of these sites a distance to the destination is evaluated. Then, a site with the minimal distance is selected as the next site to which the agent will move. If all neighboring sites are occupied it will not move. The stochasticity or noise is introduced in the model dynamics, through the uncertainty in estimating the distance from the destination. So, with probability $p$ an agent moves towards an arbitrary empty neighboring site rather than the site of minimal distance. The friction parameter $\mu$ is also introduced to control the probability that the movement of all agents involved to the same site (conflict) is denied at one time step. This friction parameter which is essential for resolving the conflict arising in parallel update simulations, is applied for pedestrian traffic problems [12, 13].

In each time step, positions, speeds and directions of all agents are updated according to the following local rules:
- with probability $p$, an agent selects one arbitrary empty neighboring site.
- with probability $(1 - p)$ agent selects an empty neighboring site corresponding to the minimal distance to the destination. If two empty neighboring sites of one agent have the same minimal distance from the destination then one of these two allowed neighbors is chosen randomly.

If two or more agents select the same site (conflicts) then:
- with probability $\mu$ none of the agents is allowed to move to their selected site.
- with probability \((1 - \mu)\) one of these agents is chosen randomly to move to its selected site; the others agents do not move.
If there exist no conflict, the agent moves to its selected site. If all neighboring sites are occupied, the agent does not move.

3 Simulation experiments and results

We carry out our computer simulations of the model by considering a square lattice of size \(L\) with periodic boundary conditions. Initially, we put randomly a number \(N\) of agents into the lattice. The density of agents is denoted as \(\rho = N/L^2\). The velocity of each agent can be either 1 or 0. The duration of each simulation run is 50,000 time steps with the first 20,000 time steps to initiate the simulation and the latter 30,000 used to generate performance statistics. Agents are only allowed to move to unoccupied nearest neighbor sites in one time step, i.e. \(v_{\text{max}} = 1\) cell/time step.

3.1 Diagrams of agents speed versus density

In figure 2, we carried out the plots of the mean velocity of agents as a function of the density, for several system size. Hence, the plots show that \(\langle v \rangle\) undergoes a sudden phase transition from a freely moving state \(\langle v \rangle \approx 1\) to jammed state \(\langle v \rangle \approx 0\) at a critical density \(\rho_c\). In the freely moving state, interaction between agents is weak and the propagation is important inside the network. In contrast, for large density, the interaction becomes strong and jamming takes place where agent movements become rare. As regards the variation with system size \(L\), we find that the critical density decreases with increasing \(L\) and the phase transition becomes sharper.

In figure 3, we plot \(\langle v \rangle\) as a function of \(\mu\) for different values of \(p\) and \(\rho\). Hence, for low density, the average speed remains almost constant if the probability \(p\) is high. However, if \(p\) is low, \(\langle v \rangle\) undergoes a sudden decrease when \(\mu\) exceeds a critical value \(\mu_c\). This corresponds to a phase transition from the freely moving state to the jamming state. Consequently, the enhancement of the friction parameter can topple over from the freely moving to the jamming of agents even at low densities. At high densities, \(\langle v \rangle\) decreases gradually with \(\mu\) for all values of \(p\). Yet, the speed \(\langle v \rangle\) remains always greater for larger \(p\). The phase diagrams of the system is depicted in figure 4, where we plotted the critical values \(\mu_c\) as a function of \(p\) for several fixed values of \(\rho\). Thus,
for low densities and for a given value of $p$, freely moving phase should exist if $\mu < \mu_c$ while jamming phase takes place if $\mu > \mu_c$. It was shown also from figure 4 that the jamming region is broaden as soon as the density is increased. Yet, when the density exceeds some value $\rho > 0.6$, the freely moving phase should never exist.

### 3.2 Spatio-temporal organizations of agents

It is clear that the density dependence of speed alone, cannot give the whole information on the phase behavior of the system. To get more information on the microscopic structure of the phases, one can determine the spatio-temporal organization of agents in the lattice. This microscopic investigations can be obtained by plotting the organization patterns of agents and the distributions of cluster sizes and travel times. The cluster and the cluster size mean here a connected bonds of unoccupied cells and a maximally number connected cells of agents respectively. The travel time is the time it would take to travel from the origin to destination.

#### 3.2.1 Self-organization patterns of agents

Figures 5(a-b) show a typical configurations of the organization of agents at low density. So, for vanished value of $p$ and for low values of $\mu$, the steady state corresponds to the freely moving. However if $\mu$ is high, agents self-organize in a large cluster with few freely moving agents at the boundary. For higher values of $p$, the freely moving phase should exist even for larger $\mu$. Hence, it seems that the role of $\mu$ is to pile up the agents into one large cluster while $p$ tends to dispatch them in all directions.

For high densities and low $p$, agents pile up into one large cluster even if $\mu$ is vanished (Fig. 5c). In the other side, this agglomeration splits up into several clusters when $p$ becomes large (Fig. 5d).

#### 3.2.2 Cluster size distributions

The cluster size distributions of the model are given in figures 6(a-b). At low density and for vanished value of $p$, only small clusters are present in the lattice whenever $\mu$ is low. This is one characteristic of the freely moving phase. From results depicted in figure 6a, we observe the bimodal nature of the cluster size distribution as $\mu$ increases. Large clusters appear in the
lattice but there are by far many more small-sized clusters than larger ones. Furthermore, with increasing $\mu$, the probability of small clusters diminished while that of large cluster increases. Another important result is the discontinuity observed from the probability distribution when $\mu$ becomes very large. As it was shown from figure 5b, almost all agents are congested in one large cluster with the exception of a few agents which are located at the boundary and moving towards their destinations. From figure 6a ($\rho = 0.1$ and $p = 0$), one can see a phase transition from the freely moving phase to the jamming phase, occurring at $\mu_c \approx 0.8$. Indeed, when $\mu < \mu_c$, the cluster size distribution is a continuous function; but it becomes discontinuous when $\mu > \mu_c$. This value of $\mu_c$ agree with that found in the phase diagram (Fig. 4). In the other hand, when $p$ is large enough, we see that large clusters cannot exist even for large $\mu$. It is important to note also that the increase of $p$ increases the probability of small-sized clusters (see figure 6b, higher part). At high densities, it is not surprising that large clusters may be usually present in the lattice. However, their sizes depend greatly to the parameter $p$. For example, the most probable cluster size (of large clusters) is shifted towards the low size region (Fig. 6b, lower part). In contrast to $p$, the effect of $\mu$ on the cluster size distribution is negligible (result not shown). If the density is high, only a small value of $\mu$ may provoke a strong congestion of agents.

3.2.3 Travel time distributions

The second quantity we look at is the travel time, i.e. the time an agent needs to travel from its origin to its destination. The probability distribution of the travel time presents a maximum which is considered as the most probable time, at which an agent finished its travel. At low density and for low $p$, the distribution is sharply peaked around its maximum, whenever $\mu$ is zero (Fig. 7a). If $p$ increases, one finds evidently a broad distribution of travel time, because agents are moving for quite long times before reaching their destinations. The higher is $p$, the higher is the travel time. As regards the variation of $\mu$, figure 7b illustrates some graphs of the travel time distributions for low densities and when we set $p = 0$. The interesting observed phenomena is the existence of a double asymptotic behavior. So, when $\mu$ is not large enough, we see that infinite travel time cannot exist. In contrast, for large $\mu$, some agent may take an infinite time to reach its destination. In this case, the agents situated in the interior of the large
cluster do not move and rest inside for an indefinite time. When $p$ is large enough and the density is low, the travel time distribution changes slightly if $\mu$ is changed (Fig. 7b). Furthermore, we see that infinite travel time does not occur even for very large $\mu$. This shows again that the state is the freely moving one at low density and for large $p$.

For higher densities, the short travel times still remain, showing the presence of some agents situated at the boundary of a large cluster but the asymptotic behavior is rather increasingly wide; reflecting the dynamics of agents inside the large clusters (see figure 7a, lower part). Indeed, in the presence of large clusters one has to distinguish between inner and outer regions of the cluster. Inside, one finds evidently a broad distribution of travel times, because agents are blocked for quite long times. In the outer region of a cluster, however, one finds shorter travel times. Furthermore, we find that infinite travel time exist even for large $p$, because the capacity of the freely moving is reduced when there was a big crowd of agents (congestion).

4 Conclusions

In summary, we have tried to identify the behavioral aspects of agents travelling from origins to destinations. The microscopic CA model presented is capable of capturing the self-organization and complex dynamics of agents. The model contains two parameters ($\mu, p$) and displays two states; namely the freely moving and the jamming state. The agents speed $\langle v \rangle$ depends strongly to the parameters ($\mu, p$). For low densities, phase transitions occur as the friction parameter $\mu$ exceeds a critical value $\mu_c$, which depends on both the density $\rho$ and the noise $p$.

In the other hand, it was found that the effect of $\mu$ is to gather different agents into a large cluster. This leads to jamming even at low density. However, the effect of $p$ is to disperse agents through the lattice. Thus, more mobility and fluidity will affect the whole system. When distinguished for different density ranges, cluster size and travel time distributions have interesting properties. Indeed, if $\rho$ and $p$ are low, a transition from the freely moving to the jamming states can occur at a critical value $\mu_c$. Thus, when $\mu < \mu_c$ agents self-organized in small clusters and only short travel times can be taken by agents. However, in the other side ($\mu > \mu_c$), agents self-organize in one large cluster with very small number of moving agents at the boundary. This is the jamming state where the travel time can be either short
and long. So, short travel times concern agents at the boundary of the large cluster while the long times concern those in the inner of the cluster. At high densities, jamming occurs for all values of $(\mu, p)$. This is due to the reduction of the capacity of a freely moving when there was a big crowd of agents in the lattice. As a result, the speed show a drastic decrease with the density. The cluster size distribution becomes a bimodal distribution which represents a coexistence of large clusters and small ones. Yet, the travel time distribution is much broad where infinite travel time exists for all values of the system parameters $(\mu, p)$. 
References


**Figures captions**

**Fig.1.** Illustration of agent movements in a square lattice with periodic boundary conditions. Circles with arrows represent agents while those without arrows represent their destination sites. Each arrow indicates the selected site that agent will choose (here, we set $p = 0$). The conflict situation is occurred for the “white” and the “gray” agents since they select the same site.

**Fig.2.** Diagrams of agents speed versus density for different values of lattice size $L$.

**Fig.3.** Diagrams of agents speed versus $\mu$ for different values of $\rho$ and $p$, ($L = 60$).

**Fig.4.** Phase diagrams of the system for several fixed values of the density: Squares, open circles, triangles and solid circles represent $\rho = 0.1$, $\rho = 0.2$, $\rho = 0.3$ and $\rho = 0.6$ respectively, ($L = 60$).

**Fig.5.** Self-organization patterns of agents. Black squares represent agents. a) $\mu = 0.2$, $p = 0.0$ and $\rho = 0.1$, b) $\mu = 0.9$, $p = 0.0$ and $\rho = 0.1$, c) $\mu = 0.0$, $p = 0.2$ and $\rho = 0.5$, d) $\mu = 0.0$, $p = 0.8$ and $\rho = 0.5$.

**Fig.6.** Cluster size distributions for several values of the system parameters, ($L = 60$).

**Fig.7.** Travel time distributions for several values of the system parameters, ($L = 60$).
$\mu = 0.5$ and $p = 0.2$

- $L = 30$
- $L = 40$
- $L = 50$
- $L = 60$
- $L = 70$
- $L = 80$
- $L = 90$
$p = 0$ and $\rho = 0.1$

- $\mu = 0.5$
- $\mu = 0.8$
- $\mu = 0.9$
$\mu = 0$ and $p = 0.5$
$p = 0.2$
$p = 0.5$
$p = 0.8$
Travelling Time vs. Probability for different values of $\mu$ and $\rho$.

- $\mu = 0.2$, $\rho = 0.1$: (top graph)
- $\mu = 0.5$, $\rho = 0.0$: (middle graph)
- $\mu = 0.5$, $\rho = 0.1$: (bottom graph)

$p = 0.0$ and $\rho = 0.1$ for $\mu = 0.5$.