Evolutionary and structural properties of mirror star MACHOs

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Abstract

There can exist a hidden sector of the Universe in the form of parallel “mirror” world which has the same particle physics as the observable world and interacts with the latter only gravitationally. Big Bang Nucleosynthesis bounds demand that the mirror sector should have a smaller temperature than the ordinary one. This implies that the mirror matter could play a role of dark matter, and in addition its chemical content should be dominated by helium. Here we study the evolutionary and structural properties of the mirror stars which essentially are similar to that of the ordinary stars but with higher helium contents. Being invisible in terms of photons, they could be observed only as MACHOs in the microlensing experiments. Using a numerical code, we compute evolution of stars with large helium abundances ($Y = 0.30 - 0.80$) and a wide range of masses, from 0.5 to 10 $M_\odot$. We found that helium dominated mirror star should have much faster evolutionary time (up to a factor $\sim 30$) than the ordinary star with the same mass. In addition, we show the diagrams of luminosities, effective temperatures, central temperatures and densities, and compute the masses of the He core at ignition and the minimum mass for carbon ignition, for different chemical compositions. The general conclusion is that mirror stars evolve faster as compared to ordinary ones, and explode earlier as type II supernovae, thus enriching the galactic halo of processed mirror gas with higher metallicity, with implications for MACHO observations and galaxy evolution.

1 Introduction

At present one of the possible candidates of dark matter is mirror matter (for reviews, see refs. [1]-[3]). The idea of mirror world – a parallel hidden sector of particles which is an exact duplicate of the observable particle sector – has been suggested a long time ago by Lee and Yang [4], and various phenomenological aspects of this hypothesis were first discussed by Kobzarev, Okun and Pomeranchuk [5]. Subsequently, different implications of mirror world for the particle physics, astrophysics and cosmology were addressed in a series of papers [6]-[30].

The basic concept is to have a hidden mirror (M) sector of the Universe which has exactly the same particle physics as that of the ordinary (O) sector. In other words, this theory is given by the product $G \times G'$ of two identical gauge factors, which could naturally emerge e.g. in the context of $E_8 \times E'_8$ superstring. (From now on all particles and parameters of the mirror sector will be marked by $'$ to distinguish from the ones belonging

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to the observable or ordinary world.) A mirror parity $G \leftrightarrow G'$ under interchanging of all fields in corresponding representations between $G$ and $G'$ factors implies that both particle sectors are described by the same Lagrangians.

In particular, since the physics of ordinary world is described by the Standard Model $SU(3) \times SU(2) \times U(1)$ with gauge fields (gluons, photons, $W$ and $Z$ bosons) coupled to ordinary quarks $q$ and leptons $l$, the physics of mirror sector should be described by the analogous gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$ with the corresponding gauge bosons (mirror gluons, mirror photons, $W'$ and $Z'$ bosons) coupled to mirror quarks $q'$ and leptons $l'$. (The corresponding Lagrangian terms in explicit form and the field transformation properties under mirror parity were presented in ref. [7].) The extension towards grand unification is straightforward: one can consider e.g. gauge theory based on $SU(5) \times SU(5)'$, $SO(10) \times SO(10)'$, etc. In any case, the mirror parity implies that all coupling constants (gauge, Yukawa, Higgs) have the same pattern in both sectors and thus their microphysics is the same. Obviously, two sectors are connected by universal gravity, but they could communicate also by other means. For example, ordinary photons could have kinetic mixing with mirror photons [8], ordinary (active) neutrinos could mix with mirror (sterile) neutrinos [9], or two sectors could have a common flavour symmetry [10] or Peccei-Quinn symmetry [11].

The different cosmological implications of the mirror world were addressed in several earlier papers [12, 13, 14]. The time evolution of the mirror Universe, starting from inflation and analyzing epochs of baryogenesis, nucleosynthesis, recombination and structure formation was consistently studied in ref. [15]. The results can be briefly summarized as follows.

If the mirror sector exists, then the Universe along with the ordinary electrons, nucleons, neutrinos and photons, should also contain their mirror partners. One could naively think that due to mirror parity the ordinary and mirror particles should have the same cosmological abundances and hence the O and M sectors should have the same cosmological evolution. However, this would be in the immediate conflict with the Big Bang nucleosynthesis (BBN) bounds on the effective number of extra light neutrinos, since the mirror photons, electrons and neutrinos would give a contribution to the Hubble expansion rate equivalent to the effective number of extra neutrinos $\Delta N_\nu \approx 6.14$.

Therefore, the BBN bounds require that at the nucleosynthesis epoch the temperature of the mirror sector should be smaller than that of the ordinary one, $T' < T$. In this case the contribution of the mirror sector translates into $\Delta N_\nu \approx 6.14 x^4$ [13, 15], where $x = T'/T$. Thus, the BBN bound on $\Delta N_\nu$ implies the upper limit $x < 0.64 (\Delta N_\nu)^{1/4}$, with rather mild dependence on $\Delta N_\nu$. E.g. $\Delta N_\nu < 0.5$ implies roughly $x < 0.5$ while $\Delta N_\nu < 0.2$ implies $x < 0.4$. Hence, in the early Universe the mirror system should have a somewhat lower temperature than ordinary particles. This situation can be realized in a most plausible way under the following paradigm:

A. At the “Big Bang” the two systems are born with different efficiency: namely, at the post-inflationary epoch the M sector is (re)heated at lower temperature than in the observable one, which can be naturally achieved in certain models [13, 15, 16].

B. At temperatures below the reheating temperature two systems interact very weakly, so that they do not come into thermal equilibrium with each other after reheating. This condition is automatically fulfilled if the two worlds communicate only via gravity. If there are some other effective couplings between the O and M particles, they have to be properly suppressed.

C. Both systems expand adiabatically, without significant entropy production, keeping nearly constant the ratio of their temperatures $T'/T$.

The parameter $x = T'/T$ plays an important role for describing the cosmological
evolution of mirror world. As far as the mirror world is cooler than the ordinary one, \( x < 1 \), in the mirror world all key epochs as are baryogenesis, nucleosynthesis, recombination etc. proceed in somewhat different conditions than in ordinary world. Namely, in the mirror world the relevant processes go out of equilibrium earlier than in ordinary world, which has many far going implications.

First, as far as baryogenesis epoch is concerned, the origin of the baryon asymmetry in both sectors is related to the same particle physics, as far as both sectors have identical Lagrangians. However, the out-of-equilibrium condition in the \( M \) sector is satisfied better than in \( O \) sector \[15\]. Therefore, it is pretty plausible that asymmetry of \( M \) baryons is bigger than baryon asymmetry in ordinary sector and hence mirror baryons could constitute dark matter, or at least its significant fraction. The situation emerges in a particularly appealing way in the leptogenesis scenario due to entropy and lepton number leaking from the hotter \( O \) sector to the cooler \( M \) sector \[17\], which leads to \( \Omega_B'/\Omega_B \geq 1 \), up to an order of magnitude. This can explain the close relation between the visible and dark matter components in the Universe in a rather natural way \[3, 17\]. (For a somewhat different scenario see also refs. \[18\]).

Second, in mirror sector radiation decouples from matter earlier than in ordinary one, at redshifts \( z_{\text{dec}}' \approx (1/x)z_{\text{dec}} \), where \( z_{\text{dec}} \approx 1100 \) is the redshift of decoupling in ordinary sector \[15, 19\]. In particular, for \( x < 0.3 \) the mirror photons decouple before the matter-radiation equality, yet in the relativistic expansion epoch. As a result, for such small values of \( x \), for the cosmological scales which still undergo the linear growth, the mirror baryons behave exactly the same way as conventional cold dark matter (CDM) \[15\]. The exact computations show that for \( x < 0.3 \) implications of the mirror baryons for the cosmic microwave background and the large scale structure of the Universe are practically indistinguishable from that of the CDM \[20\].

Third, as far as the primordial nucleosynthesis epoch is concerned, it was shown in ref. \[15\] that the mirror helium abundance should be much larger than that of the ordinary helium, and for \( x < 0.3 \) the mirror helium gives a dominant mass fraction of the mirror matter. The reason is simple. As far as \( x << 1 \), then the impact of mirror sector on ordinary nucleosynthesis is insignificant: it is equivalent to \( \Delta N_\nu \approx 6.14x^4 \), and so the BBN prediction for ordinary helium mass fraction \( Y_4 \approx 0.24 \) is not affected significantly. However, the impact of the ordinary sector on the mirror nucleosynthesis is dramatic: it is equivalent to \( \Delta N'_\nu \approx 6.14/x^4 \), and so for \( x = 0.6 - 0.1 \) the mirror helium mass fraction varies from \( Y_4' = 0.4 - 0.8 \) \[15\].

Concluding, mirror matter can be a viable candidate for dark matter of the Universe. It has the same microphysics as ordinary matter, but somewhat different cosmology, which makes it extremely interesting object for further investigations. Mirror baryons form the stable matter, exactly as their ordinary counterparts, and they should form atoms, molecules, and then even astrophysical objects, such as stars, planets, globular clusters, etc., as the ordinary matter does, however the chemical content of the mirror matter should be different from the ordinary one. In particular, it should be mainly helium dominated and in addition the heavier elements are also expected with bigger abundances than in ordinary world. Once the visible matter is built up by ordinary baryons, then the mirror baryons would constitute dark matter in a natural way. They interact with mirror photons, but not interact with ordinary photons. In other words, mirror baryons are dark for the ordinary observer, and mirror stars as well, and so the latter should be seen as Massive Astrophysical Compact Halo Objects (MACHOs) in the microlensing experiments \[13, 14\]. However, if there exists small kinetic mixing between ordinary

\[^1\]The signatures of mirror matter in the search of meteoric event anomalies \[21\] and close-in extra solar planets \[22\] were also discussed.
and mirror photons \cite{8}, the mirror particles become sort of “millicharged” particles for the ordinary observer, which suggests a very appealing possibility of their detection in the experiments for the direct search of dark matter \cite{2,23}. It is also extremely interesting that such a kinetic mixing can be independently tested in laboratory “table-top” experiments for searching the orthopositronium oscillation into its mirror counterpart \cite{24}. The mixing between the ordinary and mirror neutrinos \cite{9} can also be tested in terms of active-sterile neutrino oscillations \cite{25}. In addition, it could provide a possible mechanism for the generation of ultra high energy neutrinos \cite{16} and for the gamma ray bursts as a result of explosion of the mirror supernovae \cite{26}. Neutron – mirror neutron oscillation could provide a very efficient mechanism for transporting ultra-high energy protons and also explain their correlation with far distant sources like BL Lacs \cite{27}. Explosions of the mirror supernovae could provide a necessary energy budget for heating the gaseous part of the mirror matter in the galaxies and hence to prevent its collapse to a disk, in which case the mirror matter could form spheroidal halos in accord to observations \cite{28}. In addition, the efficiency of mirror supernovae explosions can be indirectly tested in the future detectors for the gravitational waves at the frequencies around 1 kHz. Therefore, one of the most interesting problems concerning the mirror world consists in the study of the formation and evolution of mirror stars.

The aim of this paper is to study the evolutionary and structural properties of mirror stars. As far as the mirror world should be dominated by helium, and thus it contains less hydrogen, the evolutionary properties of the mirror stars should be significantly different from that of the ordinary stars. In particular, we calculate the evolutionary times of the mirror stars in the wide range of masses for different abundances of the mirror helium, and study under which conditions they could end up as supernovae. Apart of motivations which has been already discussed above, the possible concrete applications of our analysis for the testing of the mirror matter features can be formulated as follows:

- The mirror stellar evolution is one of the crucial ingredients in non-linear astrophysical processes involved in the cosmological structure formation. In particular we need the stellar feedback in order to compute N-body simulations of structure formation at non-linear scales, and extend the matter power spectra calculated in our previous works \cite{20} to smaller scales. This would help us to understand the expected crucial differences between the mirror baryonic dark matter and CDM scenarios.

- We need mirror stellar models in order to understand the circumstances under which in the galaxy the ordinary matter forms a disk while mirror matter can form the spheroidal halo. In this analysis it is crucial to have in view the two aspects played by the mirror star formation and evolution: first, M stars are collisionless and then do not dissipate the energy falling in the disk, but stay in the halo; and second, the heavy mirror stars could explode as supernovae and could provide the necessary energy emission required for the mirror galaxy to balance the dissipation and avoid the formation of a mirror disk.

- M stars are a natural candidate for the MACHOs observed in the galaxy. At present the baryonic candidates seem unable to explain the MACHO galactic population \cite{31}, and CDM has many problems to clump in objects of stellar masses, so that the mirror hypothesis is an interesting viable alternative. The study of the features of the mirror star evolution (together with the star formation) is one of the necessary

\footnote{Also the mirror axion could provide a plausible mechanism for the gamma ray bursts and supernova type II explosions \cite{11}.}
ingredients for answering the question which fraction of the mirror matter can exist in the form of mirror MACHOs and which fraction in the form of gas or dust.

- This study is also necessary for understanding the rate and properties of mirror supernovae, which could be responsible for the gamma ray bursts observed in our sector [26] and provide gravitational waves observable by next generation detectors.

- A possibly useful collateral effect of this study is that it is applicable to very late population of ordinary stars, when a large part of hydrogen has burned into helium and new stars form with much more helium in their chemical compositions.

The plan of the paper is as follows. In next section we introduce the mirror stars as MACHO candidates. In sections 3 we present the mirror star models. Section 4 analyzes in detail the evolutionary and structural properties of the stars, showing the luminosities, effective temperatures, central densities and temperatures, helium core masses at ignition and the minimum mass for carbon ignition, as functions of the primordial helium content and of the mass. Finally, our main conclusions are summarized in section 5.

2 Mirror stars as helium dominated stars

If mirror matter exists, then the existence of mirror stars, in a certain sense, is guaranteed by the existence of ordinary stars: given that two sectors have the same microphysics, stars necessarily form in both of them. Due to different initial conditions, $x = T'/T < 1$, two sectors have different cosmological evolution, and in particular, different chemical content. Thus, the details of the galaxy formation scenario depend on the exact composition of matter and they can be different in two sectors. However, one has to take into account that during the galaxy evolution some fraction of mirror baryons would fragment into stars.

As far as the chemical contents are concerned, for a given value of $x$, the primordial helium abundances in ordinary and mirror sectors roughly can be given by the following formulas [15]

$$Y = \frac{2 \exp[-t_N/\tau(1 + x^4)^{1/2}]}{1 + \exp[\Delta m/T_W(1 + x^4)^{1/6}]}$$

and

$$Y' = \frac{2 \exp[-t_N/\tau(1 + x^{-4})^{1/2}]}{1 + \exp[\Delta m/T_W(1 + x^{-4})^{1/6}]}$$

where $\tau = 887$ s is the neutron lifetime, $\Delta m = 1.29$ MeV is the neutron-proton mass difference, $T_W \simeq 0.8$ MeV is a weak interaction freezing temperature in the standard cosmology, and $t_N \sim 200$ s is the cosmological timescale corresponding to the “deuterium bottleneck” temperature $T_N \sim 0.07$ MeV. Therefore, for $x \ll 1$ the standard BBN predictions essentially are not affected. Namely, the smaller is $x$, the prediction for ordinary helium decreases and gets closer to the standard BBN result $Y \simeq 0.24$, while the prediction for the mirror helium increases and at $x \to 0$ approaches $Y' \to 1$. In particular, for $x = 0.6$ one has $Y' \simeq 0.4$, while for $x = 0.1$, $Y' \simeq 0.8$ (for more precise computations see [15]). Therefore, in two sectors the first stars are formed with different initial abundances of helium.

The evolutionary and structural properties of stars strongly depend on the initial chemical composition, which is fixed by the helium abundance and by the global amount of heavy elements (indicated as metallicity $Z$). The primordial abundances of ordinary nuclei heavier than helium are estimated to be very small ($Z \sim 10^{-10}$). This metallicity would be
characteristic of the ‘first’ stellar population, the so-called Population III stars. Concerning
the primordial metallicity of mirror matter, it can be be some orders of magnitude higher,
however there are no reasons to expect that it will be relevant.

Meanwhile, given the complexity of the physics of galaxy formation (this process in still
to be well understood), we can make some general considerations. At a stage during the
process of gravitational collapse of the protogalaxy, it fragments into hydrogen clouds with
typical Jeans mass (for mirror matter, these gas clouds are rather the hydrogen-helium
clouds). Clouds continue to cool and collapse until the opacity of the system becomes so
high that the gas prefers to fragment into protostars. This complex phenomenon lead a
part of the protogalactic gas to form the first stars (probably very massive, with \( M \sim 10^2–
10^3 M_\odot \)). A difficult question to address here is related to the star formation in mirror
sector, also taking into account that its temperature/density conditions and chemical
contents are much different from the ordinary ones. Clearly, the details of this process
(Jeans mass, etc.) depend on these conditions, and hence should be different for ordinary
and mirror matter components. The cooling rates are mainly determined by hydrogen
atoms and molecules, while helium is much less effective. However, even in mirror sector,
unless \( x \) is extremely small, the number density of hydrogen remains significant.

The pattern galaxy evolution features should drastically depend on the mirror star
formation and evolution features. Stars play an important role: the fraction of baryonic
gas involved in their formation becomes collisionless on galactic scales, and supernova
explosions enrich the galaxy of processed collisional gas (stellar feedback). Too fast star
formation in mirror component would extinct the mirror gas and thus could avoid that
mirror baryons form disk galaxies as ordinary baryons do. If the mirror protogalaxy, at
certain stage of collapse, transforms into the collisionless system of the mirror stars, then
it could maintain a typical elliptical structure. Certainly, in this consideration also the
galaxy merging process should be taken into account. Efficient merging of mirror disks
mostly built up of stars, also would lead to ellipticals.\(^3\) As for ordinary matter, within
the dark mirror halo it should typically show up, depending on conditions of the galaxy
formation, as an observable elliptic or spiral galaxy, but some anomalous cases can also
be possible, like certain types of irregular galaxies or even dark galaxies dominantly made
out of mirror baryons. The central part of halo can nevertheless contain a large amount
of ionized mirror gas and it is not excluded that it can have a quasi-spherical form, thus
possibly avoiding the problem of cusp typical for the CDM halos. Even if mirror star
formation is very efficient, the massive mirror stars in the dense central region could fast
evolve and explode as supernovae, leaving behind compact objects like neutron stars or
black holes, and reproducing the mirror gas and dust. It is interesting to understand
whether these features could help in understanding the process of the formation of the
central black holes, with masses \( 10^6 – 10^9 M_\odot \), which are considered as main engines of
the quasars and active galactic nuclei.

The fact that dark matter made of mirror baryons has the property of clumping into
compact bodies such as mirror stars leads to a natural explanation for the mysterious
MACHOs. In the galactic halo (provided that it is the elliptical mirror galaxy) the mirror
stars should be observed as MACHOs in gravitational microlensing \([14]\). The MACHO
collaboration \([32]\) studied the nature of halo dark matter by using this technique. This
experiment has collected 5.7 years of data and provided statistically strong evidence for
dark matter in the form of invisible star sized objects, which is what you would expect if

\(^3\)In other words, one can speculate on the possibility that mirror baryons form mainly the elliptical
galaxies. For a comparison, in the ordinary world the observed bright galaxies are mainly spiral while the
elliptical galaxies account about 20% of them. Remarkably, the latter contains old stars, very little dust
and shows no sign of active star formation.
there was a significant amount of mirror matter in our galaxy. Their maximum likelihood analysis implies a MACHO halo fraction of 20\% for a typical halo model with a 95\% confidence interval of 8\% to 50\%. Their most likely MACHO mass is between 0.15M_\odot and 0.9M_\odot (depending on the halo model), with an average around M \approx 0.5 M_\odot, which is difficult to explain in terms of the brown dwarfs with masses below the hydrogen ignition limit M < 0.1M_\odot or other baryonic objects\textsuperscript{4,5} \cite{31}. These observations are consistent with a mirror matter halo because the entire halo would not be expected to be in the form of mirror stars. Mirror gas and dust would also be expected because they are a necessary consequence of stellar evolution and should therefore significantly populate the halo. Thus, perhaps MACHOs are the first observational evidence of the mirror matter.

It is also plausible that in the galactic halo some fraction of mirror stars exists in the form of compact substructures like globular or open clusters, in the same way as it happens for ordinary stars. In this case, for a significant statistics, one could observe interesting time and angular correlations between the microlensing events.

3 The stellar models

As we know, in the exact mirror symmetric scenario the microphysics of the hidden sector is exactly the same as the visible one, the only changes are due to the boundary conditions. This is a very favourable condition for the study of M stars, because the necessary knowledge is the same than for the O ones, that we know very well. This means that M stars follow the same evolutionary stages than visible ones. A very brief review of stellar evolution will be given at the beginning of the next section.

The same physics for both sectors means that the equations governing the mirror stellar evolution and the physical ingredients to put inside them (namely the equation of state, the opacity tables, and the nuclear reactions) are the same as for visible stars. The only change regards the composition of the M star. In fact, while the typical helium abundance for O stars is Y \approx 0.24, for the M stars we have Y' = 0.40-0.80. This interval is obtained considering that its lower limit is given by the primordial helium abundance coming from the mirror Big Bang nucleosynthesis studied in ref. \cite{15}. In next section we will evaluate its impact on the evolution of M stars.

If we consider a single isolated star\textsuperscript{6}, its evolutionary and structural properties depend only on the mass and the chemical composition. In particular, the latter is expressed by the abundances by mass of hydrogen (X), helium (Y), and the so-called heavy elements or metals (Z), i.e. all the elements heavier than H and He, so that the condition X+Y+Z = 1 is fulfilled.\textsuperscript{7}

We computed mirror star models using the FRANEC (Frascati RApshon Newton Evolutionary Code) evolutionary program, a numerical tool that solves the equations of stellar structures \cite{35}. As physical inputs for this code we chose the opacity tables of ref. \cite{36} for temperatures lower than 10000 K and those obtained in the Livermore laboratories

\textsuperscript{4}An interesting MACHO candidate are the Very Low Mass (VLM) stars (see ref. \cite{33} and references therein), namely stars with masses below \sim 0.4M_\odot. They have very low luminosities and temperatures, and hence are very difficult to detect and possibly contribute to the galactic dark stars, but anyway their estimated abundances cannot account for the missing mass of the galaxies.

\textsuperscript{5}Indeed the rate of MACHO events as derived from microlensing data is still a controversial issue, as emphasized in ref. \cite{34} and references therein. Clearly the outcome of the controversy is crucial for the mirror matter model.

\textsuperscript{6}We are practically neglecting, as usual, the interactions existing in systems of two or three stars.

\textsuperscript{7}From now on we will use the prime (') to indicate mirror quantities only if they appear together with ordinary ones; otherwise we don't use it, taking in mind that high Y-values refer to mirror stars and low Y-values to the ordinary ones.
and described in ref. 37 for higher temperatures, the equation of state presented in ref. 38, implemented in the low-temperature regime with a Saha equation of state, and the Eddington approximation to the grey atmosphere solution for the integration of stellar atmospheres 5. These inputs are valid over the entire ranges of temperatures and densities reached by our models.

For what concerns their evolutionary and structural properties, mirror stars are equivalent to ordinary stars with a very high helium abundance. Thus, we computed stellar models for large ranges of masses and helium contents, and for a low metallicity Z.

As far as it concerns the adopted value for this metallicity, we could adopt a value equal to zero as for ‘normal’ Pop. III stars. However, also in the O sector we still lack of any empirical evidence for low-mass Pop. III stars, i.e. of stars with metallicity equal to zero. This result has been often explained as a consequence of the peculiar initial mass function 9 of Pop. III stars: the lack of metals should make less efficient the cooling processes within the primordial clouds so their fragmentation could produce only high-mass stars (whose evolutionary lifetimes are so short that they are no more observable. In the M sector, due to strong reduction of the H abundance, due to the huge increase of the He abundance, the cooling processes inside the primordial clouds should have also a lower efficiency, so only very massive M stars should form. For such reason, being interested to stellar objects that presently can ‘work’ as MACHOs, we consider for present computation a metallicity $Z = 10^{-4}$, very low but larger than zero, characteristic of the so-called stellar population II, i.e. an old stellar population, coming soon after the first one.

4 Evolution of the mirror stars

First of all we remember that in stellar astrophysics the evolution of a star is studied in the so-called H-R (Hertzsprung-Russell) diagram, where we plot the luminosity $L$ and the effective temperature $T_{\text{eff}}$ of the star 10. In order to understand the evolutionary differences between ordinary and mirror stars, it is also necessary to give a very brief review of basic stellar evolutionary theory.

During the first fast phase of gravitational contraction at nearly constant effective temperature and decreasing luminosity, the star goes down along its Hayashi track 11, negligibly slowing down its contraction only while the structure is fast burning the few light elements (D, Li, Be, B) present. Contraction increases the central temperature $T_c$, until stars with masses $M \gtrsim 0.1 M_\odot$ 12 ignite the hydrogen burning 13 in their cores at a

$\text{8The Eddington approximation assumes local thermodynamical equilibrium with opacity independent of frequency. In this case the temperature in the stellar atmosphere is given by}$

$$T^4(\tau) = \frac{3}{4} T_{\text{eff}}^4 \left( \tau + \frac{2}{3} \right),$$

$\text{where } T(\tau) \text{ is the temperature of an atmospheric layer located at the optical depth } \tau, \text{ and } T_{\text{eff}} \text{ is the effective temperature of the star.}$

$\text{9The initial mass function gives the number of stars with mass in the range } M \div (M + dM) \text{ formed within a given stellar environment.}$

$\text{10The effective temperature } T_{\text{eff}} \text{ of a star is defined by}$

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4,$$

$\text{where } \sigma \text{ is the Stephan-Boltzmann constant, } L \text{ is the luminosity, and } R \text{ is the radius at the height of the photosphere. Thus, } T_{\text{eff}} \text{ is the characteristic temperature of the stellar surface if it emits as a black body.}$

$\text{11Hayashi track is the evolutionary track of a totally convective stellar model. It is the coldest possible track for a star of a given mass, and it is located at the extreme right of the H-R diagram.}$

$\text{12The exact value of the hydrogen burning minimum mass } M_{\text{hbmm}} \text{ is dependent on the metallicity. For our models } Z = 10^{-4} \text{ and } M_{\text{hbmm}} \simeq 0.1 M_\odot \text{, while for solar metallicity } Z = 0.02 \text{ and } M_{\text{hbmm}} \simeq 0.08 M_\odot.}$

$\text{13There are two possible ways of burning hydrogen: the first one, called PP or proton-proton chain,}$

Figure 1: Evolutionary tracks in the H-R diagram of stars with $M = 0.8 M_\odot$, $Z = 10^{-4}$ and different helium contents $Y = 0.24, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80$. The last (brightest) point along each evolutionary track corresponds to the He ignition stage (see text for more details).
temperature $T_c \sim 6 \times 10^6$ K, while the ones with lower masses do not ignite hydrogen and die as brown dwarfs. After depletion of hydrogen in the core, the burning passes to a shell at the boundary of the core, which is now made of He. At this stage the star starts to decrease its effective temperature (turn-off). Meanwhile the He core contracts until stars with $M \gtrsim 0.5 M_\odot$ reach a central temperature $T_c \sim 10^8$ K and start He-burning into C and O; stars with lower masses die as white dwarfs and start their cooling sequences. The stars with mass larger than $6-8M_\odot$ are able to ignite the successive nuclear burning processes and die exploding as type II supernovae leaving in their place a neutron star or a black hole.

A key point is the evaluation of evolutionary times. For any given stellar mass, the evolutionary phase with the longest lifetime is the one corresponding to the central hydrogen burning stage (the so-called main sequence) with time-scales of $10^{10}$ yr for masses near the solar mass. Thus, we can approximate the lifetime of a star with its main sequence time. Since both luminosity and effective temperature depend on the mass and chemical composition, clearly the lifetime too depends on them. We use now the proportionality relations valid for low mass stars \[39\]

$$L \propto \mu^{7.5} M^{5.5} \quad (3)$$

and

$$T_{\text{eff}}^4 \propto \mu^{7.5} \quad , \quad (4)$$

where $\mu$ is the mean molecular weight. From eq. \[3\] we obtain that bigger masses need higher luminosities, so that they use all the available hydrogen earlier than the lighter ones. From both eqs. \[3\] and \[4\] we know that an increase of helium abundance corresponds to an increase of the mean molecular weight and consequently in both luminosity and effective temperature. The increase in luminosity means that the star needs more fuel to produce it, but at the same time its amount is lower, because higher $Y$ values necessarily imply lower $X$ values. Both these events act to shorten the lifetime of a mirror star, which has a high He content. This can be formalized in the following relation \[39\]

$$t_{\text{MS}} \propto \frac{X}{\mu^{1.4}} \sim X \left(\frac{5X + 3}{4}\right)^{1.4} \quad , \quad (5)$$

where $t_{\text{MS}}$ is the main sequence lifetime.

The average change of the effective temperature is a consequence of the fact that the radiative opacity of a He-rich mixture is lower, and a lower opacity produces hotter effective temperatures.

After these predictions on evolutionary properties of He-rich stars, we analyze the quantitative results of our models. They can be divided into two groups. The first one is made of models with mass $M = 0.8M_\odot$ and many different $Y$ values. The second one is made instead of models with only three different He contents and a large range of masses.

We start from figure 11 where we plot the models of $0.8M_\odot$ in the H-R diagram. The models are followed until the He-burning ignition, i.e. along their main sequence, turn-off and red giant\[14\] phases, which practically occupy all their lifetimes. Our qualitative predictions are indeed confirmed. Models with more helium are more luminous and hot; for example the main sequence luminosity ratio of the model with $Y = 0.80$ to the one with $Y = 0.24$ is $\sim 10^2$. Other consequences of an He increase are a longer (in the diagram, becomes efficient at $T_c \sim 6 \times 10^6$ K, while the second one, called CNO chain, at $T_c \sim 15 \times 10^6$ K. Since their efficiencies are dominant at different temperatures, the PP chain provides energy for smaller stellar masses, and the CNO does it for bigger ones.

\[14\] A red giant is a cold giant star in the phase of H-burning in shell before the He-ignition.
Figure 2: Evolutionary times (listed in table 1) for models with $M = 0.8 \, M_\odot$, $Z = 10^{-4}$ and different helium contents $Y = 0.24, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80$. The line shows the quadratic fit.

Figure 3: Evolutionary tracks in the H-R diagram of stars with different masses $M = 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, 7.0, 10 \, M_\odot$, $Z = 10^{-4}$ and two different helium contents $Y = 0.24$ and 0.50.
not in time) phase of decreasing temperature at nearly constant luminosity, and a shorter red giant branch.

From these models we computed the evolutionary times until the He-ignition, i.e. for the entire plotted tracks, and we summarize them in table 1. As expected, the ages decrease for growing $Y$, but we see now how much high is this correlation. For $Y = 0.40$ the lifetime is already about one third compared to a visible ($Y = 0.24$) star, while for the highest value, $Y = 0.80$, it is roughly $10^2$ times lower. We can approximately say that an increase of 10% in helium abundance roughly divides by two the stellar lifetime. In figure 2 we plot the evolutionary times listed in the table. We see that, using a logarithmic scale for the stellar age, we obtain a quasi-linear relation between it and the helium content for this range of parameters. An almost perfect approximation (the standard deviation for $\log(age)$ is $\sigma_{age} = 0.0055$) can be obtained with a quadratic fit, which gives the following expression for the age expressed in Gyr

$$\log (age) = 1.61 - 1.81 Y - 1.46 Y^2$$ \quad for \quad $M = 0.8 M_\odot$. \quad (6)

Let us now extend the analysis to models covering a large range of masses, from 0.5 $M_\odot$ to 10 $M_\odot$. Since the dependence on the helium content has been already studied for the 0.8 $M_\odot$ case, we concentrate on only three $Y$ values. Figure 3 shows the evolutionary tracks for all the masses and only two helium contents, and again the models are followed up to the He-ignition. For every mass the $Y$ dependence is the same as for the above discussed 0.8 $M_\odot$ model. For models with masses $M \gtrsim 2 M_\odot$ the growth in $Y$ causes a considerable increase of the He-ignition effective temperature, together with the disappearance of the red giant branch.

In table 2 we list the lifetimes for all masses and the three indicated helium contents. The ratio of an ordinary star ($Y = 0.24$) evolutionary time to the high He-content mirror one ($Y = 0.70$) is between $\sim 30$ for 0.5 $M_\odot$ and $\sim 10$ for 10 $M_\odot$. These data are plotted in figure 4, where we see that the same dependence on the star masses holds for every helium content, with a shift toward lower ages for higher $Y$ values. In this case we perform a quadratic fit using logarithmic scales in both coordinates and with age expressed in Gyr, obtaining

$$\begin{align*}
\log (age) &= 0.738 - 3.398 \log M + 0.966 (\log M)^2 \quad \text{for} \quad Y = 0.24 , \quad (7) \\
\log (age) &= 0.016 - 3.143 \log M + 0.986 (\log M)^2 \quad \text{for} \quad Y = 0.50 , \quad (8) \\
\log (age) &= -0.630 - 2.750 \log M + 0.802 (\log M)^2 \quad \text{for} \quad Y = 0.70 , \quad (9)
\end{align*}$$

where the standard deviations $\sigma_{age}$ for $\log(age)$ are respectively 0.018, 0.010, 0.029. We can also consider together the dependencies of stellar ages on both the helium content and stellar masses, obtaining

$$\begin{align*}
\log (age) &= 1.31 - 2.16 Y - 3.73 \log M - 0.86 Y^2 + \\
&\quad +1.34 Y \log M + 0.99 (\log M)^2 - 0.28 Y^2(\log M)^2 . \quad (10)
\end{align*}$$

This is an evidence that, under large mass ranges and different boundary conditions (in terms of temperatures of the mirror sector, and thus stellar helium contents), the lifetimes of mirror stars are roughly an order of magnitude smaller than the ones of visible stars. This means that, compared to O stars, M stars evolve faster and enrich earlier the galaxy of processed mirror gas, with implications for galaxy evolution.

In fig. 5 we show the behaviour of the central temperature as a function of the central density during the main H-burning phase for the M star models with an He abundance
Table 1: Ages computed for stars of mass $M = 0.8M_\odot$ and the indicated helium contents.

<table>
<thead>
<tr>
<th>mass $(M/M_\odot)$</th>
<th>age $(10^9 yr)$ $(Y = 0.24)$</th>
<th>age $(10^9 yr)$ $(Y = 0.50)$</th>
<th>age $(10^9 yr)$ $(Y = 0.70)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>66.7</td>
<td>11.2</td>
<td>1.92</td>
</tr>
<tr>
<td>0.6</td>
<td>53.5</td>
<td>5.80</td>
<td>1.04</td>
</tr>
<tr>
<td>0.8</td>
<td>12.4</td>
<td>2.17</td>
<td>0.417</td>
</tr>
<tr>
<td>1.0</td>
<td>5.73</td>
<td>1.05</td>
<td>0.219</td>
</tr>
<tr>
<td>1.5</td>
<td>1.50</td>
<td>0.301</td>
<td>0.0902</td>
</tr>
<tr>
<td>2.0</td>
<td>0.608</td>
<td>0.140</td>
<td>0.0445</td>
</tr>
<tr>
<td>3.0</td>
<td>0.204</td>
<td>0.0564</td>
<td>0.0178</td>
</tr>
<tr>
<td>4.0</td>
<td>0.110</td>
<td>0.0313</td>
<td>0.00941</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0697</td>
<td>0.0205</td>
<td>0.00656</td>
</tr>
<tr>
<td>7.0</td>
<td>0.0366</td>
<td>0.0117</td>
<td>0.00414</td>
</tr>
<tr>
<td>10</td>
<td>0.0202</td>
<td>0.00718</td>
<td>0.00278</td>
</tr>
</tbody>
</table>

Table 2: Ages computed for stars of the indicated masses and helium contents, with a metallicity $Z = 10^{-4}$.

Figure 4: Evolutionary times (listed in table 2) for models with $M = 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, 7.0, 10 M_\odot$, $Z = 10^{-4}$ and three different helium contents $Y = 0.24, 0.50$ and $0.70$. Are also indicated the models of figure 2 and the fitted curves.
Figure 5: Behaviour of the central temperature $T_c$ as a function of the central density $\rho_c$ during the evolution for stars with different masses: $M = 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 7.0, 10 \ M_\odot$, $Z = 10^{-4}$, and $Y = 0.50$. 
Figure 6: The trend of the central temperatures $T_c$ as a function of the central density $\rho_c$ of two stellar models with masses equal to 0.8 $M_\odot$ and 2 $M_\odot$, metallicity $Z = 10^{-4}$ and initial He contents equal to $Y = 0.24$ and 0.50.
Figure 7: Evolutionary tracks in the H-R diagram of stars with $M = 2.4 M_\odot$, $Z = 10^{-4}$ and different helium contents $Y = 0.24, 0.30, 0.50, 0.70$. The filled circles along each evolutionary track correspond to the He ignition stage (see text for more details).
equal to $Y=0.50$. In order to allow to evaluate the effects of the initial He abundance on the behaviour with time of the thermal conditions at the center of the star, in fig. 6 we plot the behaviour of the central temperature as a function of the central density for two selected stellar masses, 0.8 and $2 M_\odot$, for different assumptions about the initial He content.

It is worth noticing that a larger He content has a huge effect on the central thermal conditions: larger the He content, hotter the star for any fixed value of the central density. The effect is larger for less massive stars, i.e. $M \leq 1.0 M_\odot$, with respect the more massive. This evidence has to be related to the different H-burning mechanism at work in the different ranges of mass: in low-mass stars H-burning occurs mostly through the $p$-$p$ chain which has a lower temperature dependence with respect the CNO cycle (occurring in more massive stars). So, when the initial He content increases, in order to fulfil the stellar energy requirement, the central temperature has to increase larger than in more massive stars.

One has also to note that the significant increase of the central temperature, when $Y$ increases, has the quite important consequence that for initial He abundance of the order of $\sim 0.40$ also low-mass stars with mass around $0.8 M_\odot$ reaches in their interiors the temperature required to activate the CNO cycle ($T \approx 14 \times 10^6 \ K$). When this happens the H-burning occurs in a convective core. This occurrence has a further consequence which will be discussed in the following.

The huge increase of the central temperature with the initial He content has a strong impact also on the evolutionary phases successive to the core H-burning one: as the central temperature increases the thermal conditions requested in order to ignite He-burning are achieved sooner after the central H-exhaustion. This means that intermediate-mass stars, i.e. those structures which do not develop conditions of electron degeneracy inside their He core, are able to start burning He at quite larger effective temperature as already shown (see also fig. 7).

In low-mass stars, the physical behaviour is also more complicated. In the O sector, these structures, at the end of the core H-burning phase develop conditions of strong electron degeneracy inside their He core. As a consequence they are forced to largely increase the mass of the He core, through the H-burning occurring in a shell surrounding the He core, until the energy released by the gravitational contraction does overcome the energy losses due to thermal conduction and plasma neutrinos. This occurs near the tip of the Red Giant Branch (RGB), where the thermal conditions for He ignition are achieved and the star starts to burn helium through a violent He flash\textsuperscript{15}.

In the M sector, the large initial He content has the consequence that the structures are always hotter than O stars. So, all along their evolution, low-mass M stars develop electron degeneracy at a quite lower level. This occurrence has the important effect of strongly reducing the mass of the He core at the tip of the RGB and, in turn, the brightness of the tip (so the extension in luminosity of the RGB is significantly reduced\textsuperscript{16}).

In fig. 8 and table 3 we show the trend of the mass of the He core at the RGB tip, i.e. at the He ignition, as a function of the initial He abundance: the reduction of the He core mass when increasing the He content from $\sim 0.2$ to $\sim 0.7$ is really quite large $\sim 0.15 M_\odot$. However, when increasing the value of $Y$ from 0.70 to 0.80, we notice that the He core mass does increase at odds with previous indications. This is the consequence of the fact that increasing $Y$ to quite large values, causes that the H-burning occurs via CNO cycle

\textsuperscript{15}The He ignition is a violent process since it occurs inside a He core that is under conditions of strong electron degeneracy.

\textsuperscript{16}The brightness of the point corresponding to the He ignition is strongly dependent on the mass of the He core. This occurrence provides a direct explanation of the behaviour of the brightest point along the tracks plotted in fig. 7 with the initial He content.
so in a convective core. In addition, the stellar interiors are so hot that they are no more affected by electron degeneracy. The 0.8\( M_\odot \) model computed by assuming \( Y = 0.80 \) is so hot that it burns H in a convective core and does not develop electron degeneracy at the central H-exhaustion, so its mass of the He core at the He ignition is no more fixed by the level of electron degeneracy but it depends on the maximum extension of convective core during the core H-burning phase.

In more massive stars which also in the O sector do not develop conditions of electron degeneracy in their He core, this occurrence is also more evident (see table 4 and fig. 9). In fact, increasing the initial He content, the mass size of the convective core during the central H-burning stage increases as a consequence of the larger radiative flux (we remember that He-rich stars are brighter and then more energy has to be produced via nuclear burning in the interiors), and in turn the mass of the He core at the He ignition largely increases.

For any given stellar population, i.e. for each fixed initial chemical composition, one of the most important parameters characterizing that stellar population is the value of the critical mass (\( M^{up} \)) between the stars which after the core He-exhaustion are not able to ignite the successive nuclear burning processes so becoming carbon-oxygen white dwarfs, and those structures massive enough to ignite carbon and all successive nuclear burnings so concluding their life as type II supernovae.

Figure 10 and table 5 show the trend of \( M^{up} \) as a function of the initial He content: it is worth noting the quite relevant reduction in the value of this critical mass when increasing the He content from \( \sim 0.20 \) to \( \sim 0.8 \). This occurrence is due to the convolution of two effects related to the increase of the He abundance: 1) the increase of the He core at the He ignition, 2) the larger temperatures of the stellar interiors.

The huge reduction of the value of \( M^{up} \) has the important consequence that in the M sector - when assuming an initial mass function similar to that of the O sector - the expected fraction of stars exploding as type II supernovae should be quite larger than that expected in the O sector. In the meantime, the fraction of carbon-oxygen white dwarfs - which could be potentially contributors to the MACHOs population - would be significantly reduced.

From the detailed study of this evolution together with the necessary information of the initial mirror stellar mass function, one could predict the expected population of mirror stars, in order to compare it with current MACHO observations. In addition, one could also evaluate the amount of gravitational waves expected from supernovae in the mirror sector. These are just some examples of interesting future applications of the present study.

### 5 Conclusions

In this paper we have investigated the astrophysical consequences of the existence of a mirror sector on stellar scales. In particular we studied in detail the evolutionary and structural properties of mirror stars, considered as very natural MACHO candidates.

Considering that BBN epoch in the mirror (M) world proceeds differently from the ordinary (O) one, and it predicts the mirror helium abundance in the range \( Y' = 0.4 - 0.8 \), considerably larger than the observable \( Y \simeq 0.24 \), we studied the stars made of a large amount of helium (they can be both mirror stars or ordinary stars in a very late evolutionary phase of the Universe, when a lot of hydrogen already burned in helium).

Using a numerical code, we computed evolutionary models of stars for high values of helium content and low metallicity (\( Z = 10^{-4} \)), and for a large range of masses, spanning from 0.5 to 10 \( M_\odot \).
Table 3: He-core masses computed at He ignition for stars of mass $M = 0.8 M_\odot$ and the indicated helium contents.

<table>
<thead>
<tr>
<th>$Y$</th>
<th>0.24</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{He}(M_\odot)$</td>
<td>0.509</td>
<td>0.493</td>
<td>0.465</td>
<td>0.430</td>
<td>0.387</td>
<td>0.351</td>
<td>0.378</td>
</tr>
</tbody>
</table>

Figure 8: He-core masses (listed in Table 3) computed at He ignition for models with $M = 0.8 M_\odot$, $Z = 10^{-4}$ and different helium contents $Y = 0.24, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80$.

Table 4: He-core masses computed at He ignition for stars of the indicated mass and helium content, with a metallicity $Z = 10^{-4}$.

<table>
<thead>
<tr>
<th>mass ($M/M_\odot$)</th>
<th>$M_{He}(M_\odot)$ ($Y = 0.24$)</th>
<th>$M_{He}(M_\odot)$ ($Y = 0.50$)</th>
<th>$M_{He}(M_\odot)$ ($Y = 0.70$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>0.524</td>
<td>0.760</td>
<td>1.07</td>
</tr>
<tr>
<td>5.0</td>
<td>0.660</td>
<td>0.980</td>
<td>1.39</td>
</tr>
<tr>
<td>7.0</td>
<td>0.953</td>
<td>1.48</td>
<td>2.22</td>
</tr>
<tr>
<td>10</td>
<td>1.53</td>
<td>2.50</td>
<td>3.79</td>
</tr>
</tbody>
</table>

Table 5: $M_{up}$ masses computed for models with $Z = 10^{-4}$ and the indicated helium contents.

<table>
<thead>
<tr>
<th>$Y$</th>
<th>0.24</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{up}(M_\odot)$</td>
<td>6.3</td>
<td>5.8</td>
<td>4.8</td>
<td>3.7</td>
<td>2.6</td>
<td>1.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Figure 9: He-core masses computed at He ignition for models (listed in table 4) for models with \( M = 4.0, 5.0, 7.0, 10 \, M_\odot \), \( Z = 10^{-4} \) and three different helium contents \( Y = 0.24, 0.50 \) and 0.70.

Figure 10: \( M^{up} \) masses (listed in table 5) computed for models with \( Z = 10^{-4} \) and helium contents \( Y = 0.24, 0.30, 0.40, 0.50, 0.60, 0.70 \) and 0.80.
We found that, since stars in mirror sector have very different helium abundances from the visible ones (but the physics is the same), they have much faster evolutionary times, dependent on the exact He content and thus on the temperature of the mirror sector (since they are inversely proportional). The mean life of a mirror star can be until 30 times shorter than that of an ordinary one, if we consider the most helium rich stars. Generally, we found the evidence that, under large mass ranges and different boundary conditions (in terms of temperatures of the mirror sector, and thus stellar helium contents), the lifetimes of mirror stars are roughly an order of magnitude smaller than the ones of visible stars.

This means that, compared to the ordinary ones, mirror stars evolve faster and enrich earlier the galaxy of processed mirror gas, with implications for galaxy evolution. In addition, we found the important result that the minimum mass for carbon ignition, and then for the explosion of the star as type II supernova, is inversely proportional to the helium abundance, and hence for M stars it is lower than for the O ones; this means that in the M sector there are much more supernovae than in the visible one. This has important consequences on both the star formation rate induced by the shock waves coming from the supernova explosions, and the ratio of mirror star and gas in the halo of galaxies, with implications for the stability of a spherical halo. In fact, the star formation reduces the mirror gas present, and the mirror halo looks like a collisionless system of stars, avoiding the M baryons to form disk galaxies as ordinary baryons do. At the same time, the energy injected by the mirror supernovae explosions replace the one dissipated by the friction of the mirror gas and dust still present in the mirror galaxy, and helps to maintain its typical elliptical structure. We notice that both faster evolutionary times and greater number of supernova events have the same effect of an accelerated chemical evolution on the mirror galaxy respect to the observable one.

From the evolutionary data obtained in the present work together with the necessary information on the initial mirror stellar mass function, we could predict the expected population of mirror stars, in order to compare it with current MACHO observations. In addition, we could evaluate the amount of gravitational waves expected from mirror supernovae, that are observable by the next detectors.

Acknowledgements

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