Heavy Quarkonium Production: Extending CSM and COM

J.P. Lansberg

Abstract. By questioning the applicability of the static approximation of the Colour-Singlet Model, we have seen that the production amplitude receives contributions from two different cuts. The first one in its static limit gives the colour-singlet mechanism. The second one has not been considered so far. We treat it in a gauge-invariant manner by introducing necessary new 4-point vertices, suggestive of the colour-octet mechanism. This new contribution can be as large as the colour-singlet mechanism at high \( p_T \), however these vertices are not totally constrained and when the freedom in their determination is fully exploited, we are able to reproduce the production cross-sections at the Tevatron for the \( J/\psi \), \( \psi' \) and \( \Upsilon(1S) \) and at RHIC for the \( J/\psi \).

Keywords: heavy quarkonium production, vector-meson production, gauge invariance, relativistic effects, non-static extension


1. INTRODUCTION

Ten years after the discovery of the “\( \psi' \) anomaly” by the CDF collaboration \cite{1, 2}, no totally conclusive solution has been proposed so far (for a comprehensive and up-to-date review on the subject, see \cite{3}). Even though the Colour-Octet Mechanism (COM), coming from the application of NRQCD to heavy quarkonium, is a good candidate, it appears clearly that as long as fragmentation is the dominant production contribution and the velocity scaling rules of NRQCD hold, it cannot accommodate the polarisation measurements of CDF \cite{4}, which show a non-polarised, if not slightly longitudinal, production.

In that context, we have felt the necessity to reconsider the appropriateness of the static and on-shell approximation of the Colour-Singlet Model (CSM) \cite{5}, which is still the most natural model from QCD. These approximations are also implicit in the COM, therefore any feature arising from this study should have some implication for the COM.

In order to study properly non-static and off-shell effects, we have used a vertex function as an input for the bound state characteristics, whereas the Schrödinger

2. OUR MODEL

In the case of $^3S_1$ quarkonium (noted $Q$) production in high-energy hadronic collisions, we are to consider gluon fusion $gg \to Qg$. Using the Landau equations, we have shown in [6] that there are two families of contributions (see Fig. 1 (a) and (b)): the first is the usual colour-singlet mechanism, where in the context of our model, we use a 3-point function $\Gamma^{(3)}(p, P) = \Gamma(p, P)\gamma_\mu$ at the $Q\bar{Q}Q$ vertex; the second family was never considered before. To simplify the study, we set $m > M/2$ so that the first cut does not contribute.

For the functional form of $\Gamma(p, P)$, we neglect possible cuts, and choose two opposite scenarios: a dipolar form which decreases gently with its argument, and a Gaussian form: $\Gamma(p, P) = N(1 + \frac{\vec{p}^2}{\Lambda^2})^{-2}$ and $\Gamma(p, P) = Ne^{-\frac{\vec{p}^2}{\Lambda^2}}$, both with a free size parameter $\Lambda$, and a normalisation $N$. In [8], we have shown how to fix the normalisation $N$ of $\Gamma(p, P)$ as a function of $\Lambda$.

In addition to the second family, one is driven – to preserve gauge invariance (GI) – to introduce new contributions arising from the presence of 4-point vertices. Besides restoring GI, these vertices have to satisfy specific constraints [7, 9, 10]. For the following simple choice for $\Gamma^{(4)}(c_1, c_2, P, q)$

$$-i g_s T_{ki} T_{\lambda}^a \left( \Gamma(2c_1 - P, P) + \Gamma(2c_2 - P, P) \right) \left[ \frac{c_{1\mu}}{(c_2 + P)^2 - m^2} + \frac{c_{2\mu}}{(c_1 - P)^2 - m^2} \right] \gamma_\nu, \quad (1)$$

where the momenta and indices are as in Fig. 1 (c), we got for the $J/\psi$ and $\psi'$ production at the Tevatron the results shown in Fig. 2. In the $\psi'$ case, we employed the ambiguity upon the vertex function normalisation due to the node position $a_{node}$ to describe the data at low $P_T$. Note that the slope is not that different from that
of the data. This is at variance with what is widely believed since fragmentation (with a typical 1/$P_T^4$ behaviour) processes describe the data.

However there exist different choices for the GI restoring vertex (GIRV). In the following, we present some interesting results obtained by studying the effects of autonomous vertices. The latter link different suitable choices of GIRV: they are GI alone and a priori unconstrained in normalisation. For a first study, let us restrict the choice to the three simplest possible ones \cite{9, 11} (omitting the factor $-ig_s T^K_a (\Gamma(2c_1 - P, P) - \Gamma(2c_2 - P, P))$)

\begin{align}
(a) & \alpha/(\sqrt{s}m_Q)\gamma^\mu q' & (b) & \beta'(c_1 + c_2)\gamma^\mu(c_1 + c_2)' & (c) & \xi/m_Q g^{\mu\nu} \tag{2}
\end{align}

The factors $\alpha$, $\beta'$ and $\xi$ are free constant. If we introduce these contributions in the amplitude calculation, we see in Fig. 3 that we can fit the data for some set of values for ($\alpha$, $\xi$).

\section{Conclusion}

We have shown that it is possible to go beyond the static approximation of the CSM. It may also be possible to extend the COM in the same manner. This necessitates the introduction of 4-point vertices due to the non-local 3-point vertex relevant for the non-static and off-shell contributions.

By going deeper in the analysis, we see that the form of these 4-point vertices is not absolutely constrained even after imposing necessary conditions to conserve crossing symmetry and the analytic structure of the amplitude. When this lack of constraint is used, we are able to reproduce the cross-section for the $J/\psi$, $\psi'$ and $\Upsilon(1S)$ as measured at the Tevatron by CDF (and also at RHIC by PHENIX for $J/\psi$).

In our framework, cross-sections are dominated by longitudinal $Q$, therefore by combining our approach with COM fragmentation they could agree with the polarisation measurements of CDF.
FIGURE 3. Polarised ($\sigma_T$ and $\sigma_L$) and total ($\sigma_{TOT}$) cross sections obtained: upleft – for $J/\psi$ at $\sqrt{s} = 1800$ GeV with $\alpha = 8$ and $\xi = 37.5$ to be compared with LO CSM, the fragmentation in the CSM and with the data of CDF [2]; upright – for $J/\psi$ at $\sqrt{s} = 200$ GeV with $\alpha = 8$ and $\xi = 37.5$ to be compared with LO CSM and the data of PHENIX [13]; downleft – for $\Upsilon(1S)$ at $\sqrt{s} = 1800$ GeV with $\alpha = 8$ and $\xi = 10$ to be compared with LO CSM and with the data of CDF [12]; downright – for $\psi'$ at $\sqrt{s} = 1800$ GeV with $\alpha = 27.5$ and to be compared with LO CSM, the fragmentation in the CSM and with the data of CDF [1].

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REFERENCES