NON-LINEAR STRUCTURE FORMATION IN COSMOLOGIES WITH EARLY DARK ENERGY

MATTHIAS BARTELMANN$^1$, MICHAEL DORAN$^2$ AND CHRISTOF WETTERICH$^2$

$^1$ZENTRUM FÜR ASTRONOMIE, UNIVERSITÄT HEIDELBERG, ALBERT-ÜBERLE-STR. 2, 69120 HEIDELBERG, GERMANY
$^2$INSTITUT FÜR THEORETISCHE PHYSIK, UNIVERSITÄT HEIDELBERG, PHILOSOPHENWEG 16, 69120 HEIDELBERG, GERMANY

July 12, 2005

ABSTRACT

We argue that a few per cent of “Early Dark Energy” can be detected by the statistics of nonlinear structures. The presence of Dark Energy during linear structure formation is natural in models where the present tiny Dark-Energy density is related to the age of the Universe rather than a new fundamental small parameter. Generalisation of the spherical collapse model shows that the linear collapse parameter $\delta_c$ is lowered. The corresponding relative enhancement of weak gravitational lensing on arc-minute scales lowers the value of $\sigma_8$ inferred from a given lensing amplitude as compared to $\Lambda$CDM. In presence of Early Dark Energy, structures grow slower, such that for given $\sigma_8$ the number of galaxies and galaxy clusters is substantially increased at moderate and high redshift. For realistic models, the number of clusters detectable through their thermal Sunyaev-Zel’dovich effect at redshift unity and above, e.g. with the Planck satellite, can be an order of magnitude larger than for $\Lambda$CDM.

1. Introduction

According to observations of Supernovae Ia (SNe Ia) (Riess et al. 2004), the Cosmic Microwave Background (CMB) (Spergel et al. 2003, Readhead et al. 2004, Goldstein et al. 2003, Rebolo et al. 2004) and Large Scale Structure (LSS) (Tegmark et al. 2004, Hawkins et al. 2003), the expansion of our Universe is accelerating today. A plausible candidate to drive such an acceleration is Dark Energy which may be described by an (effective, Doran & Jackel 2002) scalar field (Wetterich, 1988; Ratra & Peebles, 1988; Caldwell et al. 1998, Caldwell 2002). Such scalar field models have been shown to admit attractor solutions (Wetterich, 1988; Ratra & Peebles, 1998; Liddle & Scherrer, 1999; Zlatev & Steinhardt, 1999) where the Dark Energy component “tracks” the dominant component of the cosmological fluid. It is therefore natural to assume that Dark Energy might be non-negligible during extended periods of the evolution of our Universe.

For a mildly varying Dark Energy fraction during the whole history of the Universe, the present tiny Dark-Energy density can be linked to the huge age of the Universe. Such an “Early Dark Energy” contribution has been shown to influence the CMB (Doran et al. 2001a, 2001b) and linear structure growth (Ferreira & Jovell 1998, Doran et al. 2001b). From the CMB and LSS data, one infers a current limit on the Dark Energy contribution $\Omega_d$ during recombination and structure formation of $\Omega_d \leq 10\%$. Intriguingly, the effect of Early Dark Energy on structure formation is particularly strong, because it influences the entire structure formation period in contrast to a cosmological constant model or any other model of Dark Energy with $\Omega_d \rightarrow 0$ at earlier times.

The non-linear structure growth in Dark Energy scenarios has been subject of several investigations (see e.g. Wang & Steinhardt 1998, Ma et al. 1999, Linder & Jenkins 2003). In contrast to this work, we will focus on the implications of an Early Dark Energy component. This has two major effects. First, Early Dark Energy lowers the linear growth rate for structures. As a consequence, for a given present amplitude $\sigma_8$, the amplitude of structures at high redshift is higher as compared to $\Lambda$CDM. Second, on the non-linear level, there is more structure than one would naively expect from the linear amplitude in $\Lambda$CDM. In consequence, the abundance of collapsed objects is considerably higher at redshifts $z \sim 1$ than in a standard $\Lambda$CDM universe. The decisive quantity for the number of non-linear objects is the probability for a region containing massive particles with a total mass $M$ to collapse at a redshift $z_c$. It is proportional to

$$\exp \left( \frac{-\delta_i^2}{2 \sigma_i^2} \right),$$

where the density contrast at early times (e.g. at matter-radiation equality), $\delta_i$, is chosen such that the region collapses by redshift $z_c$, and the variance at early times, $\sigma_i$, is evaluated on a linear scale corresponding to the mass, $R \propto (M/\rho)^{1/3}$. For a given present fluctuation amplitude $\sigma_i$, the value of $\sigma_i$ is larger in cosmologies with Early Dark Energy as compared to $\Lambda$CDM. Since $\delta_i$ depends only mildly on the detailed cosmology, this enhancement of $\sigma_i$ explains the enhancement of non-linear structures relative to $\Lambda$CDM. Conversely, the value of $\sigma_i$ required to explain the number counts of non-linear objects or weak gravitational lensing on small angular scales is smaller for models with Early Dark Energy than for $\Lambda$CDM. Comparison of the linear fluctuation amplitude (e.g. from galaxy correlations or weak gravitational lensing on large angular scales) with the power in non-linear structures could thus provide clear evidence for the presence of Early Dark Energy.

The crucial quantity for Early Dark Energy is its average density fraction during structure formation, $\Omega_{d,\text{sf}}$ (see Doran et al. 2001b for a precise definition). As an example for the high sensitivity of non-linear structures to $\Omega_{d,\text{sf}}$, we present two models with $\Omega_{d,\text{sf}} = 0.04$. We should also mention that a possible coupling between Dark Energy and Dark Matter could modify our results, but will be neglected in this work.

We use a particularly simple and direct parametrisation of the Dark Energy evolution (Wetterich 2004). The parameters are the amount of Dark Energy today $\Omega_{d,0}$, the equation-of-state parameter today $w_0$, and the amount of Dark Energy at early times $\Omega_{d,e}$ to which it asymptotes for $z \rightarrow \infty$. One important feature of our parametrisation is a non-vanishing Dark-Energy contribution during recombination and structure formation. For illustration, we pick two models at random from a Monte-Carlo chain. We select models for which $\sigma_8$ is close to 0.8, the optical depth $\tau < 0.2$ and the spectral index of initial scalar perturbations is $n = 0.99$ and $n = 1.05$ respectively. The data we use for the Monte-Carlo chains are CMB (Spergel et al. 2003, Readhead et al. 2004, Goldstein et al. 2003).
2003; Rebolo et al. 2004) and LSS data (Tegmark et al. 2004) and SNe Ia (Riess et al. 2004), thus both models describe current observations well. The parameters for model (I) are: \( \Omega_{m,0} h^2 = 0.146, \Omega_{b,0} h^2 = 0.026, h = 0.67, n = 1.05, \tau = 0.18, w_0 = -0.93 \) and \( \Omega_{de} = 2 \times 10^{-4} \), leading to an effective Dark Energy contribution during structure formation (Doran et al. 2001b) of \( \Omega_{ef,0} = 0.04 \) and \( \sigma_8 = 0.82 \). Model (II) is given by \( \Omega_{m,0} h^2 = 0.140, \Omega_{b,0} h^2 = 0.023, h = 0.62, n = 0.99, \tau = 0.18, w_0 = -0.99 \) and \( \Omega_{de} = 8 \times 10^{-4} \), leading to \( \Omega_{de,0} = 0.04 \) and \( \sigma_8 = 0.78 \). We compare these models to \( \Lambda \)CDM with \( \Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0.7, h = 0.65 \) and \( \sigma_8 = 0.84 \).

2. Spherical collapse with Early Dark Energy

2.1. Solutions for early times

The qualitative features of the effects of Early Dark Energy can be understood analytically within the spherical collapse model. We consider a homogeneous, spherical overdensity which expands, reaches its maximum radius ("turn-around") at a scale factor \( a_{ta} \) and then collapses to reach virial equilibrium. We restrict the consideration to the matter-dominated era, i.e. we neglect the very slow growth of a perturbation inside the horizon during the radiation-dominated era.

Following Wang & Steinhardt (1998), we refer all quantities to their values at turn-around. Let \( \omega \) and \( \lambda \) be the density parameters, at turn-around, of the (Dark) Matter and the Dark Energy, respectively. The cosmological scale factor \( x \) and the radius \( y \) of the density perturbation are normalised to unity at turn-around,

\[
x = \frac{a}{a_{ta}}, \quad y = \frac{R}{R_{ta}}.
\]

The spherical collapse of the perturbation is then described by the two Friedmann equations

\[
\dot{x} = \left[ \frac{\omega}{x} + \lambda \zeta^2 g(x) + (1 - \omega - \lambda) \right]^{1/2},
\]

\[
\dot{y} = -\frac{\omega \zeta}{2y^2} - \frac{1 + 3w(x)}{2} \dot{g}(x)y,
\]

where dots denote derivatives with respect to the dimensionless time parameter

\[
\tau \equiv H_{ta} t.
\]

The parameter \( \zeta \) quantifies the order of the dark energy content at turn-around in a spherical volume which will collapse at a later, pre-defined time. It is determined by solving \ref{eq:tau} with the boundary conditions \( y = 0 \) at \( \tau = 0 \) and \( \dot{y} = 0 \) at turn-around, and requiring that \( y = 1 \) at turn-around. In other words, once the collapse time (or redshift) is fixed when the perturbation formally collapses to zero radius, \( \zeta \) is chosen such that the maximum radius is reached at the turn-around time, given the boundary conditions. The function \( g(x) \) quantifies the change in the Dark-Energy density with \( x \) relative to turn-around,

\[
g(x) = \exp \left[ -3 \int_1^x [1 + w(x')] d \ln x' \right] \equiv x^{-3(1+w)}
\]

where \( w \) is a suitably averaged equation-of-state parameter. For constant \( w \), one has \( \dot{w} = 0 \), but in general \( w \) and \( \dot{w} \) are functions of \( \tau \) or, equivalently, \( x \).

We assume that the Dark-Energy density is always sufficiently smaller than the matter density at early times, i.e.,

\[
g(x) \ll x^{-3}, \quad x^3 g(x) \ll 1 \quad \text{for} \quad x \to 0.
\]

Then, from \ref{eq:tau},

\[
d \tau \approx \sqrt{\frac{x}{\omega}} \left( 1 - \frac{1 - \omega - \lambda}{2\omega} x \right) dx
\]

for \( x \ll 1 \), and thus, at early times,

\[
y \approx \frac{2}{3} \sqrt{\frac{x}{\omega}} \left( 1 - \frac{3}{4 \omega} (1 - \omega - \lambda)x \right)
\]

without any effect of Dark Energy to second order in \( x \).

Upon multiplication with \( 2y \), Eq. \ref{eq:tau} becomes

\[
\frac{d(y^2)}{d\tau} = \omega \zeta \frac{d(1/y)}{d\tau} - \frac{1 + 3w(x)}{2} y \frac{d(y^2)}{d\tau}.
\]

Integration gives

\[
y^2 = \frac{\omega \zeta}{y} - \lambda \int_0^y [1 + 3w(x')] [g(x')] y' dy' + C,
\]

with the integration constant \( C \) set by the requirement at turn-around

\[
y_{y=1} = 0.
\]

which is satisfied if

\[
C = \lambda \int_0^y [1 + 3w(x')] [g(x')] y' dy'.
\]

When inserted back into \ref{eq:tau}, this yields

\[
y^2 = \frac{\omega \zeta}{y} (1 - \lambda) + \lambda \int_y^y [1 + 3w(x')] [g(x')] y' dy'.
\]

We will have to study \ref{eq:tau} at very early times, i.e. for \( x \to 0 \) and \( y \to 0 \). In that limit, the integral \ref{eq:tau} will either converge or diverge as \( x \) and \( y \) approach zero. Suppose the integral converges,

\[
\lim_{y \to 0} \int_0^y [1 + 3w(x')] [g(x')] y' dy' = I < \infty.
\]

Then, for \( y \to 0 \), Eq. \ref{eq:tau} can be approximated by

\[
\sqrt{\int_0^y dy} = \sqrt{\omega \zeta} \tau, \quad A \equiv 1 - \frac{\lambda}{\omega \zeta},
\]

which can be integrated after expanding the denominator into a Taylor series,

\[
\sqrt{\omega \zeta} \tau \approx \sqrt{\int_0^y \frac{1 + Ay}{2} dy} = \frac{2}{3 y^{3/2}} \left( 1 + \frac{3A}{10} y \right).
\]

For example, if the Dark Energy is described by a cosmological constant, \( w = -1 \) and \( g(x) = 1 \), thus \( I = -1 \) and \( A = 1 + \lambda/\omega \zeta \).

The integral \( I \) defined in \ref{eq:tau} will diverge if \( g(x) \) increases steeply as \( x \to 0 \). Then, it will be dominated by the behaviour of its integrand for \( x \to 0 \). Let

\[
\lim_{x \to 0} g(x) \equiv w_{ini}, \quad \lim_{x \to 0} g(x) \equiv y x^{-3(1+w_{ini})}
\]

with a constant \( y \ll 1 \), and we further assume \( y \) to be proportional to \( x \) at early times, \( y = \alpha x \), which will be justified in hindsight. Then,

\[
I \approx (1 + 3w_{ini}) \alpha^2 \lim_{x \to 0} \int_0^1 x^{-3(1+w_{ini})} x' dx'
\]

\[
\approx \alpha^2 \lim_{x \to 0} \left( x^{-1-3w_{ini}} \right)_{1}^{1}
\]

\[
\approx \frac{y \alpha^{3(1+w_{ini})}}{y^{1+3w_{ini}}}.
\]
I is thus expected to diverge if $w_{\text{ini}} \geq -1/3$. With (14), this yields for early times
\begin{equation}
\dot{y} = \left[ \frac{\omega \zeta}{y} - \omega \zeta + \lambda y \alpha^{3(1+w_{\text{ini}})} y^{3/3w_{\text{ini}}+2} \right]^{1/2}.
\end{equation}

According to (7), $w_{\text{ini}} \leq 0$, because otherwise the Dark-Energy density would grow above the matter density. Let $w_{\text{ini}} = 0$ first, then (20) implies
\begin{align}
\dot{y} &= \left[ \frac{\omega \zeta + \lambda y}{y} \right]^{1/2} \\
&\approx \sqrt{\frac{\omega \zeta + \lambda y}{\omega \zeta y}} \left( 1 - \frac{\omega \zeta}{2(\omega \zeta + \lambda y)} \right),
\end{align}

which can be integrated to give
\begin{equation}
\sqrt{\omega \zeta} \tau \approx \frac{2}{3} \frac{3}{2} \sqrt{B} \left( 1 + \frac{3}{10} By \right), \quad B \equiv \frac{\omega \zeta}{\omega \zeta + \lambda y y}.
\end{equation}

If $-1/3 < w_{\text{ini}} < 0$, we can replace $B$ by
\begin{equation}
B = \frac{\omega \zeta + \lambda y}{\omega \zeta + \lambda y + \alpha^{3(1+w_{\text{ini}})} y^{-3w_{\text{ini}}}},
\end{equation}

to account for the gentle change in the coefficient $B$ with $y$. Equating $\sqrt{\zeta}$ times (9) to (17) and squaring yields
\begin{equation}
\zeta x^{3} \left( 1 - \frac{3}{5} \frac{\omega - \lambda}{\omega} y \right) \approx y^{3} \left( 1 + \frac{3}{5} a y \right),
\end{equation}
while the same procedure applied to (22) implies
\begin{equation}
\zeta x^{3} \left( 1 - \frac{3}{5} \frac{\omega - \lambda}{\omega} y \right) \approx y^{3} B \left( 1 + \frac{3}{5} b y \right).
\end{equation}

Now, $B \approx 1$ to lowest order because $\gamma \ll 1$ and $\gamma \ll 1$ at early times, thus
\begin{equation}
y \approx \zeta^{1/3} x, \quad \alpha = \zeta^{1/3}
\end{equation}

while the linear density contrast at collapse time $\delta_c$ is a crucial ingredient for the Press-Schechter and related mass functions (Press & Schechter, 1974; Sheth & Tormen, 1999). We shall explain below why $\delta_c$ is substantially smaller for Early Dark-Energy models as compared to $\Lambda$CDM. This finding is a central ingredient for our results.

### 2.2. Overdensity and linear density contrast

Since the background matter density changes in proportion to $x^3$, while the matter density inside the perturbation is proportional to $y^3$, the overdensity inside the perturbation obeys
\begin{equation}
\Delta = \frac{\zeta x^3}{y^3},
\end{equation}

which, according to (24) and (25), is
\begin{align}
\Delta &= \begin{cases} 
1 + \frac{3A}{5} y + \frac{3}{5} \frac{1 - \omega - \lambda}{\omega} x & \text{for } w_{\text{ini}} < -1/3 \\
1 + \frac{3B}{5} y + \frac{3}{5} \frac{1 - \omega - \lambda}{\omega} x & \text{else}
\end{cases}
\end{align}
at early times. The two solutions join at $w_{\text{ini}} = -1/3$, as they should. For $w_{\text{ini}} = -1/3$, $I = 0$ because $(1 + 3w_{\text{ini}}) = 0$ in Eq. (19), thus $B = 1$. Also, $A \rightarrow 1$ for $w \rightarrow -1/3$ according to (15) and (16).

The overdensity $\Delta$, within virialised objects should only very weakly be affected by the (Early) Dark Energy because Dark Matter dominates the virialisation process. We verify this as described in Appendix A.

The linear density contrast $\delta$ can be used to relate the non-linear overdensity $\Delta$ with the density that would result from linear evolution of the same initial perturbation within linear evolution. With the growth factor $D_+(x)$ at a given time or normalised scale factor $x$ one has at the collapse time $x_c > 1$
\begin{equation}
\delta_c = \lim_{x \to 0} \frac{D_+(x)}{D_+(x)} [\Delta(x) - 1].
\end{equation}
The linear density contrast at collapse time $\delta_c$ is a crucial ingredient for the Press-Schechter and related mass functions (Press & Schechter, 1974; Sheth & Tormen, 1999). We shall explain below why $\delta_c$ is substantially smaller for Early Dark-Energy models as compared to $\Lambda$CDM. This finding is a central ingredient for our results.

#### 2.3. Results for $\delta_c$

We show in Fig. 1 the linear critical overdensity $\delta_c$ as a function of $\zeta$. While the results are strongly dependent on $\delta_c \approx 3/5(3\pi/2)^{2/3} \approx 1.686$ for $\Lambda$CDM and ODEDM, quite independent of the collapse redshift, they fall below for the Early-DE models, dropping even to $\delta_c \approx 1.4$ for $\zeta = 0$. We return to (33) in order to understand why $\delta_c$ is lower for Early-DE than for, e.g., the $\Lambda$CDM model. It turns out numerically that the overdensity $\Delta = 1$ is not changed much in the Early-DE compared to the $\Lambda$CDM model. For a collapse redshift of $\zeta = 0$, for instance, $\Delta = 1/x$ is lower by only 4% in the Early-DE than in the $\Lambda$CDM model. The main difference, however, is caused by the growth-factor ratio in (33), as shown in Fig. 2.

The curves show the growth factor divided by the scale factor, $D_+(a)/a$, as a function of $a$ for one of the Early-DE models and for $\Lambda$CDM. Both lower curves are normalised such that they start from unity at early times, as it should be according to (33). Obviously, the linear growth in Early-DE falls behind that in $\Lambda$CDM, starting from an overdensity of the same amplitude.

The reason is that the expansion rate in the Early-DE models is higher at early times than in $\Lambda$CDM, which increases the friction term $(\dot{a}/a)$ in the equation
\begin{equation}
\ddot{a} + 2 \frac{\dot{a}}{a} - 4\pi G \rho = 0
\end{equation}
governing the growth factor. As we shall see below, the reduced linear overdensity $\delta_c$ expected in Early-DE models has pronounced consequences for nonlinear structure formation.

The two upper curves in Fig. 2 show the growth factor normalised to unity at present, i.e. reflecting the evolution back in time of structures reaching the same amplitude today. They show that structures need to grow earlier in Early-DE models than in $\Lambda$CDM to reach the same level at the present time.

3. **Cosmological Consequences**

3.1. **Distances and age**

The influence of Early Dark Energy on global geometrical properties of the Universe is illustrated in Fig. 3, which shows the angular-diameter distance $D_{\text{ang}}(z)$ in units of $cH_0^{-1}$ as a function of redshift relative to $\Lambda$CDM. A model with constant $w = -0.8$ is shown for comparison. Note that this curve also gives the ratio of luminosity distances $D_{\text{lum}}(z)$ between Dark-Energy models and $\Lambda$CDM because $D_{\text{lum}}(z)$ and $D_{\text{ang}}(z)$ only differ by the ratio of scale factors between emission and observation.

As the figure shows, distance measures are changed only moderately, by $\lesssim 8\%$ for source redshifts below $z = 2$. The cosmic time is changed by a larger amount, as can be seen in Fig. 4. It shows the cosmic time as a function of redshift in units of $H_0^{-1}$, again relative to $\Lambda$CDM.

The increased expansion rate of the Universe compared to $\Lambda$CDM, in particular at early times, reduces the age of the Universe by approximately $(5 - 10)\%$ at low redshifts. The effect of Dark Energy with constant $w = -0.8$ is substantially less pronounced.

3.2. **The mass function**

The reduced linear density contrast $\delta_c$ necessary for spherical collapse has a pronounced influence on the mass function of Dark-Matter halos. The Press-Schechter mass function for ex-

The two upper curves in Fig. 2 show the growth factor normalised to unity at present, i.e. reflecting the evolution back in time of structures reaching the same amplitude today. They show that structures need to grow earlier in Early-DE models than in $\Lambda$CDM to reach the same level at the present time. The upper curves are normalised to unity at present ($a = 1$), illustrating the growth of structures reaching the same level today in both cosmologies. Starting from the same amplitude, the growth in the Early-DE model falls behind that in $\Lambda$CDM, largely causing the reduction in the overdensity parameter $\delta_c$. Structures have to grow earlier in Early-DE models than in $\Lambda$CDM to reach the same fluctuation amplitude today.

The figure shows the mass function proposed by Sheth & Tormen (1999), which assumes ellipsoidal rather than spherical halo collapse and reproduces the halo abundances in simulations significantly better than the original Press-Schechter mass function. As Fig. 5 shows, Early Dark Energy has a pronounced effect on the mass function. While halos with masses $\lesssim 10^{13} h^{-1} M_\odot$ have approximately equal abundances as in $\Lambda$CDM, massive clusters with $M \gtrsim 5 \times 10^{14} h^{-1} M_\odot$ are of order 1.5 - 2.5 times more abundant at the present epoch, provided the models are normalised according to the CMB temperature fluctuations. At redshift unity, however, the mass functions differ by a factors of $\sim (5 - 10)$ for massive clusters. Therefore, even if the models were normalised such as to get closer to the cluster abundance expected in $\Lambda$CDM today, they still predicted a higher number density of high-redshift clusters. As Fig. 7 shows in more detail, the normalisation parameter could be written as Press & Schechter (1974)

$$N(M, z) dM = \frac{2}{\pi} \frac{\rho_c \delta_c}{\sigma_R D_z(z)} \frac{d \ln \sigma_R}{dM} \times \exp \left(-\frac{\delta_c^2}{2\sigma_R^2 D_z^2(z)}\right) \frac{dM}{M},$$

showing that $\delta_c^2$ is compared in the argument of the exponential function to the variance $\sigma_R^2 D_z^2(z)$ of the Dark-Matter density field smoothed on scales $R$ corresponding to the mass $M$, with $D_z(z) = 1$. Linear quantities occur in the exponential of the Press-Schechter mass function because it predicts the distribution of the later non-linear fluctuations from their linear distribution at early times. Even small reductions of $\delta_c$, as they are illustrated in Fig. 6, lead to a noticeable increase in the mass function for a fixed normalisation parameter $\sigma_8$ of the Dark-Matter power spectrum. The increase of the mass function in the two Early Dark Energy models compared to $\Lambda$CDM is illustrated in Fig. 5.
would have to be lowered to $\sigma_8 \approx 0.69$ for the Early-DE models to reproduce the halo abundance at $z = 0$ of a $\Lambda$CDM model with $\sigma = 0.84$.

The earlier development of the mass function naturally leads to enhanced dynamical activity of Dark-Matter halos, in particular at the highest cluster masses. The extended Press-Schechter formalism [Bond et al. 1991] allows the calculation of the merger probability,

$$ d^3p \over d\ln \Delta M dt (M, z) , $$

i.e. the probability for a halo of mass $M$ to merge within time $t$ at redshift $z$ with another halo of mass $\Delta M$ (Lacey & Cole 1993, 1994). Multiplying (36) with the mass function and the differential cosmic volume $dV$ corresponding to a redshift interval $dz$, and integrating over a mass range from $M_1$ to $M_2$ and over the time $t$ corresponding to a redshift interval $z - \delta z$ to $z$ yields the total number of mergers between a halo of mass $M$ and another halo of mass between $M_1$ and $M_2$, within the time interval from $t(z + \delta z)$ and $t(z)$,

$$ \delta N(M, z) = N(M, z) \left| {dV \over dz} \right| \int_{\ln M_1}^{\ln M_2} d\ln M \times \int_{t(z+\delta z)}^{t(z)} \left( {d^2\delta p(M, z) \over d\ln M dt} \right) . $$

We set $M = 5 \times 10^{14}$ $h^{-1} M_\odot$, $M_1 = M/4$, $M_2 = M$, and $\delta z = 0.2$. In other words, we compute using expression (37) the number per unit redshift of cluster-sized Dark-Matter halos at redshift $z$ that have undergone a major merger within $z$ and $z + 0.2$. Figure 6 shows the ratio between the merger number in the two Early-DE models relative to $\Lambda$CDM.

The two curves in the figure show that major mergers at moderate and high redshifts of cluster-sized halos are substantially more frequent in the two Early Dark-Energy models compared to $\Lambda$CDM. This reflects our previous result that the halo mass function grows earlier in the Early-DE models, leading to enhanced merger activity at earlier times.

Of course, these results on the abundance of Dark-Matter halos depend sensitively on the normalisation of the power spectrum, as the rising curves in Fig. 6 illustrate. Much less dependent on the amplitude of the power spectrum is the expected ratio of halo numbers above two different redshifts. The falling curves in Fig. 6 give examples.

The curves show for three cosmological models in dependence of the normalisation parameter $\sigma_8$ by how much the number of cluster-sized halos with mass $M \geq 10^{14}$ $h^{-1} M_\odot$ grows between redshifts unity and zero. For $\sigma_8 = 0.8$ in $\Lambda$CDM, there are about four times more clusters today than there were at redshift unity, and about two times more in the Early-DE models. This corroborates that clusters are more abundant at high redshifts in the Early-DE models compared to $\Lambda$CDM. Since CDM is a hierarchical model of structure formation, a similar effect appears for lower-mass halos at higher redshift. In the Early-DE models, for instance, the number of galaxy-sized halos grows much less than in $\Lambda$CDM above redshifts 2 and beyond.
Earlier work has found that Dark-Matter halos tend to be more concentrated if they form earlier, where the concentration $c$ is the ratio between the halo’s virial and scale radii (Navarro et al. 1997; Bullock et al. 2001; Eke et al. 2001). While the central slope of Dark-Matter density profiles in halos is still under some debate (cf. Power et al. 2003), numerical simulations consistently show that halos in (Cold) Dark Matter have a steep density profile outside, and a flat density profile inside the scale radius. The definition of the virial radius changes in the literature, depending on whether the mean density enclosed by the virial radius is supposed to be 200 or another factor times the mean or the critical cosmic density. Different recipes have also been given for the statistical relation between the virial mass and the concentration of halos. They were tested against numerical results on Dark-Energy models by Dolag et al. (2004), who found that the algorithm described by Eke et al. (2001) worked for models with constant $w \neq -1$ without adaptation.

All algorithms have in common that they have to define a collapse redshift $z_c$ for a halo of given mass $M$ at redshift $z$, and a ratio between the central density of the halo and the mean or critical cosmic background density at $z_c$. Eke et al. (2001) define the collapse redshift by requiring that, at $z_c$, the amplitude of the power spectrum at the mass scale of the halo reaches a given fixed value. We show in Fig. 8 the expected concentration according to Eke et al.’s prescription as a function of halo mass at redshifts zero (upper curves) and unity (lower curves).

In agreement with earlier studies (Bartelmann et al. 2002; Klypin et al. 2003; Dolag et al. 2004; Weinberg & Kamionkowski 2003), halos in cosmologies with dynamical Dark Energy tend to be more concentrated than in $\Omega_c M_{\odot}$, reflecting their earlier growth on a cosmological background with higher mean density. For galaxy-sized halos in the Early-DE models, the concentration increases by $\approx 20\%$ at redshift zero.

3.3. Halo properties

3.4. Gravitational lensing

The efficiency of gravitational lensing by isolated lenses such as galaxies or galaxy clusters is given by the critical surface-mass density

$$\Sigma_c = \frac{c^2}{4\pi G} \frac{D_s}{D_{ls} D_{ns}},$$

which contains the angular-diameter distances $D_{ls}$ from the observer to the lens, the source, and from the lens to the source, respectively (e.g. Schneider et al. 1993). The effective lensing distance $D_{\text{eff}} = D_D D_{ls}/D_s$ thus measures the geometrical efficiency of a given mass distribution.

Dark Energy reduces the effective lensing distance compared to $\Lambda$CDM. For sources at redshift $z_s = 2$, as assumed for the figure, identical mass distributions are $\approx 5\%$ less efficient at $z_l \approx 0.4$, and $\approx 10\%$ less efficient at $z_l \approx 1.0$, in Early-DE than in $\Lambda$CDM models. Compared to the increased concentration of the Dark-Matter halos, this effect is less important for strong lensing by galaxies, groups or clusters (Meneghetti et al. 2004).

In sufficient approximation, weak gravitational lensing by large-scale structures can be described by the power spectrum $P_s(l)$ of the effective convergence $\kappa$ (see Bartelmann & Schneider 2001 for a review). Earlier structure growth also affects $P_s$, as shown in Fig. 10.

The weak-lensing power is illustrated there plotting $l^2 P_s(l)$ as a function of angular scale for the two Early-DE models, relative to the $\Lambda$CDM model. Two pairs of curves are given for the two source redshifts $z_s = 1$ and 1.5. The curves tend to unity at angular scales above 10 arc minutes, indicating that weak lensing will not be modified compared to $\Lambda$CDM on large angular scales. On small angular scales $l > 1$ however, the weak-lensing power in the Early-DE models exceeds that in the $\Lambda$CDM model by $\approx 40\%$ for $z_s = 1$ and by $\approx 55\%$ for $z_s = 1.5$. This would imply that $\sigma_8$, as derived from weak-lensing measurements could be $\approx 1.4^{-1/2} \approx 0.8$ times smaller than inferred assuming a $\Lambda$CDM model.
The most significant observable consequence of Early-DE models identified so far is that the counts of cluster-sized halos should decrease significantly more slowly than in CDM (cf. Figs. 5 and 7). While statistically complete galaxy-cluster samples are notoriously hard to identify, future cluster surveys based on the thermal Sunyaev-Zel’dovich (SZ) effect ([Sunyaev & Zeldovich, 1972] should provide ideally suitable data for testing this prediction of Early-DE models.

The upcoming Planck satellite, due for launch in 2007, will observe the entire microwave sky in frequency bands between 30 and 857 GHz with an angular resolution of down to 5 arc minutes. One of its frequency bands is centred on 217 GHz where the (non-relativistic) thermal Sunyaev-Zel’dovich effect vanishes. Planck’s frequency bands are thus well-placed for identifying the unique spectral signature of the thermal SZ effect which reduces the CMB intensity below, and increases it above, 217 GHz. Planck’s high sensitivity and comparatively high angular resolution will enable it to detect a huge sample of clusters reaching high redshift.

Analytic estimates raise the expectation that Planck may find of order 30,000 galaxy clusters on the full sky ([da Silva et al, 2004; Bartelmann et al, 2003], while simulations taking realistic foreground contamination, noise patterns and full-sky, multiband filtering techniques into account arrive at lower numbers, \( \lesssim 10,000 \) ([Schaer et al., 2004]). Nonetheless, the cluster sample expected from Planck will be enormous. Following the analytic description of the Planck cluster sample given in [Bartelmann et al, 2003], we show in Fig. 8 the cumulative redshift distribution of clusters in the sample expected from Planck in the Early-DE models, normalised by the expectation in the CDM model.

While the total number of detections is moderately increased by a factor of two to three, the number of high-redshift clusters detectable in Early-DE models is substantially higher than in CDM. The number of clusters above redshift unity is already an order of magnitude larger. Thus, the number of clusters detected by Planck will provide a highly sensitive test for Early-DE compared to CDM models.

Interestingly, observations of the CMB on small angular scales (\( \lesssim 5' \)) with the Cosmic Background Imager (CBI) indicate a much higher temperature fluctuation amplitude than expected in CDM with the normalisation \( \sigma_8 \approx 0.8 \) typically inferred from other observations ([Padin et al., 2001; Mason et al., 2003]. If due to the thermal SZ effect, these measurements require \( \sigma_8 \gtrsim 1 \) in the CDM model ([Bond et al., 2005]), which would stretch other normalisation constraints to their limits. Cosmological models with Early Dark Energy such as those used here as examples could naturally explain this “CBI anomaly”.

4. Summary and Conclusions

We have investigated expectations for the growth of non-linear structures in a model universe with Early Dark Energy. Such models assume that the accelerated cosmic expansion is driven by a scalar field which has non-negligible energy density all through cosmic history. We have selected two examples for such models with Early Dark Energy whose parameters are chosen such as to comply with the temperature fluctuations measured in the cosmic microwave background.

We have generalised the spherical-collapse model such that Dark Energy with variable equation-of-state parameter \( w \) can be taken into account. In our models the Dark Energy follows for \( z \gtrsim 1 \) the density of the dominant component in the mixture of cosmic fluids. Modified in this way, the spherical collapse model predicts that the linear density contrast necessary for collapse, \( \delta_c \), is lowered compared to CDM models or Dark-Energy models with constant \( w \).

This first result, which may appear unexpected, can be understood as follows. The requirement of having virialised halos at a given redshift sets the initial overdensity inside the spherical perturbation idealising the later halo. Early Dark Energy reduces the linear growth factor compared to CDM due to its higher expansion rate in the young universe. The initial overdensity required for later collapse is thus extrapolated to lower linear overdensity at collapse time. Lower linear density contrast is thus sufficient for non-linear collapse, which increases the number of fluctuations capable of forming halos.

The effects of Early Dark Energy on the geometrical properties of the Universe are moderate. Assuming the same Hubble constant today, such models are \( \sim 5\% \) younger than comparable CDM models today, and the angular-diameter distances are
lowered by no more than \( \sim 10\% \) even to high redshifts. Yet, these effects are noticeably stronger than in Dark-Energy models with constant \( w \); typically, they are roughly twice as large than in a model universe which has \( w = -0.8 \).

Since halos tend to grow earlier in Dark-Energy compared to \( \Lambda \)CDM models, their core densities are higher, and so are their concentration parameters. Using the prescription of Eke et al. for computing expected halo concentrations, we found that halos at redshift zero are more concentrated than in \( \Lambda \)CDM by \( \sim 20\% \) for galaxy masses, and by \( \sim 15\% \) for cluster masses. This increase is visible, but smaller for halos forming at higher redshift.

Strong gravitational lensing profits non-linearly from higher halo concentrations. On the other hand, the effective lensing distance (proportional to the inverse critical surface density for lensing) is lowered by Early Dark Energy, albeit weakly. At redshifts which are typical for strong-lensing galaxies or clusters, \( \sim 0.3\text{--}0.8 \), say, we found a reduction of the effective lensing distances of order \( \sim 8\% \) compared to \( \Lambda \)CDM. The power of weak gravitational lensing by large-scale structures, however, is more substantially changed. While there is no difference in the weak-lensing power between Early Dark-Energy and \( \Lambda \)CDM models on large angular scales \( \gtrsim 1^\circ \), it is increased by \( \sim 40\% \) on arc-minute scales for sources at redshift \( \sim 1 \). This implies a steeper increase with decreasing angular scale of, e.g. the two-point correlation function of the cosmic shear than expected in \( \Lambda \)CDM. It also implies that the normalisation of the Dark-Matter power spectrum, \( \sigma_8 \), inferred from weak lensing should be lowered by a factor of \( \sim 0.8 \) as compared to its value for a \( \Lambda \)CDM model.

The most pronounced effect, however, regards the present number density of massive halos and its evolution towards higher redshift. The Early Dark-Energy models we have studied here, normalised to the CMB temperature-fluctuation measurements, predict approximately the same number density of galaxy-sized halos today as expected in a \( \Lambda \)CDM universe, and a number density of cluster-sized halos which is \( \sim 40\% \) higher. Given the uncertainties in cluster counts even at low redshift, this appears tolerable, although a very moderate reduction of \( \sigma_8 \) would establish complete agreement at redshift zero between the halo counts in Early Dark-Energy and \( \Lambda \)CDM models. However, this present cluster population shrinks much less quickly towards high redshift than in \( \Lambda \)CDM, implying that many more of the clusters existing today were already present at redshifts of order unity.

The cluster population above a mass limit of \( 10^{14} h^{-1} M_\odot \) expected for \( \sigma_8 = 0.8 \) in Early Dark-Energy models shrinks by a factor of \( \sim 2.5 \) at redshift unity compared to redshift zero, but by a factor of \( \sim 4 \) in \( \Lambda \)CDM models. This discrepancy increases rapidly for more massive clusters. Raising the mass limit to \( 5 \times 10^{14} h^{-1} M_\odot \) makes the cluster population shrink by a factor of \( \sim 9 \) in the Early Dark-Energy models, but by a factor of \( \sim 28 \) in an equally-normalised \( \Lambda \)CDM model.

It appears reasonable here to distinguish two possibly different values of \( \sigma_8 \) extracted from linear and non-linear structure growth. As the mass function [15] shows, lowering \( \delta_c \) due to Early Dark Energy can be compensated by equally lowering \( \sigma_8 \). This implies that approximately the same halo number at all redshifts as in the \( \Lambda \)CDM model can be reproduced in the Early Dark Energy models by suitably lowering \( \sigma_8 \). This \( \sigma_8 \) would then properly describe the abundance of non-linear, collapsed structures. This can be tested against the amplitude of linear structures, which is measurable for instance by the galaxy power spectrum on large scales or weak gravitational lensing on scales \( \gtrsim 10^\circ \). In other words, if the value of \( \sigma_8 \) extracted from non-linear structures within a \( \Lambda \)CDM model turns out systematically higher than the \( \sigma_8 \) inferred from larger structures, this would clearly hint at the presence of Early Dark Energy.

For a given \( \sigma_8 \) measured from linear fluctuations, the Early Dark Energy models predict a substantially slower evolution of the halo population as in the \( \Lambda \)CDM model. This prediction has several immediate observable consequences. First, the evolution of the X-ray cluster luminosity function towards redshift unity should be somewhat flatter than expected in a \( \Lambda \)CDM universe. Current data are probably still insufficient for quantitative tests. Second, the dynamical activity of clusters near redshift unity should be substantially higher than expected in \( \Lambda \)CDM. The number of major mergers experienced by cluster-sized halos near redshift unity is predicted to be \( \sim 5 \) times higher than
in ACDM. Third, and most prominently, the number of clusters expected to be found in thermal Sunyaev-Zel’dovich surveys should increase substantially. Applying a simple model for cluster detection by the Planck satellite, we found that ~2 times more clusters should be found at arbitrary redshifts by Planck than in an ACDM universe, but ~10 times more above redshift unity. Leaving the question aside of how redshifts could be measured for clusters detected exclusively by their thermal Sunyaev-Zel’dovich effect, this prediction will be testable very soon. We note that this consequence of Early Dark Energy may naturally explain the high fluctuation amplitude of CMB temperature fluctuations on arc-minute scales, the so-called “CBI anomaly” (Padin et al. 2001; Mason et al. 2003; Bond et al. 2003).

Given these results, we arrive at the main conclusion that models for Early Dark Energy will leave a measurable imprint on weak-lensing measurements and determinations of the distance-redshift relation which is measured using type-Ia supernovae, but their most distinguishing signature is the slow evolution of the cluster abundance between redshifts ~ 1 and zero. Structure formation being hierarchical, this also implies a similar effect on the abundance evolution of galaxy-sized halos at substantially higher redshifts, z ~ 2 – 3.

**Acknowledgements**

We are grateful to E. Thommes for helpful discussions.

**REFERENCES**


Bartelmann, M. & Schneider, P. 2001, Physics Reports, 340, 291


Caldwell, R., Dave, R., & Steinhardt, P. 1998, Physical Review Letters, 80, 1582


Doran, M. & Jackel, J. 2002, PRD, 66, 043519


Doran, M., Schwinn, J., & Wetterich, C. 2001b, Phys. Rev. D, 64, 123520


Ferreira, P. & Joyce, M. 1998, PRD, 58, 023503


Liddle, A. & Scherrer, R. 1999, PRD, 59, 023509


Ratra, B. & Pebbles, P. 1988, PRD, 37, 3406


Schneider, P., Ehles, M., & Palombara, M. 1992, Gravitational Lenses (Springer Verlag, Heidelberg)


Spergel, D., Zeldovich, Y. 1972, Comments on Astrophysics and Space Physics, 4, 173


Wetterich, C. 1988, Nuclear Physics B, 302, 668


**A. The virial overdensity**

The conventional calculation used in many papers (see, e.g. Lahav et al. 1991; Kitayama & Suto 1996)

Wang & Steinhardt 1998; Weinberg & Kamionkowski 2003

Horellou & Berge 2005

Maor & Lahav 2005

starts from the Friedmann equation for the second time derivative of the perturbation radius and converts it to an energy equation after multiplication by . This energy equation contains the cosmological constant or Dark-Energy density in a term which appears analogous to a potential-energy term. Energy conservation is then used by equating the total potential energy of the overdense sphere at turn-around to the sum of kinetic and potential energies at virialisation, which can be expressed solely in terms of the potential energy by means of the virial theorem.

Following this approach, assuming that the Dark Energy does not clump and therefore not participate either in the virialisation of the collapsing overdensity (see Maor & Lahav 2005 for a thorough discussion), we calculate and confirm that it is only weakly modified by Early Dark Energy compared to, e.g. the ACDM result.

We show in Fig. 12 the virial overdensity calculated in this way as a function of the collapse redshift for two models of Early Dark Energy and compare it to the results for a ACDM with and an OCDM model with . While , is somewhat lower at moderate and high for the Early-DE models than for ACDM, the difference between them remains moderate. For high , they approach the “canonical” value of ~ 178 and fall towards , ~ 10 for . As shown for comparison, shows a substantially flatter behaviour with in the OCDM model.
energies of particles in bound orbits. Dark-matter particles orbiting within collapsing overdensities, however, will not feel any force from the Dark Energy because the latter is homogeneously distributed. Thus, it appears dubious to assign a potential to the Dark Energy whose gradient should appear as a conservative force term in the equation of motion. Rather, the Dark Energy should only appear acting on the expansion of the background, and therefore as a dissipative force term which should time-average out of the virial theorem. In this picture, the concept of energy conservation also needs to be revisited. Since none of our later results depends on $\Delta v$, we leave the discussion at that point, showing $\Delta v$ as the conventional approach predicts it for the Early Dark-Energy models, but expressing qualms regarding the validity of the underlying physical concepts.