UHE neutrino damping in a thermal gas of relic neutrinos

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Abstract

We present a calculation of the damping of an ultra-energetic (UHE) cosmic neutrino travelling through the thermal gas of relic neutrinos, using the formalism of finite-temperature field theory. From the self-energy diagram due to Z exchange, we obtain the annihilation cross section for an UHE neutrino interacting with an antineutrino from the background. This method allows us to derive the full expressions for the UHE neutrino transmission probability, taking into account the momentum of relic neutrinos. We compare our results with the approximations in use in the literature. We discuss the effect of thermal motion on the shape of the absorption dips for different UHE neutrino fluxes as well as in the context of relic neutrino clustering. We find that for ratios of the neutrino mass to the relic background temperature $10^{-2}$ or smaller, the thermal broadening of the absorption lines could significantly affect the determination of the neutrino mass and of the characteristics of the population of UHE sources.

1 Introduction

One of the predictions of Big Bang Cosmology is that the Universe is filled with a background of neutrinos, analogous to the cosmic microwave background, but with a lower temperature $T_{\nu 0} \approx 1.95 \text{ K}$ ($1.69 \times 10^{-4} \text{ eV}$), and a number density $n_{\nu 0} \approx 56 \text{ cm}^{-3}$ per species [1]. The direct detection of this cosmological relic background is extremely difficult because of the very small interaction cross-section of low energy neutrinos. It is therefore interesting to explore the possibility of probing the cosmic neutrino background ($C\nu B$) with ultra high energy neutrinos (UHE$\nu$). At high energies, the Z resonance in the s channel for the process $\nu \bar{\nu} \rightarrow X$ enhances the probability for the interaction of an UHE neutrino with the $C\nu B$ [2, 3, 4, 5, 6]. This process results in absorption dips in the UHE neutrino spectrum that, if they can be observed at Earth with the appropriated resolution, would enable us to perform relic neutrino spectroscopy, thereby providing us with evidence for the existence of the $C\nu B$ and with an independent way of determining the absolute neutrino mass [7, 8].

The resonant production of Z through $\nu \bar{\nu}$ annihilation has also been proposed as a possible mechanism for generating UHE cosmic rays through the hadronic decay of the Z boson ("Z-burst" mechanism [9, 10, 11, 12]).

In the literature, the interaction of an UHE neutrino with the $C\nu B$ is calculated with different approximations. In this paper, we compute the dominant contribution of the interaction of an UHE$\nu$ with the $C\nu B$ using finite temperature field theory (FTFT). This formalism ensures that we take all the effects of the neutrino background into account. In section 2, we evaluate the damping of a UHE$\nu$ beam
travelling through the CνB, using the self-energy diagram corresponding to a Z exchange. From the damping we determine the absorption probability for an UHEν neutrino emitted at a given redshift.

We then present in section 3 some illustrations of our calculations in realistic physical contexts. First, the shape and position of the absorption lines in the spectrum of UHEν from interactions with the CνB depend on the mass of the neutrinos and on the type and distribution of sources for UHEν. We explore various combinations of parameters to investigate the differences between the finite temperature calculation and previous approximations and to determine the regimes in which those approximations break down and thermal effects become significant. Then we further illustrate our results in the context of relic neutrino clustering for different hypothesis on the density and scale of the clusters.

Conclusions are presented in section 4.

2 Damping rate and transmission probability of an UHEν

2.1 Self-energy in the relic neutrinos thermal background

The dispersion relation of a particle that propagates through a medium is determined from the linear part of the effective field equation. In momentum space, for a neutrino with four-momentum \( k^\mu \) and mass \( m_\nu \), it takes the form

\[
(\frac{k}{-m_\nu} - \Sigma_{\text{eff}}) \psi = 0,
\]

where \( \Sigma_{\text{eff}} \) corresponds to the retarded self-energy and embodies the background effects. For Dirac neutrinos, the chiral nature of neutrino interactions implies that, to one-loop order,

\[
\Sigma_{\text{eff}} = (a k + b \bar{\psi}) L,
\]

where \( L = \frac{1 - \gamma_5}{2} \) and \( u^\mu \) is the velocity four-vector of the medium; in its own rest frame \( u^\mu = (1, \vec{0}) \) and \( k^\mu = (\mathcal{E}, \vec{K}) \). The coefficients \( a \) and \( b \) are complex functions of the scalars

\[
\mathcal{E} = k.u, \quad K = \sqrt{\mathcal{E}^2 - k^2},
\]

with \( K = |\vec{K}| \). In the present context \( \Sigma_{\text{eff}} \) corresponds to the Feynman diagram of fig. 1, where the loop contains one relic (anti)neutrino from the thermal bath, with four-momentum \( p^\mu = (E_p, \vec{p}) \), and a Z boson with a blob indicating that we consider its decay width to all possible channels.

A consequence of the presence of a self-energy term in the equation of motion is to modify the dispersion relation of the incoming neutrino to

\[
\mathcal{E}_k = \mathcal{E}_r - \frac{i\gamma}{2},
\]

where \( \mathcal{E}_k, \mathcal{E}_r \), and \( \gamma \) are functions of \( K \). The real part, \( \mathcal{E}_r \), is in general not equal to \( \sqrt{K^2 + m_\nu^2} \) but has some additional correction reflecting the dispersive interactions that can take place in the medium, while the imaginary part corresponds to the damping factor, or else said, to the total reaction rate [13]. This means that the survival probability of an UHE neutrino travelling through the relic neutrino background can be simply expressed as

\[
P_T(\tau) = e^{-\gamma \tau}
\]

as a function of the propagation time \( \tau \). The damping factor is directly related to the imaginary part of the self-energy \( \Sigma_i \).
In the real-time formalism of the FTFT, both the propagators and the self-energies become $2 \times 2$ matrices. As shown in ref. [14], $\Sigma_i$ can be expressed in terms of the off-diagonal elements of the self-energy matrix as:

$$\Sigma_i = \frac{i}{2} (\Sigma_{12} - \Sigma_{21}).$$  \hspace{1cm} (6)

The vertices of the theory are also doubled compared to the vacuum case; the subscript 1 denotes the normal vertices of the Standard Model while the vertices labelled 2 get an extra factor -1 sign. In our case, $\Sigma_{12}$ and $\Sigma_{21}$ are calculated from the Feynman diagram of fig. 1, giving, for $c,d=1,2$ or $2,1$,

$$-i \Sigma_{cd} = \left( \frac{q}{2 \cos \theta_W} \right)^2 \int \frac{d^4 p}{(2\pi)^4} i D^\mu\nu_{dc}(q) \gamma_\mu L i S_{cd}(p) \gamma_\nu L,$$  \hspace{1cm} (7)
i.e., each term is evaluated with the corresponding propagators.

Since the temperature of the medium is small compared to the boson mass, we may discard the thermal contribution to the propagator. Consequently we have that

$$D^\mu\nu_{12}(q) = 2i \text{Im} [D^\mu\nu(q)] \theta(-q.u)$$  \hspace{1cm} (8)

$$D^\mu\nu_{21}(q) = 2i \text{Im} [D^\mu\nu(q)] \theta(q.u)$$  \hspace{1cm} (9)

where $\theta$ is the step function. For $D^\mu\nu(q)$ we adopt the usual prescription for the $Z$ propagator around the resonance [15] [16] [17]:

$$D^\mu\nu(q) = \frac{-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M^2}}{q^2 - M^2 + i q^2 \Gamma}.$$  \hspace{1cm} (10)

Here $M$ is the $Z$ mass and $\Gamma$ is the total width for $Z$ decaying to fermion pairs. It can be expressed analytically at lowest order (neglecting fermion masses) as [15]:

$$\Gamma = \sum_f \sqrt{2} G_F \frac{M^3}{6\pi} N_c(f) (I_3(f))^2 - 2 I_3(f) Q(f) \sin^2 \theta_W + 2 Q(f)^2 \sin^4 \theta_W,$$  \hspace{1cm} (11)

with $N_c(f) = 1(3)$ for leptons (quarks) and $I_3$ and $Q$ are respectively the fermion isospin and charge. The numerical values for $\Gamma$ and $M$ are taken from [18]. We also assume here that the $Z$ boson decays in vacuum.
The propagators for the relic neutrino entering eq. (10) directly reflect the presence of a thermal bath,

\[ S_{12}(p) = 2\pi i (p^2 - m_{\nu}^2) \left[ \eta_F - \theta(-p.u) \right] (\hat{p} + m_{\nu}); \]

\[ S_{23}(p) = 2\pi i (p^2 - m_{\nu}^2) \left[ \eta_F - \theta(p.u) \right] (\hat{p} + m_{\nu}), \]

where

\[ \eta_F = \frac{\theta(p.u) f_\nu(P)}{e^{\beta p - \alpha} + 1} + \frac{\theta(-p.u) f_{\bar{\nu}}(P)}{e^{\beta p + \alpha + 1}} \]

selects the distribution function of either the neutrino or the antineutrino. The distribution of background relic (anti)neutrinos assumes the well-known, relativistic Fermi-Dirac form\(^1\), with \( \beta = 1/T_\nu \) and \( \alpha = \mu/T_\nu \), \( \mu \) being the chemical potential for neutrinos and \( T_\nu \) the temperature of the relic neutrino bath.

After replacing the expressions above in eq. (6), expanding the delta function and neglecting contributions of order \( m_{\nu}^2/M^2 \) coming from the term proportional to \( q_\mu q_{\bar{\nu}} \) in (10), one gets that

\[ \Sigma_F(k) = R \gamma_\mu T^\mu(k) L, \]

with

\[ T^\mu(k) = \left( \frac{g}{2 \cos \theta_W} \right)^2 \int \frac{d^3p}{(2\pi)^3} \frac{\gamma_\nu}{2E_p} \left\{ D_{\mu\nu}(p+k) f_\nu(P) + \right. \]

\[ + \left. \Im [D_{\mu\nu}^\dagger(p-k)] \left[ \theta(\xi_k - E_p)(1 - f_\nu(P)) + \theta(E_p - \xi_k) f_\nu(P) \right] \right\}, \]

where we replace \( p_0 \) by \( E_p \) everywhere.

The first term corresponds to the resonant production of a Z boson through \( \nu - \bar{\nu} \) annihilation, the factor of \( f_\nu(P) \) reflects the Pauli blocking acting on the antineutrinos in the background. The second and third terms correspond to the emission of a Z-boson respectively by the incoming neutrino (\( \xi_k > E_p \)) or by the background antineutrino (\( E_p > \xi_k \)). Both processes are kinematically forbidden and we drop them from this point on.

### 2.2 Ultrarelativistic approximation

Let us assume that the neutrino travelling through the relic neutrino thermal bath is ultrarelativistic and that we can neglect the background effects on its energy, i.e., \( \xi_k \simeq K \). In that case, we can use the following expression for the damping:

\[ \gamma \simeq -2 \Im[b(K, K)] = -\frac{2}{K} (k_\mu T^\mu). \]

This expression was derived in ref. [13] for the case of a massless fermion; we verified that it remains valid for the case of massive neutrinos, provided the incoming neutrino is ultrarelativistic. The term proportional to \( q_\alpha q_\beta \) in the \( Z \) propagator gives contributions of order \( m_{\nu}^2/M^2 \) in the contraction \( k_\mu T^\mu \); if we discard them, then the damping rate corresponding to the \( \nu \bar{\nu} \) annihilation process reads

\[ \gamma(K) = \frac{g^2}{\cos^2 \theta_W} \Gamma_M \int \frac{d^3p}{(2\pi)^3} \frac{f_\nu(P)}{2K E_p} \frac{(k + p)^2(k.p)}{(1 + \xi)(k + p)^4 - 2M^2(k + p)^2 + M^4} \bigg|_{p_0 = E_p}, \]

\[ (19) \]

\(^1\)It is worth remarking here that, although relic neutrinos are not relativistic anymore at present time, their distribution maintains a relativistic form since their interactions froze out at their decoupling time, corresponding to \( T_{\nu d} \sim 1 \text{ MeV} \). [19]
energies for increasing relic neutrino momentum $P$ constant over the relevant range of momenta, i.e., the peak in the cross-section gets broader as $P$ increases, and shifts to smaller UHE $s_0$ corresponds to the bare resonant energy $K$ to the thermal distribution of background neutrinos and the position of the peak $P$ where $G_\nu$ will be selected by the C.P corresponds to the total energy in the center-of-mass, and $\sigma_{P}$ particular value of the relic neutrino momentum $C$ become more and more important as the ratio between the neutrino mass and the rewritten in terms the momentum integration of the cross-section for the process $\nu \bar{\nu} \rightarrow Z$, weighted by the corresponding statistical factor:

$$\gamma(K) = \int_0^\infty \frac{dP}{2\pi^2} P^2 f_\nu(P) \sigma_{\nu \bar{\nu}}(P,K),$$

with $P = (E_p^2 - m^2)^{-1/2}$. Neglecting the chemical potential of the relic (anti)neutrinos, we use the following expression for the antineutrino distribution function:

$$f_\bar{\nu}(P) = f_\nu(P) = \frac{1}{e^{P/T_{00}} + 1}.$$

The cross-section reads

$$\sigma_{\nu \bar{\nu}}(P,K) = \frac{G_F \Gamma M}{\sqrt{2} 2K^2 P E_P} \int_{s_-}^{s_+} ds \frac{s(s - 2m^2)}{(s - M^2)^2 + \xi s^2},$$

where $G_F$ is the Fermi constant. The integration variable is $s = (k + p)^2$, which corresponds to the total energy in the center-of-mass, and $s_\pm = 2m^2 + 2K(E_p \pm P)$.

The integral in eq. (22) can be done in a closed form. For $m_\nu \ll M, K$, we get

$$\sigma_{\nu \bar{\nu}}(P,K) = \frac{2\sqrt{2} G_F \Gamma M}{2KE_p} \left\{ \frac{1}{1 + \xi} + \frac{M^2}{4KP(1 + \xi)^2} \ln \left( \frac{1 + \xi}{1 + \xi} \right) \right\}.$$

In fig. 2, we plot the cross-section as a function of the UHE$\nu$ energy $K$, for neutrino masses $m_\nu$ ranging from $10^{-1}$ eV to $10^{-4}$ eV. Each curve corresponds to a particular value of the relic neutrino momentum $P$. As expected, the thermal effects become more and more important as the ratio between the neutrino mass and the C$\nu$B temperature (which we take here as $T_{00} = 1.69 \times 10^{-4}$ eV) decreases. Comparing with the value of the cross-section in the approximation of relic neutrinos at rest$^2$, $\sigma_{\nu \bar{\nu}}(0,K)$, we see that as long as the neutrino mass is sufficiently large, down to $\approx 0.01$ eV, the cross-section does not vary much on the range of $P$ corresponding to the thermal distribution of background neutrinos and the position of the peak corresponds to the bare resonant energy $K_{res} = M^2/(2m_\nu)$. For smaller masses, the peak in the cross-section gets broader as $P$ increases, and shifts to smaller UHE$\nu$ energies for increasing relic neutrino momentum $P$. The peak value is approximately constant over the relevant range of momenta, i.e., the range of momenta which will be selected by the C$\nu$B distribution function (the maximum of $P^2 f_\nu(P)$ is in $P_{max} \approx 3.75 \times 10^{-4}$ eV), and is about one order of magnitude lower than $\sigma_{\nu \bar{\nu}}(0,K_{res})$.

The relevant quantity in the problem is the total damping, obtained from eq. (24) after the integration on the relic neutrino momentum. We will now show that the net

$^2$Here $\sigma_{\nu \bar{\nu}}(0,K)$ is obtained by taking the limit of $\sigma_{\nu \bar{\nu}}(P,K)$ for $P \rightarrow 0$.
effect of thermal broadening is a loss of efficiency and of definition of the damping, which affects the transmission probability and the depth and shape of the absorption dips.

Equations (20) and (24) give the damping of an UHE$\nu$ propagating through a thermal bath of relic neutrinos, caused by the $\nu - \bar{\nu}$ resonant annihilation into a Z boson. It is plotted in fig. 3 (in black, continued curve) as a function of the energy $K$ of the incoming neutrino, for the same range of neutrino masses as the cross-sections.

A simpler expression for $\sigma_{\nu\bar{\nu}}$ can be obtained by evaluating the integral in eq. (22) at the midpoint of the integration interval, $\bar{s} \approx 2KE_p$, using the mean value theorem. Taking into account that $s_+ - s_- = 4KP$ we get the following expression:

$$\bar{\sigma}_{\nu\bar{\nu}}(K,P) = \bar{\sigma}_{\nu\bar{\nu}}(\bar{s}) = 2\sqrt{2}G_F \Gamma M \frac{\bar{s}}{(\bar{s}-M^2)^2 + \xi \bar{s}^2},$$

(25)

where all the dependence on $P$ is implicit in $\bar{s} = 2K\sqrt{m^2 + P^2}$.

A common approximation consists in neglecting the momentum of the background neutrinos [5, 7]. The corresponding expressions can be recovered in our formalism by evaluating the cross-section (25) in $P = 0$, i.e., in $\bar{s} = s_0 = 2mK$, or equivalently by taking the limit for $P \rightarrow 0$ of the general expression (24). The damping reads in this case:

$$\gamma_{\nu\bar{\nu}}^0(K) = \bar{\sigma}_{\nu\bar{\nu}}(s_0) n_\nu = 2\sqrt{2}G_F \Gamma M \frac{2Km}{4(1 + \xi)K^2m^2 - 4M^2Km + M^4} n_\nu,$$

(26)

where we have identified the remaining integral over $P$ with the background neutrino
density (assuming that \( n_\nu = n_\bar{\nu} \)). Using the full expression for the total Z width to fermions, eq. (11), one can easily recover the result from [5], while the expressions of [7] can be recovered by further evaluating the cross-section exactly at the pole of the resonance, i.e. in \( s_0 = 2Km = M^2 \) (narrow-width approximation).

We show the corresponding results in fig. 3 where we directly compare the approximated damping factor, eq. (27), with the exact expression obtained from eqs. (20) and (24), which we have integrated numerically. The comparison is done for different values of the neutrino mass.

Two effects combine: the modification of the cross-section peak due to its dependence on \( E_p \), and the presence of the thermal distribution which selects relic neutrino momenta comparable to the temperature of the CνB. Both effects contribute to smearing out the resonance peak, and in shifting its maximum value to lower energies. As long as the neutrino mass is \( \gtrsim 0.01 \) eV, the position of the peak is not significantly affected and the net effect is to lower the damping efficiency by some factor. For smaller masses, the effect of thermal broadening is much stronger and the damping gets spread over a larger range of UHE neutrino energies, resulting in a worse definition of the absorption dip.

One could also ask whether the approximation of eq. (25), easier to handle than the full cross-section formula of eq. (24), is not sufficient to keep track of the thermal effects on the damping factor. It appears however from the fig. 3 where it is represented by the dashed (blue) curves, to be satisfactory only for very small neutrino masses, outside the range which is favoured by oscillation experiments (at least for one of the flavours). In the subsequent analysis we therefore keep on working with the full expression for the cross-section \( \sigma_{\nu\bar{\nu}}(K, P) \), eq. (24).

![Figure 3: Damping factor due to \( \nu - \bar{\nu} \) annihilation, in eV, as a function of UHE neutrino energy \( K \); each plot correspond to a given value of the neutrino mass (10^{-1} \text{ eV}, 10^{-2} \text{ eV}, 10^{-3} \text{ eV}, 10^{-4} \text{ eV} \) from left to right and from top to bottom) and displays the integrated damping, \( \gamma_{\nu\bar{\nu}} \), from the full cross-section eq. (24) (black, continued curve) and from the approximated cross-section eq. (25) (blue, dashed curve), as well as the approximation for neutrinos at rest, \( \gamma_{\nu\bar{\nu}}^0 \), from eq. (27) (red, dotted curve). The three curves are superposed for \( m_\nu = 0.1 \) eV, and so are the two approximations for \( m_\nu = 0.01 \) eV.


2.3 Transmission Probability across the relic neutrino background

High energy neutrinos can travel cosmological distances almost without interacting. To calculate the damping by the CνB one has to take the expansion of the Universe into account. The standard treatment of the effect is to derive the transmission probability by integrating the damping factor over the UHE neutrino path, in terms of the redshift $z$, back to the source position $z_s$. The formula (5) is then modified to

$$P_T(K_0, z_s) = \exp \left[ - \int_0^{z_s} \frac{dz}{H(z)(1 + z)} \gamma_{\nu\bar{\nu}}(K_0(1 + z)) |_{T_\nu \to T_0\nu(1+z)} \right],$$

(28)

where the subscript 0 denotes quantities in today’s Universe. We take for the Hubble factor $H(z) = H_0 \sqrt{0.3(1 + z)^3 + 0.7}$ as suggested by recent observations [20], with the numerical value of $H_0$ from [18].

Eq. (28) encompasses two effects due to the expansion of the Universe. First, the UHE neutrino energy gets red-shifted, $K \to K_0(1 + z)$ (the subscript 0 denotes present time quantities). This has a broadening effect on the absorption dip even in the approximation of relic neutrinos at rest, since the resonance energy changes along the UHE\(\nu\) neutrino path. Second, the temperature of the CνB gets red-shifted, $T_\nu \to T_0\nu(1+z)$: the thermal bath of relic neutrinos is hotter at earlier times. This directly affects the ratio between $m_\nu$ and $T_\nu$ and results in a modification of the absorption properties of the CνB respect to the UHE neutrinos that cross it.

Figure 4: Transmission probability $P_T(K_0, z_s)$ as a function of the incident neutrino energy as detected on Earth, $K_0$, for an UHE neutrino source located at redshifts $z_{\text{source}} = 1, 5, 10, 20$ (from top to bottom in each plot) and for a neutrino mass $m_\nu = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ eV. The continued, black curves corresponds to the full $\gamma_{\nu\bar{\nu}}$ from eqs. (20) and (24), while the dotted (red) curves are for the approximation of relic neutrinos at rest, $\gamma_{\nu\bar{\nu}}^0$, from eq. (27).

In fig. 4, we show the transmission probability of an UHE neutrino emitted at a fixed redshift 1, 5, 10 or 20, as a function of its present energy, $K_0$, and compare it with the results obtained in the approximation of relic neutrinos at rest.
As long as \( m_\nu / T_\nu \gtrsim 10^3 \), the shape of the absorption dip is not affected by thermal broadening and is rather sharply delimited, at high energies, by the bare resonant energy for the propagating neutrino, \( K_{\text{res}}^0 = M^2 / (2m) \), and at low energies by the red-shifted resonant energy \( K_{\text{res}}^0 / (1 + z) \). Evaluating the position of these points would in principle allow us to determine the value of the neutrino mass as well as the redshift at which the UHE\( \nu \) neutrino was emitted. As \( m_\nu / T_\nu \) decreases, the absorption dips gets shallower and broader, and the position of the minimum transmission probability is shifted to lower energies. The effect of thermal motion also increases with the redshift \( z \) since UHE\( \nu \) neutrino from more distant sources are emitted in hotter backgrounds.

3 Applications

The results presented so far deal with a monoenergetic source of UHE\( \nu \) neutrinos located at a given redshift; let us now illustrate the potential effects of thermal motion on the process of absorption of UHE neutrinos in two more realistic contexts of physical relevance.

3.1 Absorption lines in a realistic UHE neutrino flux

The flux of UHE neutrinos at Earth depends on one hand on the mechanisms at work in the sources, which determine the injection spectrum of the UHE neutrinos, and on the other hand on the spatial and temporal distribution of the sources themselves. We follow here the lines of [7] and express the UHE\( \nu \) flux in function of the present neutrino energy \( K_0 \) as:

\[
F_\nu(K_0) = \frac{1}{4\pi} \int_0^\infty \frac{dz}{(1+z)H(z)} P_T(K_0,z) L_\nu(K_0,z),
\]

(29)

where \( L_\nu(E_0,z) \) is the neutrino source emittivity distribution, expressed in terms of the redshift \( z \) and the present energy of the UHE neutrino. In the hypothesis of identical injection spectra for all sources, one can factorize the dependence in the red-shift and write

\[
L_\nu(K_0,z) = \eta(z) J_\nu(K_0),
\]

(30)

where \( \eta(z) \) describes the distribution of the sources in the Universe, and \( J_\nu(E_0) \) gives the number of neutrinos emitted per unit of energy by each of these sources. We use the following, standard ansatz [21, 7]:

\[
\eta(z) = \eta_0 (1+z)^n \theta(z - z_{\text{min}}) \theta(z_{\text{max}} - z);
\]

(31)

\[
J_\nu(K) = j_\nu K^{-\alpha} \theta(K_{\text{max}} - K).
\]

(32)

Eq. 31 is suitable for an approximate description of UHE neutrino sources distribution in models ranging from astrophysical acceleration sites (“bottom-up” mechanisms, for which we can take \( n \approx 4 \) and \( z_{\text{max}} \leq 10 \)) to exotic, non-accelerator sources (which have \( n \approx 1 / 2 \) and may extend to a much larger \( z_{\text{max}} \)). In both cases, we take the lower bound for the source distribution to be \( z_{\text{min}} = 0 \). The spectral index \( \alpha \) typically ranges between 1 and 2, depending on the production mechanism considered; we assume here that it is constant over the range of energies examined and do not consider the possibility of broken power-law spectra above the GZK energy. We also suppose that \( K_{\text{max}} > K_{\text{res}}^0 (1+z) \) in all our analysis.
Figure 5: UHE neutrinos fluxes in presence of damping, $F_\nu$, normalized to the corresponding flux in absence of interactions, $F_{\nu 0}$, for a neutrino mass $m_\nu = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ eV (from top to bottom). The black (continued) curves are for the exact expression using eq. (24) while the dotted, red curves are for the approximation of neutrinos at rest, eq. (27). The left column is for $n - \alpha = 2$ and $z_{\text{max}} = 2, 5, 10$ (from top to bottom in each plot), while the right column is for $n - \alpha = 0$ and $z_{\text{max}} = 10, 20$. 
Under these assumptions, as pointed out in [7], the only dependence on the spectral indexes \( \alpha \) and \( n \) enters through a difference \( n - \alpha \). We consider here, for the purpose of illustrating our results, two distinct situations: \( n - \alpha = 2 \) which could describe the UHE neutrino flux expected from an astrophysical, bottom-up-type source, and \( n - \alpha = 0 \) which would rather be associated to UHE neutrino fluxes produced in top-down processes. Results are presented in fig. 5 which displays the UHE\( \nu \) flux, eq. (29), as a function of the present energy \( K_0 \) of the UHE\( \nu \), after normalization to the flux in the absence of absorption effects, \( F_{\nu 0} \) (obtained by replacing \( P_T(K_0, z) = 1 \) in eq. (29)). The first column corresponds to \( n - \alpha = 2 \) and for each value of \( m_\nu \) we show several curves corresponding to different redshift limits for the source population, \( z_{\text{max}} = 2, 5, 10 \), while the second column corresponds to \( n - \alpha = 0 \) and redshift limits \( z_{\text{max}} = 10, 20 \).

As long as \( m_\nu / T_{\nu 0} \gtrsim 10^2 \) one can see that thermal effects do not significantly affect the shape of the absorption dip in the UHE neutrino flux. In particular, the endpoint of the dip at high energies, corresponding to \( K^\text{res}_0 \), is well-defined and can be used to estimate the absolute value of \( m_\nu \); provided the absorption dip is not too shallow nor too narrow to be resolved experimentally. The global shape of the dip clearly depends on the value of \( n - \alpha \). Assuming one can determine the spectral index \( \alpha \) from the measurements of the UHE\( \nu \) spectrum in the range of energies which is not affected by the absorption, one could obtain information on \( n \) and therefore on the development of the source population. From the endpoint of the absorption dip at low energies, which corresponds to the resonance energy of the neutrinos emitted at the largest redshift, \( K_0(1 + z_{\text{max}}) \), we can also estimate the epoch at which the UHE\( \nu \) sources appeared.

For smaller values of the ratio \( m_\nu / T_{\nu 0} \), the situation is significantly complicated by the effect of thermal motion. As a result of the broadening of the transmission probability, the dips get shallower and can extend over several orders of magnitude in energy, depending on the maximum redshift chosen for the source distribution. This might complicate their observation, especially at small redshifts. From the figure one sees indeed that for \( z_{\text{max}} \leq 2 \), the flux won’t be depleted more than a 5 %. Results will also be more difficult to interpret in terms of a prediction for the neutrino mass and the maximum redshift for the population of sources, since the position of the endpoints of the absorption dip is not so clearly defined anymore.

On the other hand, maximum absorption is now achieved at lower energies \( K_0 \). The shift is significant, even more than an order of magnitude for very small masses, \( m_\nu \approx 10^{-4} \) eV. In view of the inverse-power-law form of the UHE\( \nu \) energy spectrum, this could significantly help improving the detection potential, although we are speaking here of energies of the order of \( 10^{23} - 10^{24} \) eV, which are beyond the reach of the majority of UHE\( \nu \) experiments planned so far [7].

### 3.2 Absorption lines due to relic neutrino clustering

The possibility that massive relic neutrinos cluster onto dark matter halos has been intensively studied, in particular in relation with the generation of the UHE cosmic rays through the Z-burst mechanism [9, 10, 11]. Recent works [22, 23] have presented revised estimations of the density profiles and typical spatial extension of the neutrino clusters, giving overdensities of the order of \( 10 - 10^4 \ n_{\nu 0} \) that can extend on scales \( L \sim 10^{-2} - 1 \) Mpc, depending on the neutrino mass, on the mass of the attracting halo and on its velocity dispersion (which is typically of the order of 200 km/s for a galaxy and 1000 km/s for a galaxy cluster). This last parameter also constrains the epoch at which clustering can start, as neutrinos cannot be efficiently trapped as long as their mean velocity,

\[
\langle v_\nu \rangle \simeq 1.6 \ 10^2 \ (1 + z) \left( \frac{eV}{m} \right) \ km \ s^{-1}, \tag{33}
\]
is larger than the velocity dispersion of the attracting galaxy or galaxy cluster \[23\]. For light neutrinos \((m_\nu \lesssim 1 \text{ eV})\), clustering will thus take place at very small redshifts and we can safely ignore the effect of the expansion of the Universe in this analysis. Other important limiting factors to the clustering of neutrinos on large scales are Pauli blocking and the limit on the maximum phase-space density, as described in \[24\]. They actually imply that only neutrinos with mass \(m \gtrsim 1 \text{ eV}\) will efficiently cluster on galactic halos, on typical scales \(L_G \sim 50 \text{ kpc}\), while neutrinos with mass \(m \gtrsim 0.1 \text{ eV}\) can cluster on scales as big as \(L_C \sim 1 \text{ Mpc}\) in halos associated to (super-)clusters of galaxies \[9, 22, 23\].

The analysis here above mentioned, performed within the approximation of neutrinos at rest, globally agree that the dominant contribution to the absorption of the UHE\(\nu\) flux comes from the annihilation on the uniform relic neutrino background, whose density increases with redshift. In comparison, the effect of neutrino clustering is limited to small scales and the overdensities are not large enough to have a significant incidence on the damping of UHE\(\nu\) travelling on cosmological distances, except maybe in the case of galaxy superclusters like Virgo \[25\].

In the present context, the clustering process can be rephrased in terms of an increase of the temperature of the relic neutrino gas. Supposing that the thermal profile of the neutrino distribution, eq. \[21\], is maintained through the clustering process, which seems reasonable in view of the results of \[23\], we can use the simple relation

\[
n_\nu(T_\nu) = \frac{3\zeta(3)}{4\pi^2} T_\nu^3
\]

between density and temperature of the relic neutrinos, so that the ratio between the neutrino mass and the temperature of the (clustered) background \(R_\nu = m_\nu/T_\nu\) can be directly related to the cluster density.

We assign the neutrino cluster a constant density \(n_\nu^t = \mu n_\nu^0\), where the overdensity factor \(\mu\) ranges between 10 and \(10^4\) according to the above mentioned results, and a spatial extension \(L_\nu^t\). Here we limit ourselves to considering the effects of one cluster on an incident flux of UHE\(\nu\), without trying to build a realistic situation with a distribution of clusters of different scales and densities, as we are rather interested in investigating the potential effect of thermal motion on the absorption lines. In view of the above arguments, we only consider neutrinos with large masses, \(m_\nu = 1, 10^{-1} \text{ eV}\).

With overdensity factors \(\mu \approx 10 \div 10^4\), the minimum ratio achievable is \(R_\nu \gtrsim 240\) for \(m_\nu = 1 \text{ eV}\) and \(R_\nu \gtrsim 24\) for \(m_\nu = 0.1 \text{ eV}\). As expected, the effect of the thermal motion of the relic neutrinos is in general negligible or small due to the relatively small overdensities achievable. We have to saturate the bounds on the parameters to get a significant effect, as shown in fig. 6a, which displays the transmission probability with and without thermal effects, for a cluster of relic neutrinos with mass \(m_\nu = 0.1 \text{ eV}\) and extension 1 Mpc. For a maximal overdensity \(\mu = 10^4\) the thermal motion reduces the maximum absorption probability across the cluster from \(\approx 55\%\) to \(\approx 25\%\), contributing to reducing its effect respect to the non-clustered CMB absorption probability shown in fig. 4.

A quite different situation could arise in the context of exotic models allowing a stronger clustering of the neutrinos, for example in hot clouds at very small scales (\(\sim 10^{-5} \div 10^{-1} \text{ pc}\), i.e. as small as the Solar system) through non-standard interactions \[26\]. In this case the densities achievable are huge, of the order of \(10^9 \div 10^{15} n_\nu^0\), and although the spatial extension of the cluster is very small compared to the astrophysical scales considered in the previous case, the effects can be large as the temperature inside the cloud is \(10^3\) to \(10^5\) times greater than the temperature of non-clustered relic neutrinos. This is illustrated in the fig. 6b for a neutrino cloud with spatial extension 10 pc, several possible densities, and a neutrino mass as big as 1 eV. Here the approximation of eq. \[27\] should not be
used at all, as the temperature of the thermal bath is comparable to the neutrino mass, and the ratio $R_\nu$ achieves values of the order of 1 to 5. In this context, and for the upper range of allowed densities, the thermal effects could even make the observation of the absorption dip easier, as it is much broader than in the approximation of relic neutrinos at rest. In particular, the transmission probability is dropping to nearly 0 on a significant range of values of $K$.

4 Conclusions

Within the formalism of finite-temperature field theory we have calculated the damping factor of an UHE$\nu$ propagating through the C$\nu$B including the effects of the thermal motion of the relic neutrinos in a systematic way. This allowed us to generalise the expressions for the transmission probability $P_T$ that are commonly used in the literature.

From the exploration of the parameter space allowed by cosmological and astrophysical constraints as well as by current limits on the neutrino mass, we see that thermal effects significantly affect the shape and position of the absorption dips in a realistic UHE$\nu$ flux as soon as the ratio between the neutrino mass and the C$\nu$B temperature goes below $\approx 10^2$, i.e., well before the relic neutrinos become relativistic. As expected, the effect essentially consists in smearing out the dip and shifting its minimum value to lower energies. This will complicate the observation of the dip in a real experiment measuring neutrino fluxes, especially if the UHE$\nu$ source population is concentrated at small redshifts, producing rather shallow and extended dips. The shift of the absorption dip to lower energies, where neutrino fluxes are expected to be higher, increases in principle the potential of detection of the effect respect to the case of a neutrino at rest with the same mass. Still, the situation could become more intricate if the pattern of neutrino mass eigenstates is such that their combined effect results in a superposition of dips with different depths and extensions.

In the context of neutrino mass spectroscopy, we see from the examples that thermal effects do not affect the determination of the endpoint of the absorption dip, and hence the possibility of extracting information on the absolute neutrino mass, as long as $m_\nu \gtrsim 0.01$ eV, which is verified by at least one neutrino in the currently favoured mass schemes. As the absorption dips get broader and shallower, the prospects for determining efficiently the resonance energy get worse
as the endpoint of the absorption dip is no longer sharply defined.

Finally, as another application of our formalism, we have also investigated the transmission probability for UHE neutrinos propagating through a relic neutrino cluster. In the standard context of neutrino clustering around galaxies or galaxy clusters, we found that the thermal motion can further reduce the absorption effect of the cluster (which is in any case limited due to the fact that clusters form at very small redshift while the absorption effects for an UHE neutrino travelling on cosmological distances are dominated by the interactions occurring at the beginning of its life). The situation can be drastically different if some exotic model provides us with extremely dense neutrino clusters, like the neutrino clouds of \cite{25}. In that case thermal effects can not be neglected at all as the relic neutrino could even be relativistic. The resulting absorption dips are broad and, for certain values of the cloud density, can drop to zero on a significant range of energies of the incident UHE neutrino, making it easier to detect.

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